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DR AND CG SEMINAR (5TH) BEST ESTIMATE OF TRAJECTORY TECHNIQUES. (U)
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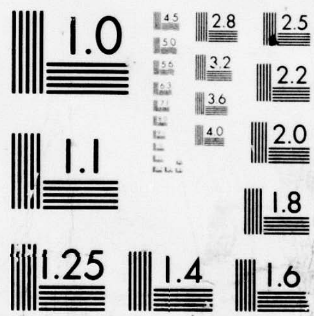

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

DR&CG

LEVEL II

5TH SEMINAR (12)

BEST ESTIMATE OF TRAJECTORY TECHNIQUES

AD A077170

2 OCTOBER 1978

SPACE & MISSILE TEST CENTER-EASTERN TEST RANGE
PATRICK AFB, FLORIDA

DATA REDUCTION AND COMPUTING GROUP
RANGE COMMANDERS COUNCIL

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REPORT DOCUMENTATION PAGE

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Best Estimate of Trajectory (BET), black boxes, instrumentation/computer systems, guidance and weapon delivery systems, sophisticated weaponry, accuracy problems, rigid regression, underspecified models, N Interval Trajectory Estimation (NITE), GPSBET, BET development.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) PREFACE

The Best Estimate of Trajectory (BET) Seminar was conceived when the Range Commanders Council Executive Committee and the Data Reduction and Computing Group (DR&CG) mutually agreed that since the DR&CG had not formally addressed BET in several years, a reexamination of this technique would be worthwhile. The DR&CG membership felt that a seminar where theoreticians knowledgeable in this area could compare and contrast BET techniques in use at the various ranges would provide the best vehicle for bringing new material on this subject

over/

to the surface.

The resulting seminar, which was held on 2 October 1978, was a valuable exercise. All speakers agreed that although theoretical BET problems were solved years ago, the use of any BET program still requires careful analysis of the model under consideration. Thus, particular attention must be paid to determining if enough information is provided to solve the problem at hand and whether the geometry used, combined with the instrumentation available, will lead to an ill-conditioned problem. It was noted that BET programs cannot presently be treated as "black boxes." However, as guidance and weapon delivery systems become more accurate, the search for instrumentation/computer systems with comparable accuracies that can be used as a standard against which to test this sophisticated weaponry becomes increasingly more challenging. Since BET techniques comprise in most cases the only method which can be applied to solving such accuracy problems, their importance is expected to increase appreciably in the coming years.

(12)

(6) 5TH DR&CG SEMINAR (5th)

BEST ESTIMATE OF TRAJECTORY TECHNIQUES

DATA REDUCTION AND COMPUTING GROUP
RANGE COMMANDERS COUNCIL

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A paper entitled "KMR BET Analysis," by Tom Keeney, was not received in time for publication.

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AGENDA

DATA REDUCTION AND COMPUTING GROUP
OF THE
RANGE COMMANDERS COUNCIL

BEST ESTIMATE OF TRAJECTORY (BET) SEMINAR

2 OCTOBER 1978

MONDAY - 2 OCTOBER 1978

0800	OPENING REMARKS - DANNY WEDDLE, NATC, CHAIRMAN
0805	NITE-N INTERVAL TRAJECTORY ESTIMATION - C. W. WELSH, ETR/RCA
0900	KMR BET ANALYSIS - TOM KEENEY, KMR
1000	GPS BET - ROBERT W. MAI, YPG
	LUNCH
1230	TAPP - TRAJECTORY PREDICTION PROGRAM - G. TRIMBLE, SAMTEC/FEC
1330	BET DEVELOPMENT AT WSMR - BILL AGEE, WSMR
1430	RIDGE REGRESSION WITH UNDERSPECIFIED MODELS - BOB ROWE, ETR/RCA
1530	CLOSING REMARKS - DANNY WEDDLE, NATC, CHAIRMAN

PREFACE

The Best Estimate of Trajectory (BET) Seminar was conceived when the Range Commanders Council Executive Committee and the Data Reduction and Computing Group (DR&CG) mutually agreed that since the DR&CG had not formally addressed BET in several years, a reexmaination of this technique would be worthwhile. The DR&CG membership felt that a seminar where theoreticians knowledgeable in this area could compare and contrast BET techniques in use at the various ranges would provide the best vehicle for bringing new material on this subject to the surface.

The resulting seminar, which was held on 2 October 1978, was a valuable exercise. All speakers agreed that although theoretical BET problems were solved years ago, the use of any BET program still requires careful analysis of the model under consideration. Thus, particular attention must be paid to determining if enough information is provided to solve the problem at hand and whether the geometry used, combined with the instrumentation available, will lead to an ill-conditioned problem. It was noted that BET programs cannot presently be treated as "black boxes." However, as guidance and weapon delivery systems become more accurate, the search for instrumentation/computer systems with comparable accuracies that can be used as a standard against which to test this sophisticated weaponry becomes increasingly more challenging. Since BET techniques comprise in most cases the only method which can be applied to solving such accuracy problems, their importance is expected to increase appreciably in the coming years.

NITE - N INTERVAL TRAJECTORY ESTIMATION

by

C. W. WELSH
RCA, Patrick AFB, Florida

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NITE - N INTERVAL TRAJECTORY ESTIMATION

C. W. Welsh
RCA, Patrick AFB, Florida

ABSTRACT

NITE is the "Best Estimate of Trajectory" computer program developed at SAMTEC Det #1 (ETR), Patrick AFB, Florida. NITE is a minimum variance (weighted least squares) trajectory and error model coefficient estimation program designed to process tracker and guidance data from single or multiple powered flight and/or free fall trajectories from launch to impact in a single computer run. Tracking instrumentation may be either earth-fixed or referenced to moving ships, aircraft and satellites whose trajectories may also be simultaneously estimated (differentially corrected). The program will accept either preprocessed or raw data and will perform a variance Geodetic Dilution of Precision (GDOP) analysis either as a primary task or as a byproduct of the trajectory and error model coefficient estimation. The variance propagation considers both estimated (modeled) and enforced (unmodeled) systematic error terms.

NITE has been used to provide trajectory, instrumentation error model coefficient, station location, ship location, aircraft location, drag and lift coefficients for a broad class of launch vehicles, reentry vehicles and satellites such as Minuteman III, Titan III, Polaris, Poseidon, Trident, Thor-Delta, Atlas-Centaur, Saturn, Pershing, SRAM, Geos A, Geos B, Pageos, Transit and Echo. It is presently being used to process data at SAMTEC-Western Test Range (WTR).

NITE was originally written for the IBM 7094 computer in Fortran IV with a special IBSYS monitor. It has also been modified for operation on the IBM 360/65 using a modified OS monitor, and for operation on the CDC Cyber 74 and CDC 6500 using the NOS monitor.

I. INTRODUCTION

NITE is an acronym used for the N-Interval Trajectory Estimation computer program to produce the "Best Estimate of Trajectory" (BET) at the Eastern Test Range. The N-Interval concept evolved from the desire to combine several missile and/or satellite trajectories into one solution to enhance the estimation capability of surveys and/or tracker error model coefficients. However, most of the BET's produced at ETR are of the single interval type. Sufficient flexibility has been designed into the program to allow the use of earth-fixed and/or moving trackers, inertial and/or earth fixed data, with a wide variety of types of measurements. The locations of all (moving or fixed) trackers may be simultaneously estimated (differentially corrected). The input measurements may be either preprocessed or raw (decoded) data. In the latter case, NITE will correct data for refraction, transit time, scaling and a comprehensive error model. NITE will also perform a Variance or Geodetic Dilution of Precision (GDOP) analysis as a primary product or as a byproduct of the trajectory and error model coefficient estimation. The variance propagation considers both the estimated (modeled) and enforced (unmodeled) systematic error terms.

NITE's prime task has been to provide trajectory, instrumentation error model coefficient, station location, ship location, aircraft location, lift and drag coefficient estimates for all major ETR launches. NITE is also used at WTR.

The mathematical basis for NITE is minimum variance (weighted least squares) which can be found in most statistics textbooks. References (2) through (8) contain related discussions of trajectory and error model coefficient estimation theory. The program document (1) gives a detailed description of the mechanics of the NITE estimation and error analysis techniques. Only enough mathematics necessary for describing instrumentation error models and for interpreting results will be presented in this paper.

The design of NITE is the result of many interacting forces. Primary among these forces has been the experience gained by ETR personnel in processing and analyzing data from thousands of missile launches and satellite passes over the past 18 years and in the development of earlier BET programs such as BETA, BETY, GLAD, TRAD and TEEM. Another significant influence in the program's design has been the many requirements and suggestions from RCA Technical Analysis, PAA Tech Staff, TRW Systems, The Aerospace Corporation, NASA and from RCA and PAA consultants. Many similar computer programs developed by other agencies have been studied for worthwhile ideas applicable to ETR requirements. The programs studied include those developed by the PAA Tech Staff, The Aerospace Corporation, JPL, Lockheed, TRW Systems, The Mitre Corporation, and The Wolf Research and Development Corporation bearing the titles ERAN, ORAN, TRACE, REMP, TRAP, EMBET, DPODP and MARLOCK.

NITE was originally written for the IBM 7094 computer in Fortran IV using a special IBSYS monitor. It was later modified for operation on the IBM 360/65 using a special OS monitor. It has been recently modified to run on the CDC 6500 and CDC CYBER-74 using the CDC NOS monitor.

II. THE PURPOSE OF THE NITE PROGRAM

The original purpose of the ETR BET programs was to combine all of the tracker data from a missile launch to provide the range user with better trajectory estimates than those from a single tracker. Although this is still the primary goal, the BET concept has been extended to cover many other tasks such as the following:

A. To estimate (differentially correct) multiple powered flight, orbital, sub-orbital and reentry trajectories in a single run while simultaneously estimating

   1. coefficients of large tracker and inertial guidance instrumentation error models;

   2. geodetic locations of earth fixed trackers;

   3. trajectories of shipboard, airborne, and satellite-borne trackers;

   4. drag and/or lift error model coefficients;

   5. geopotential coefficients;

   6. impact points;

   7. observation (measurement) weights;

   8. underwater transponder locations;

   9. relative positions and velocities of multiple reentry bodies;

   10. other quantities whose partial derivatives can be computed externally such as

       a. missile pitch, roll, yaw and thrust decay coefficients;

       b. ship or aircraft pitch, roll, flexure and inertial navigation coefficients;

       c. atmospheric pressure, electron density, temperature, wind velocity;

d. point mass geopotential terms;

e. second order instrumentation error model terms.

This capability often eliminates the need for developing a special analysis-type program when a new tracker system or a different physical phenomenon requires investigation.

B. To perform a variance (GDOP) analysis for the quantities being estimated or unmodeled, either as a byproduct of the estimation process or as a product of a special GDOP run, using theoretical trajectories and the theoretical trackers. The latter type runs are of value in range planning.

C. To provide intersystem data comparisons.

D. To perform orbital prediction accuracy analysis.

E. To show the effect of ignoring selected error model terms during the estimation process (using unmodeled error analysis).

F. To apply various constraints on or between quantities being estimated.

G. To correct measurements for known errors such as refraction, transit time and erroneous calibrations.

III. NITE PROGRAM CAPABILITIES

A. Measurement Types

1. Azimuth position, velocity and acceleration.
2. Elevation position, velocity and acceleration.
3. X-Y mount position, velocity and acceleration.
4. Range position, velocity and acceleration.
5. Range sum position, velocity and acceleration.
6. Range difference position, velocity and acceleration.
7. Inertial guidance position and velocity.
8. Ballistic camera plate coordinates.
9. Topocentric and geocentric right ascension and declination.
10. Earth fixed cartesian coordinate position and velocity components.
11. Latitude, longitude and height.

B. Modes of Operation

1. EDIT Mode

   Input observations may be compared with a transformed reference trajectory and may be corrected for most types of systematic errors. The corrected output of the EDIT mode run may be used directly as input to an estimation mode run.

2. ESTIMATION Mode

   A minimum variance (weighted least squares) estimate of the trajectory and error model coefficients is computed from the input observations. GDOPs, residuals and partial derivatives are byproducts of the estimation mode.

Coefficients of instrumentation, drag, lift, geopotential, or other error models may be estimated or may be unmodeled. Unmodeled is a term used for the process that applies known coefficients (not estimated) and propagates a given uncertainty.

3. SIMULATION Mode

Random and systematic errors may be applied to measurements computed from a reference trajectory. The output of the simulation mode may be used as input to estimation or edit mode runs. Thus the ability of the program to estimate known systematic errors under various conditions may be estimated.

4. ORBITAL PREDICTION Mode

Free-fall initial conditions are determined from data during early satellite passes. The trajectory is then generated and compared with data acquired on the same satellite many passes later.

5. GDOP Mode

Only the error propagation features of the program are utilized. Estimates of variance of the input observations (including a priori estimates of variance in survey and other error model terms as well as in instrumentation observations) are transformed into estimates of variance in the parameters being estimated. The magnitude of the GDOPs is solely a function of the assumed trajectory, the assumed instrumentation combination, assumed random observation errors and the a priori knowledge of the systematic error components. The GDOP mode may thus be used to evaluate hypothetical instrumentation systems operating under hypothetical conditions.

C. Data Handling

1. Data may be on either tape or cards or both.
2. Data may be on one or several files. Tapes will be automatically rewound if necessary.

k. Dynamic lag

l. Radar mislevel, collimation, nonorthogonality, droop

m. Tracker survey error terms of several types

(1) Movement of latitude, longitude and weight

(2) Movement of X, Y, Z rectangular cartesian coordinates

(3) Translation only with fixed rotation (for astronomically oriented systems)

(4) Both translation and rotation

(5) Movement of a central site, with related stations moving as an array

(6) Base line azimuth, elevation and range from the central site

(7) Combinations of items 1-6, allowing both "external" and "internal" survey adjustments

n. Tracker velocity if on moving aircraft, ship or satellite

(1) Latitude, longitude and altitude rates

(2) Heading and speed

o. Random errors for simulation

3. Products of terms may be formed. This allows the following additional partials:

a. The product of a timing error partial and a reinitialization partial, allowing correction for a timing error that occurs within a data span.

b. The products of survey partials and time polynomial partials allow ship or aircraft movement to be expressed as polynomials in time.

c. The product of trigonometric functions allowing modeling of second order radar errors.

d. The products of values from the input tape and built-in partials can be used to expand instrumentation error models.

4. The need for estimating a zero set term in free fall can be eliminated while still considering its effect on the solution by defining a measurement as a reinitialization type. The measurement is then defined as a difference between the current value and the value at the first time the measurement entered the solution. This feature was designed mainly for long arc work, but can also be used for SRN-9 system ship location.

E. Free-Fall Long and Short Arc Capabilities

1. Up to n short arc initial conditions may be estimated (where n is the number of intervals: unlimited for the 7094 version of NITE, limited to 14 for the 360/65 version and limited to 9 for the CYBER/6500 version). A long arc (a long arc is defined as one that extends over more than one interval; see F-2 below) may be preceded by any number of short arcs or powered flight trajectories but must be the last arc estimated in the run.

2. Ten different body types defined by weight, cross-sectional reflection area, coefficients of drag, lift and reflectivity may be handled in one computer run. A drag table (mach number vs drag coefficient, altitude vs drag coefficient or time vs drag coefficient) may be applied to each object. Lift tables may be entered in the same manner. Up to 300 entries are allowed in each table in the 360/65 and CYBER/6500 versions of NITE.

3. Any desired geopotential model may be read in on cards. The maximum size is order 24 degree 24 for the 7094 and 360/65 version and order 30 degree 30 for the CYBER/6500 version. The 7094 version has 6 built-in geopotential models, the 360/65 version has 21 built-in geopotential models and the CYBER/6500 version has 5 built-in geopotential models.

   All models may be truncated by the program user. Up to 15 terms may be estimated or modified through the station constants. The built-in models are updated as improved models become available.

4. Up to 15 of the constants associated with free fall (drag, lift, cross-sectional area, geopotential coefficients, etc.) may be estimated or have their a priori variances propagated into the GDOPs on one computer run.

5. The Cowell Method, using a Gauss-Jackson with automatic step size selection, is used in orbital integration. The integration is eighth order.

6. The built-in atmospheric model is the Patrick reference below 30 km., U.S. Standard (1962) Model between 30 km and 120 km, and a modified Jacchia 1964 (Dynamic) model above 120 km. Atmosphere models may also be read in through the station constants either as card-by-card samples (standard weather deck) or as polynomials. Wind speed and wind direction profiles may also be entered.

7. Extraterrestrial potential (lunar, solar and planetary) and solar radiation pressure may be applied to all objects.

8. Initial conditions (initial position and velocity vector) used as input may be of 10 different types.

9. A self-start capability allows initial conditions to be computed from two position vectors.

10. Six types of orbital elements are output options.

11. Discontinuities in position and velocity (velocity kicks) may be entered through the error model coefficients.

12. Trajectories may be constrained to pass through known impact points or to impact at specified times.

F. Number of Parameters and Measurements Allowed

1. By building up the normal equation coefficients one at a time and by making use of the fact that element i, j of the normal equation coefficient matrix is zero if parameter i is never in the solution at the same time or during the same arc as parameter j, the only limit to the number of measurements instrumentation error model terms, trajectory coordinates or free-fall initial conditions which may be estimated during a computer run is the number that can be defined in the n intervals.

3. Data from one NITE run may be used as input to other NITE runs.

4. Data may be eliminated from solutions through start-stop time control by elevation angle cutoffs, or through a missing data indicator (all bits for the input data tape word).

5. Data may be generated from either the input or output trajectories.

D. Error Model Terms

1. Any terms whose partial derivatives can be computed externally and placed on the NITE input tape. Inertial guidance terms, many of which require numerical integration, have been handled in this manner. Complex atmospheric models, thrust models and second-order instrumentation error models may also be handled in this way.

2. The following terms are built into the NITE partial derivative subroutine:

   a. Zero sets for all the instrumentations

   b. Reinitialization of measurements at specified times. This allows compensation for shifts in the input measurements in addition to the original bias.

   c. Velocity measurement biases

   d. Acceleration measurement biases

   e. Rate biases caused by an error in the local frequency reference

   f. Polynomials in time for trajectories and all input measurements

   g. Scale factor (to compensate for errors in wave length, speed of light or sound, focal length, etc.)

   h. Refraction correction of the range, range sum, range difference, elevation and ballistic camera plate coordinates

   i. Timing biases

   j. Transit time corrections

2. In order to make efficient use of the sparse nature of the error model normal equation coefficient matrix, the program breaks a computer run into intervals during which specified error model terms do not occur simultaneously in the solution with other terms. This allows the inversion, by partitioning, of a matrix of an order limited only by the number of intervals. The largest number of error model terms which can be handled during any interval is 100. If m specified terms (such as survey and geopotential) are in the solution during more than one interval, 100-m additional terms may be estimated in each of the intervals. The maximum number of terms which can be estimated during a computer run broken into n intervals would then be m+n (100-m). Thus, if each term were in the solution during more than one interval, the limit for the computer run would be 100 terms.

3. Three, six or nine (position, position and velocity, position and velocity, and acceleration) trajectory parameters may be estimated at each timepoint.

4. Forty position, 40 velocity and 40 acceleration type measurements may be used at each time, and in each interval. Thus, if there were m times per interval, n intervals and 40 position, 40 velocity and 40 acceleration measurements at each time, the total number of measurements for the run would be 120xnxm.

G. Other Features

1. The weighting of measurements may be performed in several ways:

   a. Weights may be input from tape

   b. Weights may be input by station constants

   c. Weights may be computed by the program as functions of error model terms

   d. Any combination of a., b. and c. may be used

2. A full a priori covariance matrix for the first ten instrumentation error models may be input.

3. Long running 7094 computer runs may be interrupted and restarted at a later time. This is accomplished by saving all information accumulated up to the time of interruption. The 360/65 version has been adapted to run from a CRT terminal, where data are displayed on the CRT during the run. The CYBER/6500 version can also be run from a terminal.

4. Dual tracking points may be assumed for range sum and range difference type measurements.

5. Tracking points may be switched at specified times. This is sometimes necessary when different antennas are used after staging events.

6. Attitude information necessary in computing tracking point corrections may either be derived from velocity vectors or input as pitch, roll and yaw from tape. Attitude rate data used in correcting velocity data may also be input.

7. Errors in parameter estimates due to unit errors in unmodeled error coefficients may be computed as an option.

8. Improved estimates of instrumentation measurement weights may be computed from residuals as an option.

9. Both convergence and divergence tolerances are tested in order to prevent unnecessary calculations.

10. Partials of residuals with respect to unmodeled errors may be output.

11. Residual analysis output includes means, standard deviations and ranges of residuals.

12. Reentry data may be output: altitude, mach number, atmospheric density, coefficients of drag and lift, lift vector angle, ballistic coefficient, drag and lift accelerations, and reentry angle.

13. Ship pitch, roll and heading and their velocities may be input to correct shipboard or airborne position and velocity measurements.

14. Ships or aircraft may be constrained to follow either great circle or rhumb line (constant heading).

15. Plots of the residuals may be output on printer.

IV. INPUT REQUIREMENTS

The input requirement can be explained by following the structure of the station constants, which are segmented into groups.

A. Group 1 - General constants common to all intervals

These are constants for decision logic such as the number of intervals; outer iteration divergence tolerance; position only, position and velocity, or position and velocity and acceleration; mode of run (GDOP, EDIT, SIMULATION or ESTIMATION); output in feet or meters, right handed or left handed; maximum number of words to read from the input tape; outer iteration convergence tolerance; number of outer iterations; moving origin differencing; frequency of printed output; frequency of input; number of inner iterations; and EDIT tolerances.

B. Group 2 - Master Coordinate System

This defines the location of the output trajectory, its orientation and spheroid.

C. Group 3A - Independent and Adjustable Auxiliary Origins

Up to 10 surveys can be defined in this group by geodetic and astronomic coordinates.

D. Group 3B - Survey Covariance Matrices

Survey covariance matrices may be defined for the surveys in Group 3A.

E. Group 4A - General Free-Fall Constants

This group defines constants (logical and data) for the operation of the free-fall constraints.

F. Group 4B - Estimation of Free-Fall Parameters

These are the constants to define up to 15 free-fall parameters to be estimated, their a priori value and their a priori standard deviation.

G. Group 5A - Adjustable System Parameters

Similar to 4B, up to 100 adjustable system parameters are defined, and their a priori values and standard deviations are given. Any systematic parameters to be used for weights are defined, as are those parameters that will not be estimated, but whose uncertainties will be propagated.

H. Group 5B - Covariance of Parameters

The covariances of up to 10 parameters defined in Group 5A may be input.

I. Group 6A - Independent, Nonadjustable Auxiliary Origins

Up to 20 geodetic and astronomic origins may be defined for all types of systems including inertial systems. Aspect angles relative to particular surveys may be called.

J. Group 6B - Dependent, Nonadjustable Auxiliary Origins

Up to 20 origins may be defined that are related to Group 3A or Group 6A origins by a fixed cartesian coordinate relationship.

K. Group 6C - Camera Orientation

Up to 20 ballistic camera orientations may be defined.

L. Group 6D - Moving Origins

Up to 3 moving origins may be defined. Each may be related to origins in Groups 3A, 6A or 6B. Moving origins may be read in from tape or generated from initial position and rate data. Attitude (pitch, roll and yaw) data of ship motion may be read in.

M. Group 7 - Start and Stop Times

This group controls the time spans of the solution.

N. Group 8 - Refraction Tables

Up to 10 refraction index tables may be input. A ray tracing technique is used to apply tropospheric and ionospheric refraction corrections.

O. Group 9 - Measurement Definitions

Up to 40 measurements (each having position, position and velocity, position, velocity and acceleration) of the types mentioned in Section III A may be defined and base line time delays may be defined.

P. Group 10 - Measurement Model

Measurement error models of any combination of those listed in Section III D may be defined. As many as 12 terms may be defined for each of the 40 measurements provided the total number of estimable terms is less than 100.

Q. Group 11 - Additional Error

Estimates of additional random error may be defined.

R. Group 12 - Measurement Tape Words

The locations on the input tape or input cards of the 40 measurements and their random errors defined in Group 9 are defined in this group.

S. Group 13 - System Parameter Estimates for this Interval

System parameter estimates and their a priori estimates of error with identification numbers less than 101 may be estimated or propagated. Those with identification numbers larger than 100 are not estimated, but the estimates of error may be propagated.

T. Group 14 - System Control

Control of measurement entry into the solution is available via time spans and/or minimum elevation cutoff. Scaling of input measurements is also available.

U. Group 15 - Tracking Points

Up to 10 tracking point vector pairs may be defined.

V. Group 16 - Initial Approximations

Up to 5 sets of initial approximations may be defined as word positions of the input XYZ or AER (position, position and velocity or position and velocity and acceleration) or right ascension and declination. Or, any or all of the 5 sets may be single points input on cards.

W. Group 17 - Tracking Point Orientations

Up to 20 sets of missile attitude information may be input to be used in correcting data for tracking points.

X. Group 18A - Free-Fall Constants the Current Interval

Julian day number, GMT of T-0, free fall start and stop times, initial condition data and integration criteria are input.

Y. Group 18B - Solar Flux and Geomagnetic Index

Tables are entered.

Z1. Group 18C - Drag - Lift Information

Tables of drag and lift (up to 300 entries each) may be entered, along with cross-sectional area, weight and reference time for drag and lift error model polynomials. The tables may be functions of time, altitude or mach number.

Z2. Group 18D - Weather - Atmosphere Data

The built-in atmosphere profile (density and sonic speed as functions of altitude for the Patrick standard from 0 to 30 km, and U.S. Standard 1962 from 30 to 700 km) may be called, or any profile of speed of sound, density, wind speed and wind velocity may be entered. These may be entered as sets polynomials in altitude or as tables.

Z3. Group 18E - Gravity Model

Built-in geopotentials may be called or any geopotential model may be read in on cards. Size limits are order 24 degree 24 for the 7094 version and the 360/65 version, and order 30 degree 30 for the CYBER/6500 version. The models may be unnormalized, APL normalized or Kaula normalized. The models may be truncated at specific missile or trajectory altitudes.

Z4. Group 19 - Plot Scales

Printer plots of residuals may be output with selectable scales.

V. VALIDATION TECHNIQUES

A. Program Certification

Air Force/RCA policy requires that each production computer program (any program that processes data for external dissemination) must be certified. The certification procedure requires, among other things, that the validity of the program results be verified by some means external to the program such as hand checkpoints or comparison with a certified program. All production versions of NITE are certified.

B. Individual Computer Run Validation

The validity of the results of each NITE run is verified by evaluation of the following:

1. Convergence.
   Obviously, the results are of no use unless the least squares iteration process has converged. The "outer iteration" is the name given to that part of the weighted least squares process that estimates the systematic error model terms (Groups 4B, 5A, 13 and the initial conditions in 18A). The results of each iteration are printed, i.e., the corrections applied to the estimates of the previous iteration, and the estimates of the current iteration. The analyst in charge of the run reviews the magnitude of the corrections and finds the run acceptable if the magnitudes of the corrections become insignificant relative to the magnitude of the current estimate and its uncertainty.

2. The analyst reviews the magnitude of the final systematic error estimates relative to the a priori and computed systematic error estimate uncertainties. Outliers are singled out for further analysis to justify their values. Usually the computed uncertanties of the estimates is significantly smaller than the a priori uncertainties, a reflection of the strength of the solution.

When the results show small or no reductions in uncertainties, the run requires more analysis to verify the results of these "weak" solutions.

3. The trajectory is the result of a weighted least squares combination of the data corrected for systematic error model terms in what is called the "inner" iteration loop. The values of the computed trajectory are analyzed for reasonableness in the sense of noise content and, if necessary, comparisons with other estimates of the trajectory (as computed from other single sensors, or other NITE runs).

4. The correlation matrix of the systematic error model terms is reviewed. Values greater than 1 are indicators of matrix inversion errors which can happen if the matrix is ill conditioned in a weak solution. High correlations (less than or equal to one) are acceptable, but are indicators of poor separability among coefficients.

5. The magnitudes of the residuals are reviewed. The residual analysis outputs means, standard deviations, maximums and minimums of the residuals. Means of the residuals significantly different than zero require analysis for the cause:

   a) A priori uncertainties of the error model terms may be small enough to give the a priori value too much weight, preventing full resolution of the systematic error. Action: raise a priori uncertainty or change the estimate of the systematic error term.

   b) The dynamic weighting may be weighting out trended residuals. Action: verify weighting.

   c) The systematic error model may be incorrect. Action: review trends in residuals, find justification for new model from analysis of system performance.

6. Review the Distribution of the Residuals

This is best accomplished by review of the plots of the residuals. Ideally, the residual distribution should be random about zero. Any deviations should be justified by analysis of the weighting (high random error estimates hence low weights indicate marginal system performance, low elevation tracking being contaminated by refraction errors, multi-path, clutter, etc.), and a review of the modeling .

VI. A PRIORI INFORMATION

A. Estimates of the uncertainty of error model coefficients are derived primarily by the Technical Analysis group. Historical statistical histories are maintained of the results of all BET and satellite solutions. The error model coefficients and their uncertainties are evaluated, summarized and the results are produced in a "Quarterly Accuracy Report" (QAR). Uncertainties of any terms not presented by the report are derived by the analyst in charge and are based upon the facts available, such as the expected magnitude of the coefficient, the strength of the solution, the quality of the measurement and the information presented by NITE in the partial derivative output.

B. Random Error Estimates

Estimates of random error used in weighting are usually derived from scaled differences between smoothed and unsmoothed measurements. The smoothing span of the filter is selected to ensure maximum smoothing without the removal of missile motion. The scaled differences are augmented by as much as 5% of the refraction correction, to ensure that range and elevation are properly weighted to account for residual refraction, multipath and ground clutter errors. The random error estimates are monitored to ensure that they are within the range of past performance of that system as indicated in the Tech Analysis QAR. Occasionally, a system is used that cannot be smoothed. QAR estimates are used if available; if not, estimates are derived from comparisons with more accurate systems, or by analysis of NITE residuals where the system in question is weighted out (provided, of course, that the BET is known to have less noise than the system in question). If data are collected while the tracked object is in free fall, and free-fall constraints are used in NITE, a good estimate of random noise can be found in the residuals.

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GPSBET PROGRAM DESCRIPTION

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ABSTRACT

This paper describes the background for the development of US Army Yuma Proving Ground (USAYPG) Best Estimate of Trajectory (BET) program. The program is called GPSBET which employs optimal estimation techniques to combine measurements from several sources and arrive at a single trajectory estimate. The estimation technique employed is Kalman filtering and smoothing. The implementation of the filter and smoothing equations is described in this paper, as well as the ancillary processing performed to prepare the measurements for filtering. The input to and output from GPSBET, in addition to the validation procedures, are also described. The performance of GPSBET in terms of accuracy and computer run time is also documented,

1. HISTORY

In the pre-laser era, YPG used a multistation cinetheodolite system to obtain data for a Best Estimate of Trajectory. The trajectory was obtained from an odle solution (minimizing the sum of squares of distance residuals), followed by a Davis solution (minimizing the sum of squares of angular residuals), followed by a second degree moving arc smoothing routine. In 1973, YPG received a PATS (Precision Aircraft Tracking System) Laser. Upon comparison of laser derived trajectories with cinetheodolite derived trajectories, it became apparent that the laser had the potential for providing position data in a very automated mode and with accuracy as good as cinetheodolites but dependent only on distance from the PATS rather than being complicated by geometry. A software development effort was initiated in the fall of 1973 to compute trajectory using some combination of laser and cinetheodolite data.

The software development was based on Kalman filtering of measurements. The Kalman filter was ideal for ease of accepting any combination of range or azimuth or elevation measurements subject only to the restriction that they come in chronological order. Subsequent analysis of trajectory data and measurement residuals revealed that the range measurements from the laser were the strength of any laser/cine solution. YPG then purchased two additional lasers. Further tests and simulations demonstrated that cinetheodolite data contributes little or nothing to the accuracy of a 3-laser solution and was very expensive to acquire and reduce and it slowed data turnaround tremendously.

In mid-1974, the Aerospace Corporation, in support of the NAVSTAR Global Positioning System, helped YPG with the development of Kalman filter equations including the incorporation of INS (Inertial Navigation System) and IMU (Inertial Measurement Unit) data. INS data consists of sensed velocities in three components and IMU data consists of gimbal angles which reflect the pitch, roll and yaw of an aircraft. The purpose of INS measurements was to lower the uncertainty in velocity estimates. The purpose of the IMU measurements was to improve lever arm corrections by providing accurate attitude information.

2. TERMINOLOGY

YPG's Best Estimate of Trajectory program is called GPSBET. The estimation is accomplished using Kalman filter equations and the Rauch-Tung-Striebel recursive smoothing equations. It is appropriate therefore to define the components of the state vector which are estimated by the Kalman filter/smoother. The state vector consists of X, Y, Z components of position, velocity and acceleration plus a bias state for each measurement type from each instrument. For a 3-laser solution, the state vector has 18 components:

1	X	Position	
2	Y	Position	
3	Z	Position	
4	X	Velocity	
5	Y	Velocity	
6	Z	Velocity	
7	X	Acceleration	
8	Y	Acceleration	
9	Z	Acceleration	
10	Range bias		Laser 1
11	Azimuth bias		Laser 1
12	Elevation bias		Laser 1
13	Range bias		Laser 2
14	Azimuth bias		Laser 2
15	Elevation bias		Laser 2
16	Range bias		Laser 3
17	Azimuth bias		Laser 3
18	Elevation bias		Laser 3

When INS/IMU measurements are used then 6 more bias states are added:

19	Orientation bias	East axis
20	Orientation bias	North axis
21	Orientation bias	Vertical axis
22	Velocity bias	East axis
23	Velocity bias	North axis
23	Velocity bias	Vertical axis

The state vector estimates of position, velocity and acceleration are in a right handed cartesian coordinate system with origin at a specific point on the ground at YPG. The bias estimates for each laser are in a spherical coordinate system centered at the respective lasers with zero azimuth being True North at each laser and zero elevation being horizontal at each laser. The orientation biases are relative to the gyro wander angle reference frame as are the velocity bias estimates.

3. ANCILLARY PROCESSING

The purpose of trajectory estimation at YPG is generally to provide a position, velocity and acceleration time history for a particular point on a particular target in a standard coordinate system. Therefore, a trajectory estimation program must include modules for atmospheric corrections, systematic error correction, coordinate conversion, and lever arm corrections, in addition to the noise reduction and state vector estimation algorithms. Also, the actual measurements going into a BET program are never perfect and the imperfections are not time stationary so that a considerable amount of checking for wild points and for changes in noise distribution must be done. Sections 3.1 through 3.7 discuss some of the special data handling and evaluation procedures that are included in GPSBET to help ensure that the Kalman filter parameters adequately describe the data being filtered.

3.1 ATMOSPHERIC CORRECTIONS

GPSBET corrects all laser range and elevation measurements for atmospheric refraction. The model currently being used is a two parameter model where the parameters are computed from RAWINDSONDE data obtained generally within two hours of the time for which a trajectory estimate is desired. The model assumes a constant temperature lapse rate so that when a target is located in or near a low level temperature inversion, significant errors can result (where significant means anything over .5 meter).

3.2 TRACKING SYSTEM ERRORS

The laser measurements are corrected for:

Range bias	-	R_b
Range scale factor	-	R_s
Azimuth bias	-	A_b
Elevation bias	-	E_b
North tilt	-	NT
East tilt	-	ET
Collimation	-	CØL

These corrections are made by computing coefficients during calibration. Calibration includes collecting at least 400 measurements from each laser as each lases on eight targets at 45^o increments around each laser.

The equations for correcting measurements (R_m, A_m, E_m) to (R_c, A_c, E_c) are:

[1] $$R_c = R_m + R_s * R_m + R_b$$

$$E_c = E_m + E_b$$

[2] $$A_c = A_m + A_b + \sin^{-1}\left[\frac{\sin(COL)}{\cos(E_c)}\right]$$

[3] $$E_c = \sin^{-1}\left[\sin(E_c) * \cos(COL)\right]$$

Mislevel of each laser is accounted for by rotation of the predicted state X, Y, Z from the common cartesian coordinate system to the laser misleveled system (which includes earth's curvature from the selected origin to the laser sites). This is a convenient and precise method of obtaining azimuth and elevation predictions, A_p and E_p, which are in the same reference system as the A_c and E_c computed in [2] and [3].

Other sources of tracking system errors have been investigated. Encoder eccentricity errors are observable and although they are small, will be included in the BET when the calibration software has been modified to compute coefficients. Lens sag is negligable for the YPG lasers. Servo lag is negligable as long as mount acceleration is significantly less than 80 milliradians/sec^2 (which is generally the case for aircraft tracking). GPSBET does estimate range, azimuth and elevation rates and accelerations which can be used to model servo response. The model appropriate for YPG's lasers is:

$$L = K * A,$$

where

L = Angular lag

K = Constant dependent upon gain of servo system

A = Angular acceleration experienced by the mount

3.3 EARTH'S CURVATURE CORRECTIONS

GPSBET produces state vector estimates in a cartesian coordinate system with origin at input latitude, longitude and altitude. The geodetic coordinates of the origin, as well as the tracking instruments, are input relative to a reference ellipsoid whose parameters can be varied. The model of the earth used is Clarke's Spheroid of 1866 with the WGS 72 Datum. At each measurement time, the estimate for the state vector at that time is corrected for earth's curvature by rotating the X, Y, Z through the difference in latitude and longitude from the input origin to the instrument location (plus the mislevel of the instrument). The rotated X, Y, Z can then be transformed to spherical coordinates (range, azimuth, elevation) and compared with the measurements (which have been corrected for tracking system errors as described in 3.2). The Kalman filter uses the difference between predictions and measurements (called residuals) to update state vector and covariance matrix estimates and to make edit checks.

3.4 ATTITUDE ESTIMATION AND LEVER ARM CORRECTIONS

Range instrumentation can not usually provide measurements of the position or velocity of the particular point on a target for which a reference trajectory is required. In the case of laser trackers, a retroreflector package must be installed on the target and the location of that package cannot coincide with the desired reference point. For radar, the same situation exists for beacon tracking. In the case of cinetheodolite tracking, the desired reference point is not always identifiable on film. When very accurate position and velocity information is required for a particluar point on a target (such as an antenna or the center of gimbals in a gyro package), then corrections must be made to allow for the separation of the instrumentation reference point and the desired reference point. On aircraft, the separation may be on the order of 10 meters. When .5 meter accuracy is required, then 10 meter separation of reference points must be carefully removed. This is done in GPSBET by continually updating estimates of the attitude matrix. The attitude matrix is a rotation matrix to go from the aircraft body coordinate system to the reference cartesian coordinate system. It includes pitch, roll, yaw information about the aircraft plus earth's curvature from the aircraft to the origin of the reference cartesian system.

To accomplish lever arm corrections, the following vectors and matrices must be computed or input:

M = Rotation matrix to correct for Earth's curvature from the origin of the reference cartesian system to a cartesian system with aircraft as origin.

M^T = Matrix transpose of M.

PRY = Rotation matrix to correct from pitched, rolled and yawed aircraft body coordinate system to locally horizontal XY-plane (Z up) at the aircraft.

A = Attitude matrix to express rotation from aircraft body coordinate system to reference cartesian system.

$\begin{bmatrix} XLEV \\ YLEV \\ ZLEV \end{bmatrix}$ = Vector containing distances in aircraft body coordinate system from desired reference point to instrumentation reference point.

$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$ = Kalman filter estimates of position of desired reference point on aircraft.

$\begin{bmatrix} XI \\ YI \\ ZI \end{bmatrix}$ = Position of instrumentation reference point on the aircraft.

To incorporate a measurement from a particular instrument, GPSBET does the following for lever arm correction:

$$\begin{bmatrix} XI \\ YI \\ ZI \end{bmatrix} = A * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

where

$$A = M^T * PRY$$

and

$$M = \begin{bmatrix} cDL & sDLsPH & -sDLcPH \\ sPHIsDL & cPHIcPH + sPHIcDLsPH & cPHIsPH - sPHIcDLcPH \\ cPHIsDL & sPHIcPH - cPHIcDLsPH & sPHIsPH + cPHIcDLcPH \end{bmatrix},$$

$$PRY = \begin{bmatrix} cBcP & -cBsPsR + sBcR & -cBsPcR - sBsR \\ -sBcP & sBsPsR + cBcR & sBsPcR - cBsR \\ sP & cPsR & cPcR \end{bmatrix}$$

sXXX - sin of the angle XXX

cXXX - cos of the angle XXX

DL - difference in longitude of aircraft and origin

PH - Latitude of origin

PHI - latitude of aircraft

P - pitch of aircraft

R - roll of aircraft

B - heading of aircraft with respect to True North

The pitch, roll and heading of the aircraft are obtained from gimbal angle measurements when available. Otherwise, velocity and acceleration estimates are used to estimate pitch, roll and heading.

The attitude matrix, A, computed at each measurement time is an important part of the output of GPSBET. It can be used to determine the position, velocity, acceleration of a point on the aircraft other than that estimated by the state vector.

3.5 RETROREFLECTOR IDENTIFICATION

The NAVSTAR Global Positioning System project is currently the principal user of YPG's laser tracking instrumentation and the GPSBET program. For GPS testing, two retroreflector packages are mounted on some aircraft (e.g., C-141 and F-4). The purpose is to eliminate blockage of retroreflectors by the wings, fuselage, etc. This results in virtually complete coverage from three laser trackers throughout all maneuvers. Since the retros are separated by 5-10 meters from each other, the

GPSBET program needs an algorithm to decide which retro is being tracked by a given laser so that the appropriate lever arm corrections might be applied. The alogrithm being employed involves computing the look angle from a given laser to the axis of each retroreflector package. The retro packages used are hemispherical and have been mounted on the top, pointing up, and the bottom, pointing down, of an aircraft. The retro whose axis makes the smallest angle with the line of sight to a given laser is the retro identified as the most likely source for returning the light to the laser and lever arm corrections are made from the reference point chosen for the state vector to the selected retro.

The use of two retroreflector packages does solve the blockage problem (i.e., all lasers almost always have line of sight to one retro or the other), but causes degradation of accuracy since the identification is imperfect and lasers can get light returned from both retro packages at the same time. The identification scheme is very sensitive to errors in the estimated roll of the aircraft and this gives rise to the erroneous identification of retro switches which in turn puts noise on velocity and acceleration estimates.

3.6 EDITING AND REINITIALIZATION

The GPSBET program accomplishes editing by checking the magnitude of measurement residuals prior to the incorporation of the measurement into the filter. The procedure includes the following steps:

1. Compute the difference between the measurement and the filter's prediction for the measurement (this difference is called the measurement residual and is computed as described in 3.3).

2. Compute the expected variance of the residual by adding the

variance in the prediction (computed from the covariance matrix of the state vector) to the variance of the measurement noise (a priori information required as input to GPSBET).

3. Divide the residual by the uncertainty in the residual (the square root of the variance described in 2.).

4. If the residual is more than N times the uncertainty in the residual (where N is an input constant, usually set to 5) then the measurement is not included, otherwise the Kalman filter equations are executed to correct the predicted state vector and covariance matrix estimates.

5. If editing has occurred, then counters are incremented for:

- total number of edits of measurements of this type
- number of consecutive edits of measurements of this type
- number of consecutive edits of measurements from this instrument.

The edit counters described in 5. are checked throughout the filtering of data and when input limits are exceeded, some action is taken, e.g.,

- the filter is reinitialized when at least two of the three types of measurements from all lasers have been edited for an input number of seconds
- the filter is reinitialized when at least two of the three measurement types have been edited for an input number of consecutive measurement times for a given laser
- the filter is reinitialized when a given measurement is edited consecutively more often than an input limit
- printing of edit messages is stopped when an input limit is reached.

Reinitialization is accomplished by converting the range, azimuth, elevation data to X, Y, Z in the common cartesian system for each of the lasers. When three consecutive second differences are within an input criterion for X, Y and Z of a given laser, then a second degree fit is made to those five X coordinates, five Y coordinates and five Z coordinates. The polynomial and its derivatives are evaluated at the time of the first measurement in the fit. This provides X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\ddot{X}$, $\ddot{Y}$, and $\ddot{Z}$ which are used as initial state vector components for the Kalman filter. The covariance matrix is reinitialized to the same values that were input at the beginning of the estimation. This scheme for reinitializing the filter is usually used to get initial state vector estimates at the beginning of the trajectory estimation.

An eleven-point fit is an option for initialization and reinitialization, but is rarely required; since the Kalman filter initialization transients dampen before highly accurate estimates are required. In most cases, the initial uncertainties in the state vector are input as large numbers (e.g., 1000 meters, 500 meter/sec, 100 meters/sec/sec for position, velocity, acceleration components) so that the Kalman filter will accept measurements even though the initialization was not very good.

3.7 RESIDUALS SUMMARIES

Probably the most important output from the GPSBET program from an analytical standpoint, (i.e., the output that provides the best measure of filter performance, measurement quality and quantity and accuracy of a priori information) are residuals summary reports.

These summaries are output at a rate determined by input parameters and can be selected to come at a particular time interval or after a certain number of measurements have been received. After a set of measurements has been processed for a given instrument (e.g., range, azimuth, elevation from laser 1), the estimates for X,Y,Z are translated to the appropriate instrument, corrected for earth's curvature, corrected to the instruments misleveled state, converted to spherical coordinates and differenced with the calibrated measurements to form residuals. Only for those measurements not edited are the residuals tabulated. The tabulated residuals are summed, their squares are summed and the number of unedited measurements of each type is counted. At requested intervals the mean of the residuals, RMS of the residuals and number of measurements used since the last report is printed for each measurement type for each instrument. The number of measurements used since the last report tells how much editing and data dropouts were occurring. The RMS of residuals is an excellent measure of the noise on the measurements and is used to validate the a priori information. The mean of residuals should be near zero, but when it isn't points to problems such as erroneous calibrations, misidentification of retros, lack of sufficient filter response to process noise, inconsistent measurement sources, etc.

4. KALMAN FILTER

The program GPSBET uses Kalman filter equations to derive a trajectory estimate at each time for which a measurement is made available. Section 4.1 explains the notational conventions to be used in 4.2 to describe the input parameters controlling the behavior of the Kalman filter and in 4.3 to outline the implementation of the Kalman filter equations in GPSBET.

4.1 NOTATION

Capitol letters (e.g., A,M,X) will be used to represent matrices. Sometimes these matrices have only one column or one row, in which case they will be referred to as vectors, but no notational distinction will be made. Time will be denoted by t, and t_n refers to the n^{th} time in a sequence. Δt will refer to a time interval from some time t_m to another time t_n where the order of the subtraction will be made clear in the text. When a capital letter is subscripted such as X_n this will refer to the estimate of X at time t_n. A vector (matrix) symbol with a carrot over it (e.g., $\hat{X}_n,\hat{P}_n$) refers to a predicted value for the vector (matrix) at time t_n. In all equations, the symbol * refers to multiplication. Lower-case letters (e.g., a,b,n,j) refer to scalars. The lower case letters i,j,k,l,m,n are reserved for integer scalars.

The scalars $x,y,z,\dot{x},\dot{y},\dot{z},\ddot{x},\ddot{y},\ddot{z}$ are reserved for the position, velocity, acceleration components in a particular state vector estimate.

The notation $b*X$ refers to multiplication of a scalar and a matrix (in particular, $\Delta t * Q$ is the multiplication of the matrix Q by the scalar Δt which is a time interval). Multiplication, addition and subtraction in matrix equations implies that the number of rows and columns in each matrix allows such operations. The notation H^T refers to the transpose of the matrix H.

X_n - Kalman filter estimate for the state vector after incorporation of measurements from a given instrument at time t_n (the first 9 components are $x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z}$)

P_n - Kalman filter estimate for the covariance matrix of the state vector X_n (the diagonal of P_n contains the estimates of the variance of the components of the state vector)

$S_{n,n+1}$ - Transformation matrix to estimate a value for X_{n+1} at time t_{n+1} given X_n and $\Delta t = t_{n+1} - t_n$

$\hat{X}_{n+1}$ - Prediction of X_{n+1} given X_n and $\Delta t = t_{n+1} - t_n$

$$\hat{X}_{n+1} = S_{n,n+1} * X_n$$

Q - Effect of process noise on covariance matrix P per unit of time

$\hat{P}_{n+1}$ - Prediction of P_{n+1} given P_n and $\Delta t = t_{n+1} - t_n$

$$\hat{P}_{n+1} = S_{n,\,n+1} * (P_n + \Delta t * Q)\, S^T_{n,\,n+1}$$

θ_n - Measurement taken at time t_n

$\hat{\theta}_n$ - Prediction of θ_n given $\hat{X}_n$

A_n - Transformation matrix to go from state vector domain to measurement domain

$$\hat{\theta}_n = A_n * \hat{X}_n$$

NOTE: In GPSBET, this transformation is accomplished with the trigonometric/algebraic relationships between cartesian coordinates (X, Y, Z) and spherical coordinates (range, azimuth, elevation). The transformation of state vector uncertainties to uncertainties in θ_n are estimated by $H_n * \hat{P}_n * H_n^T$ where H_n is described below.

H_n - Partial derivatives of measurement to be incorporated at time t_n (i.e., θ_n) with respect to the state vector:

$$H_n = \frac{\partial \theta_n}{\partial \hat{X}_n} = \begin{matrix} \frac{\partial \theta_n}{\partial X} \\ \frac{\partial \theta_n}{\partial Y} \\ \frac{\partial \theta_n}{\partial Z} \\ \frac{\partial \theta_n}{\partial \dot{X}} \\ \vdots \\ \vdots \end{matrix}$$

K_n - Gain vector for Kalman filter computed for incorporating the measurement θ_n taken at time t_n

r - Scalar denoting the measurement residual i.e.,

$$r = \hat{\theta}_n - \theta_n$$

4.2 KALMAN FILTER INPUT PARAMETERS

This section will describe only those input parameters that are required in the Kalman filter implementation in GPSBET. Many other parameters are input to the program for controlling data flow, manipulating input data, selecting processing options, selecting output options, etc. These other parameters will be described in Appendix A so as not to clutter the main body of this report.

D - An array containing the standard deviations of the noise on each measurement type for each instrument. It is double dimensioned so that D (i, j) is the standard deviation of the i^{th} measurement type of the j^{th} instrument. For example, D (2, 3) = .000200, could mean the standard deviation of the azimuth measurements for the third laser is 200 microradians.

Q - A matrix the same size as the covariance matrix P, but which contains the increased variance expected in the state vector due to process noise expected in 1 second.

P_0 - Initial values of the covariance matrix, usually set very large so that data is accepted and only consistent data will drive the diagonal of P to small values.

X_0 - Initial values for the state vector which can be input or computed from laser data as described in Section 3.6.

4.3 FILTER EQUATIONS

To describe the implementation of the Kalman filter equations in GPSBET, this section will start with a state vector estimate X_n and an estimate of its covariance matrix P_n at time t_n. The equations and intermediate steps performed by GPSBET to incorporate a measurement θ_{n+1} taken at time t_{n+1} will be described. The estimates X_n and P_n may have been initial estimates (denoted X_0 and P_0) as described in Section 4.2. The measurement θ_{n+1} is assumed to have been corrected for atmosphere effects as described in Section 3.1 and for tracking system errors as described in Section 4.2.

Step 1: The estimates for the state vector, X_n, and covariance matrix, P_n, at time t_n, are projected ahead (or back if measurements come in reverse chronological order) to obtain predictions X_{n+1}, P_{n+1} for their value at time t_{n+1}:

$$\Delta t = t_{n+1} - t_n$$

$$\hat{X}_{n+1} = S_{n,\, n+1} * X_n$$

$$\hat{P}_{n+1} = S_{n,\, n+1}^{*} (P_n + \Delta t * Q) * S^T_{n,\, n+1}$$

$$
S_{n,\,n+1} = \begin{bmatrix}
1 & 0 & 0 & t & 0 & 0 & \frac{t^2}{2} & 0 & 0 & 0 & \cdots \\
0 & 1 & 0 & 0 & t & 0 & 0 & \frac{t^2}{2} & 0 & 0 & \cdots \\
0 & 0 & 1 & 0 & 0 & t & 0 & 0 & \frac{t^2}{2} & 0 & \cdots \\
0 & 0 & 0 & 1 & 0 & 0 & t & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & t & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & t & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots
\end{bmatrix}
$$

<u>Step 2</u>: The predicted value for the state vector, $\hat{X}_{n+1}$, is transformed to a predicted value, $\hat{\theta}_{n+1}$, for the measurement, θ_{n+1}. The computation of $\hat{\theta}_{n+1}$ is accomplished with algebra/trigonometry rather than matrix multiplication. For example, if θ_{n+1} is an azimuth measurement from laser 1, then the X, Y, Z components of $\hat{X}_{n+1}$ are rotated and translated to the local cartesian system at laser 1 (call them X', Y', Z'). Then $\hat{\theta}_{n+1}$ is computed: $\hat{\theta}_{n+1} = \sin^{-1}\left(\frac{x'}{\sqrt{(x') + (y')^2}}\right) + b$, where b is the component of $\hat{X}_{n+1}$ that represents the filter's estimate of azimuth bias for laser 1. The measurement bias estimates remain zero unless the initial uncertainty in the measurement bias states is large enough and measurements from other sources are good enough to allow the Kalman filter gain for measurement bias to cause non-zero biases to be estimated. Since YPG's laser trackers are calibrated for every mission with surveyed targets, the GPSBET program is usually forced not to estimate laser measurement biases by setting the initial uncertainty very small. For INS velocity measurements on the other hand biases of 1.-2. meters/

second are usually estimated. These biases result from gyro drift and are modeled rather carefully using the gyro specifications and knowledge of the dynamics experienced by the gyro.

The above procedure for computing $\hat{\theta}_{n+1}$ from $\hat{X}_{n+1}$ will be denoted subsequently by

$$\theta_{n+1} = A_{n+1} * X_{n+1} ,$$

although, as described above, a matrix A_{n+1} does not have to be found; since algebraic and trigonometric relationships can be used instead, and these relationships provide an exact transformation.

Step 3: The difference between the measurement θ_{n+1} and the prediction $\hat{\theta}_{n+1}$ is computed:

$$r = \hat{\theta}_{n+1} - \theta_{n+1}$$

The scalar r is called the residual.

Step 4: The standard deviation of r, denote it by s_r, is computed next. Since r is the difference of two quantities presumed to be statistically independent (reasonable since the measurement at time t_{n+1} has not been used at all to arrive at $\hat{\theta}_{n+1}$), then

$$s_r^2 = d^2 + \left[D\ (i,\ j)\right]^2$$

where

D (i, j) - a priori estimate of the standard deviation of measurement type i from instrument j, and θ_{n+1} is a type i measurement from instrument j.

d - uncertainty in the estimate $\hat{\theta}_{n+1}$, which can be computed from the uncertainty in $\hat{X}_{n+1}$ and the partial derivatives of measurement θ_{n+1} with respect to $\hat{X}_{n+1}$:

$$d^2 = H_{n+1} * \hat{P}_{n+1} * H_{n+1}^T$$

Thus

$$s_r = \left\{\left[H_{n+1} * P_{n+1} * H_{n+1}^T\right] + \left[D\ (i,\ j)\right]^2\right\}^{\frac{1}{2}}$$

Step 5: An edit check is made next. If

$$\frac{r}{s_r} < \ell, \qquad \ell \text{ is input)}$$

then the remaining Kalman filter equations are executed. If

$$\frac{r}{s_r} \geq \ell, \qquad \ell \text{ is input)}$$

then the measurement θ_{n+1} is edited. The limit on the number of standard deviations allowed (denoted by ℓ above) is an input parameter (often set to 5.). When editing is indicated all appropriate edit counters are incremented, an edit message may be printed, the following assignments are made:

$$X_{n+1} = \hat{X}_{n+1}$$
$$P_{n+1} = \hat{P}_{n+1}$$

and the Kalman filtering cycle for measurement θ_{n+1} is ended.

Step 6: If editing does not occur, then the Kalman filter gain vector, K_{n+1}, is computed:

$$K_{n+1} = 1./s_r^2 * H_{n+1} * \hat{P}_{n+1}$$

The gain vector is the measure of how much the residual

$$r = \theta_{n+1} - \theta_{n+1}$$

should affect the state vector.

NOTE: If a Kalman filter were designed to incorporate the three measurements (range, azimuth, elevation) from a given laser at the same time, the result would be the same as incorporating them one at a time;

but the quotient $1./s_r^2$ in the gain computation would be a 3x3 matrix inversion.

Step 7: The state vector estimate $\hat{X}_{n+1}$ is now corrected optimally to include the information received from measurement θ_{n+1}:

$$X_{n+1} = \hat{X}_{n+1} - K_{n+1} * r$$

where

$$r = \hat{\theta}_{n+1} - \theta_{n+1}$$

Step 8: The covariance matrix estimate $\hat{P}_{n+1}$ is corrected:

$$P_{n+1} = \hat{P}_{n+1} - K_{n+1} * H_{n+1} * \hat{P}_{n+1}$$

At this point, a summary of the key equations from the above steps will be written for ease of reference. The equations assume that a measurement has been received at time t_{n+1} and that the transition matrix to go Δt seconds from time t_n to t_{n+1} is available.

$$\hat{X}_{n+1} = S_{n,\ n+1} * X_n$$

$$\hat{P}_{n+1} = S_{n,\ n+1} * (P_n + \Delta t * Q) * S^T_{n,\ n+1}$$

$$\hat{\theta}_{n+1} = A_{n+1} * \hat{X}_{n+1}$$

$$r = \hat{\theta}_{n+1} - \theta_{n+1}$$

$$s_r^2 = H_{n+1} * \hat{P}_{n+1} * H_{n+1} + [D\ (i,\ j)]^2$$

$$r/s_r = \text{edit check}$$

$$K_{n+1} = 1/s_r^2 * H_{n+1} * \hat{P}_{n+1}$$

$$X_{n+1} = \hat{X}_{n+1} - K_{n+1} * r$$

$$P_{n+1} = \hat{P}_{n+1} - K_{n+1} * H_{n+1} * \hat{P}_{n+1}$$

5. RAUCH-TUNG-STRIEBEL SMOOTHER

The smoothing equations use a backwards recursion involving the filter estimates for the state vector and covariance matrix to correct earlier estimates based on information received from later measurements.

To accomplish smoothing, the following information must be recorded during the Kalman filtering of the measurements:

$\hat{X}_n$ - predicted state vector at time t_n

X_n - corrected state vector using all measurements up to time t_n

$\hat{P}_n$ - predicted covariance matrix

P_n - corrected covariance matrix using all r measurements up to time t_n

$S_{n,\ n+1}$ - transition matrix expressing process effect on X_n to go from t_n to t_{n+1}

The smoothing equations correct the state vector and covariance matrix at time t_n based on the difference between their projection to time t_{n+1} and the smoothed values at time t_{n+1}. Suppose the last time for which data was available for the Kalman filter was t_{n+1} and that all measurements have been filtered through time t_{n+1}. Let $\bar{X}_{n+1}$, $\bar{P}_{n+1}$ denote smoothed estimates for the state vector and covariance matrix at time t_{n+1}. The first step in smoothing is:

$$\bar{X}_{n+1} = X_{n+1}$$
$$\bar{P}_{n+1} = P_{n+1}$$

To obtain the smoothed estimates at time t_n, first compute the gain matrix G:

$$G = P_n * S^T_{n,\ n+1} * \left[\hat{P}_{n+1}\right]^{-1}$$

where

P_n - filtered estimate of covariance matrix at time t_n

$S^T_{n\ \ n+1}$ - transpose of the transition matrix to project the state vector from time t_n to time t_{n+1}

$\left[\hat{P}_{n+1}\right]^{-1}$ - inverse of the covariance matrix predicted for time t_{n+1} based on P_n, the process noise from t_n to t_{n+1} and the transition matrix $S_{n,\ n+1}$

NOTE: GPSBET recomputes $S_{n, n+1}$ and $\hat{P}_{n+1}$ during the smoothing pass by recording $\Delta t = t_{n+1} - t_n$ and the deweighting matrix Q during the filtering pass. This saves on the amount of intermediate reading and writing since the off diagonal elements of Q are zero and specifying Δt is all that is required to construct $S_{n,n+1}$ (see Section 4.3). The smoothed estimate of the state vector at time t_n is:

$$\bar{X}_n = X_n + G * (\bar{X}_{n+1} - \hat{X}_{n+1})$$

The smoothed covariance matrix at time t_n is:

$$\bar{P}_n = P_n + G * (\bar{P}_{n+1} - \hat{P}_{n+1}) * G^T$$

This backwards recursion continues to the initialization time of the Kalman filter. If a reinitialization occurred during filtering then the smoothing process starts over just prior to reinitialization.

During the smoothing pass, an improved attitude matrix is computed from the smoothed velocities and acceleration (unless IMU measurements were used).

Also during the smoothing pass, measurement residuals are computed and recorded on magnetic tape. Measurement residuals are computed by converting the smoothed estimates of cartesian coordinates to the measurement domain as described in 4.3. The differences between these predictions and measurements (which are passed to smoother with the state vector and covariance matrix estimates) are computed.

6. OUTPUT AVAILABLE

Output from GPSBET is in the form of a line printer listing and an output tape. The output tape has two files for each trajectory. The first file contains all the program control information such as options selected, instrument locations, a priori information input to the Kalman filter, etc. The second file is a data file. The line printer listing contains the control information in the same form as the first file on the output tape. The data output on the line printer is controlled by input parameters.

Line printer output during the Kalman filter pass can include any or all of the following data types:

a. state vector estimates;
b. one-sigma uncertainties in state vector estimates;
c. full covariance matrix estimates;
d. measurement residuals;
e. summaries of measurement residuals over selected intervals;
f. edit messages when editing occurs;
g. identification of retro switches.

Data types f. and g. are printed whenever they occur. Types a. through e. are printed at selected time intervals and/or after filtering a particular number of measurements. The time intervals and points between output are selected by changing input parameters.

During the smoothing pass line printer output consists of the state vector and sigma vector (1-sigma uncertainties in state vector estimate) as computed during the filter pass and as computed in the smoothing pass plus differences in the state vector and the F-ratio of the variances.

A complete description of the output tape from GPSBET is contained in Appendix B. A sample of the line printer output is in Appendix C.

7. VALIDATION

The GPSBET program has undergone extensive validation to ensure that the Kalman filter and smoothing equations are implemented correctly and that the a priori information is incorporated properly. The validation was accomplished in two stages. First, YPG tested and debugged the program to the extent that computation of a trajectory from a single PATS laser compared favorably with YPG's previous reduction programs. The next stage of validation was called the end-around-check. The end-around-check was a joint effort of The Aerospace Corporation and YPG. The Aerospace Corporation was under contract to the Space and Missile Systems Organization (SAMSO) of the US Air Force. SAMSO's interest in YPG's BET program stemmed from the NAVSTAR Global Positioning System (GPS) project. GPS intended to (and does now) use GPSBET output as a reference trajectory for comparison with the NAVSTAR solution for position and velocity.

The end-around-check included:

a. The Aerospace Corporation generated simulated trajectories similar to the flight paths to be flown during the GPS project. Three laser locations were simulated and the X,Y,Z coordinates of the simulated trajectories were converted to range, azimuth, elevation from each of the three laser locations. Random number generators were used to generate noise which was added to the measurements. A small percentage of the measurements were changed to random numbers to simulate data dropouts.

b. The simulated laser measurements were transmitted to YPG via magnetic tape in the same format as actual laser measurements were being recorded at YPG.

c. YPG personnel processed the simulated laser measurements with the GPSBET program, producing an output tape and line printer listing.

d. The output tape from GPSBET was sent to The Aerospace Corporation.

e. The Aerospace Corporation differenced YPG's output tape with the true trajectory from which the measurements were generated.

f. The Aerospace Corporation analyzed the differences between truth and the estimates made by the GPSBET program.

g. The Aerospace Corporation produced reports of the differences, their analysis of the differences and suggestions for changes to GPSBET.

h. The above cycle was repeated until The Aerospace Corporation, SAMSO and YPG were confident that GPSBET was filtering and smoothing data as well as Kalman filter theory allows (subject to some time and money constraints).

i. All of the above steps were repeated with the same trajectories but using simulated inertial platform measurements in addition to laser measurements.

The results of the end-around-checks are summarized in Tables 1 and 2 which contain the RMS of the difference between truth and GPSBET estimates for two trajectories called I and II and for two types of data A and B. Trajectory I was a gentle .5g turn, whereas trajectory II was a figure-8 with a 4.5g turn on one end and a 3.5g turn on the other in addition to 1g vertical climbs and descents. Cases IA and IIA were using only simulated laser data whereas Cases IB and IIB were using simulated data from 3 lasers plus simulated inertial platform data. Table 1 summarizes how well the filter only estimates compared with truth, whereas Table 2 summarizes the results of filtering and smoothing. Timing results in Section 8.3 show how expensive the smoothing pass is.

TABLE 1*

FILTER ONLY

RMS OF DIFFERENCES WITH TRUTH

	UNITS	IA	IB	IIA	IIB
X	m	.07	.02	.40	.24
Y	m	.03	.01	.30	.17
Z	m	.03	.01	.48	.24
$\dot{X}$	m/sec	.18	.01	1.40	.45
$\dot{Y}$	m/sec	.08	.01	1.01	.33
$\dot{Z}$	m/sec	.02	.00	1.23	.28
$\ddot{X}$	m/sec^2	.27	.07	3.24	1.76
$\ddot{Y}$	m/sec^2	.11	.03	2.13	1.22
$\ddot{Z}$	m/sec^2	.02	.00	1.99	.91

* These are results of analysis by The Aerospace Corporation and were taken from DPDWG Technical Memorandums prepared by H. Bernstein, The Aerospace Corporation.

TABLE 2*

FILTER AND SMOOTH

RMS OF DIFFERENCES WITH TRUTH

	UNITS	IA	IB	IIA	IIB
X	m	.02	.01	.23	.11
Y	m	.01	.01	.17	.08
Z	m	.01	.01	.28	.11
$\dot{X}$	m/sec	.01	.00	.38	.10
$\dot{Y}$	m/sec	.01	.01	.30	.09
$\dot{Z}$	m/sec	.01	.00	.40	.09
$\ddot{X}$	m/sec^2	.03	.02	1.13	.64
$\ddot{Y}$	m/sec^2	.02	.01	.89	.50
$\ddot{Z}$	m/sec^2	.01	.00	.90	.43

* These are results of analysis by The Aerospace Corporation and were taken from DPDWG Technical Memorandums prepared by H. Bernstein, The Aerospace Corporation.

8. PERFORMANCE USING ACTUAL DATA

The performance of GPSBET using actual data is monitored very closely since YPG customers depend on the absolute accuracy of the reference trajectory that GPSBET produces. YPG maintains extensive histories of laser calibration coefficients, monitors measurement noise, computes process noise, studies measurement redisuals, plots inter-laser differences, plots single laser solutions against GPSBET solutions, etc., to ensure that GPSBET output is as good as the measurements allow. In addition to the internal checks mentioned above, YPG invites comparisons with instrumentation and reduction procedures external to the laser systems and YPG software. Sections 8.1 and 8.2 describe some examples of the internal and external comparisons and analysis that YPG has done and will continue to do to verify the accuracy of GPSBET estimates.

8.1 INTERNAL ANALYSIS

Extensive analysis of GPSBET accuracy must be performed since range users expect 0.5 meter accuracy in position estimates. The most important requirement for attaining this accuracy is pre- and post-mission calibration of all lasers to update calibration histories and allow analysis of potential problems. Another requirement is complete meteorological data for accurate atmospheric corrections. A priori information about measurement noise and process noise must be verified. YPG's real-time data collection, computation and display facility provides a means of acquiring the information required for an accurate trajectory estimate.

After a real-time mission, the standard procedure is to generate quick-look plots of the trajectory data. These plots include at least:

X	vs Y
Y	vs Z
X,Y,Z	vs time
$\dot{X},\dot{Y},\dot{Z}$	vs time
RTE source	vs time

The quick-look plots are generally available within 4 hours after a 2-hour mission. These plots can be used to adjust the a priori information for GPSBET.

In many cases, the Real-Time Estimate (RTE) of the trajectory of an aircraft is more than adequate to meet customer accuracy requirements. In particular, when a customer uses only a single laser for data acquisition, then the RTE is almost as accurate as the GPSBET for position and velocity. The Kalman filter and smoother output is however, less noisy and contains good acceleration estimates. Figure 1 shows an X vs. Y and Y vs. Z plot of a segment of a helicopter trajectory for which some special analysis was done. A single laser (Site 7 laser) was forced to be used for the RTE (the source selection algorithm used in real time was manually overriden) and the GPSBET was run using data only from Site 7 laser. Figures 2, 3, 4 show the X, Y, Z differences between the RTE and BET processing when both used data from Site 7 laser only. Differences are attributable to laser data dropouts, noise in the estimates, some differences in the level of sophistication in atmospheric correction and computational errors due to roundoff and truncation.

For a 200-second segment of the trajectory shown in Figure 1, a three laser BET was computed. The three laser BET was differenced with each of the individual laser solutions as computed in real time. Plots

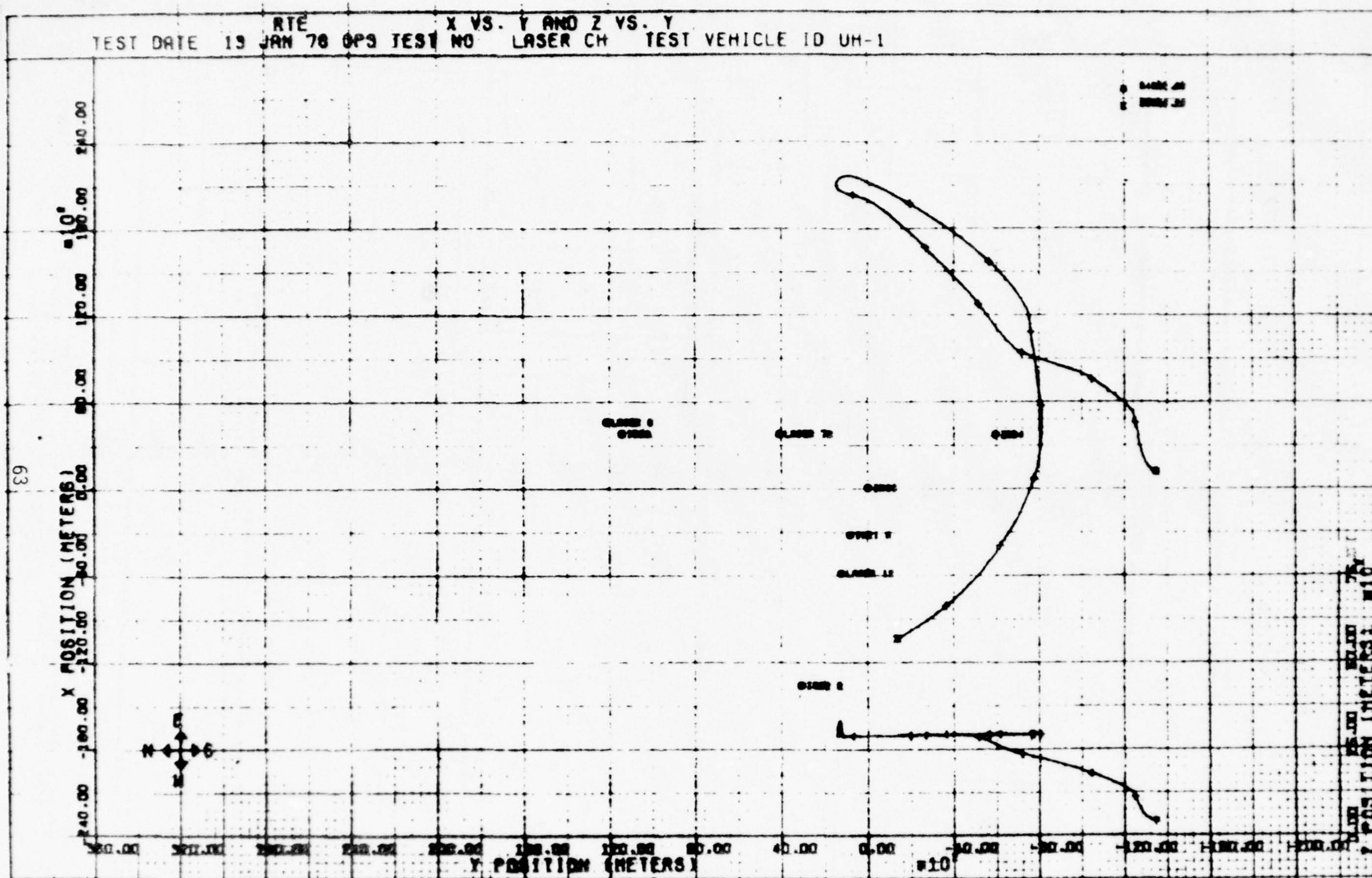

RTE
X VS. Y AND Z VS. Y
TEST DATE 19 JAN 78 OPS TEST NO LASER CH TEST VEHICLE ID UH-1
X POSITION (METERS)
Y POSITION (METERS)
Z POSITION (METERS)

Figure 1

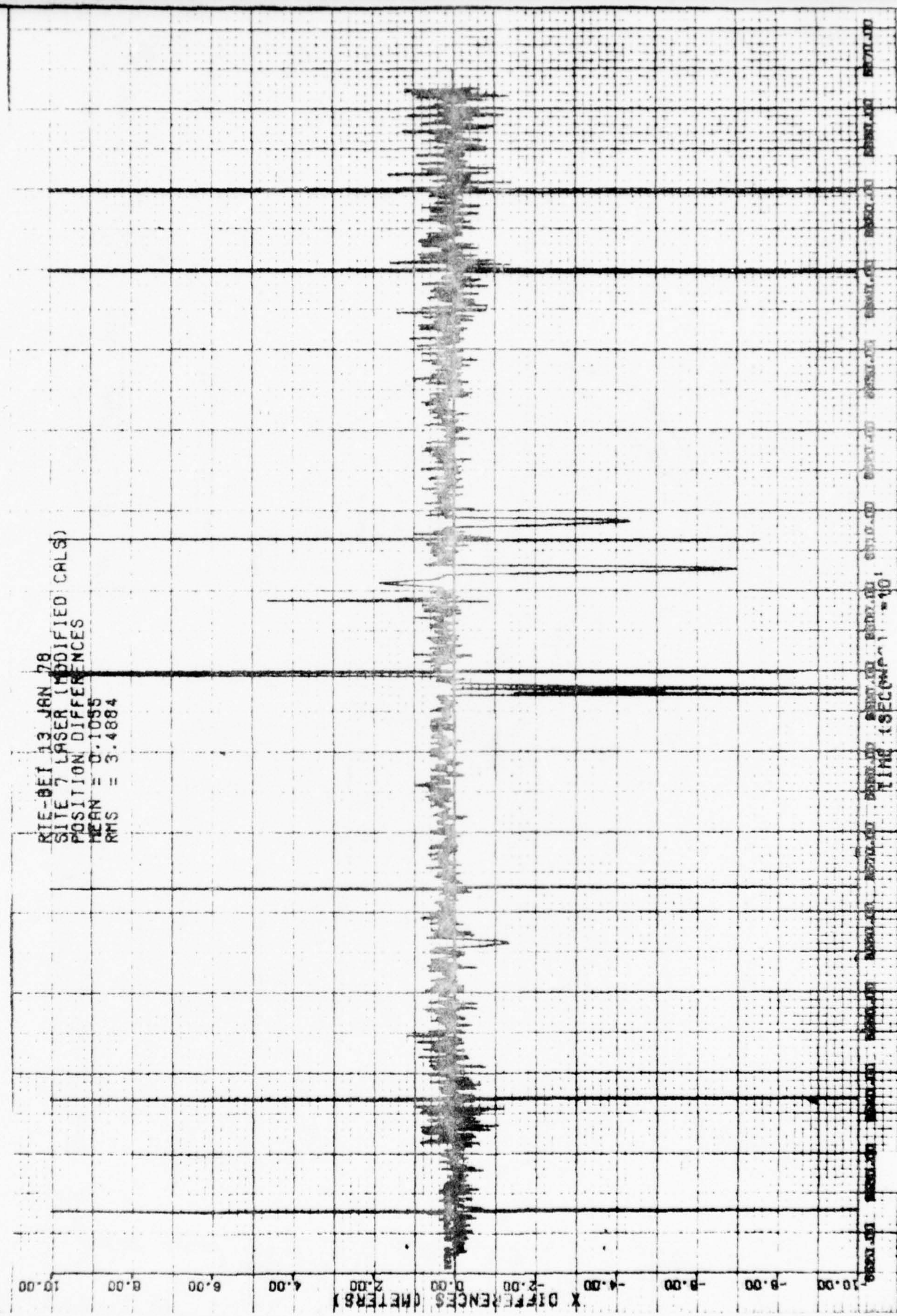

RIE-BET 13 JAN 78
SITE 7 LASER (MODIFIED CALS)
POSITION DIFFERENCES
MEAN = 0.1055
RMS = 3.4884
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
X DIFFERENCES (METERS)
TIME (SECONDS) *10

Figure 2

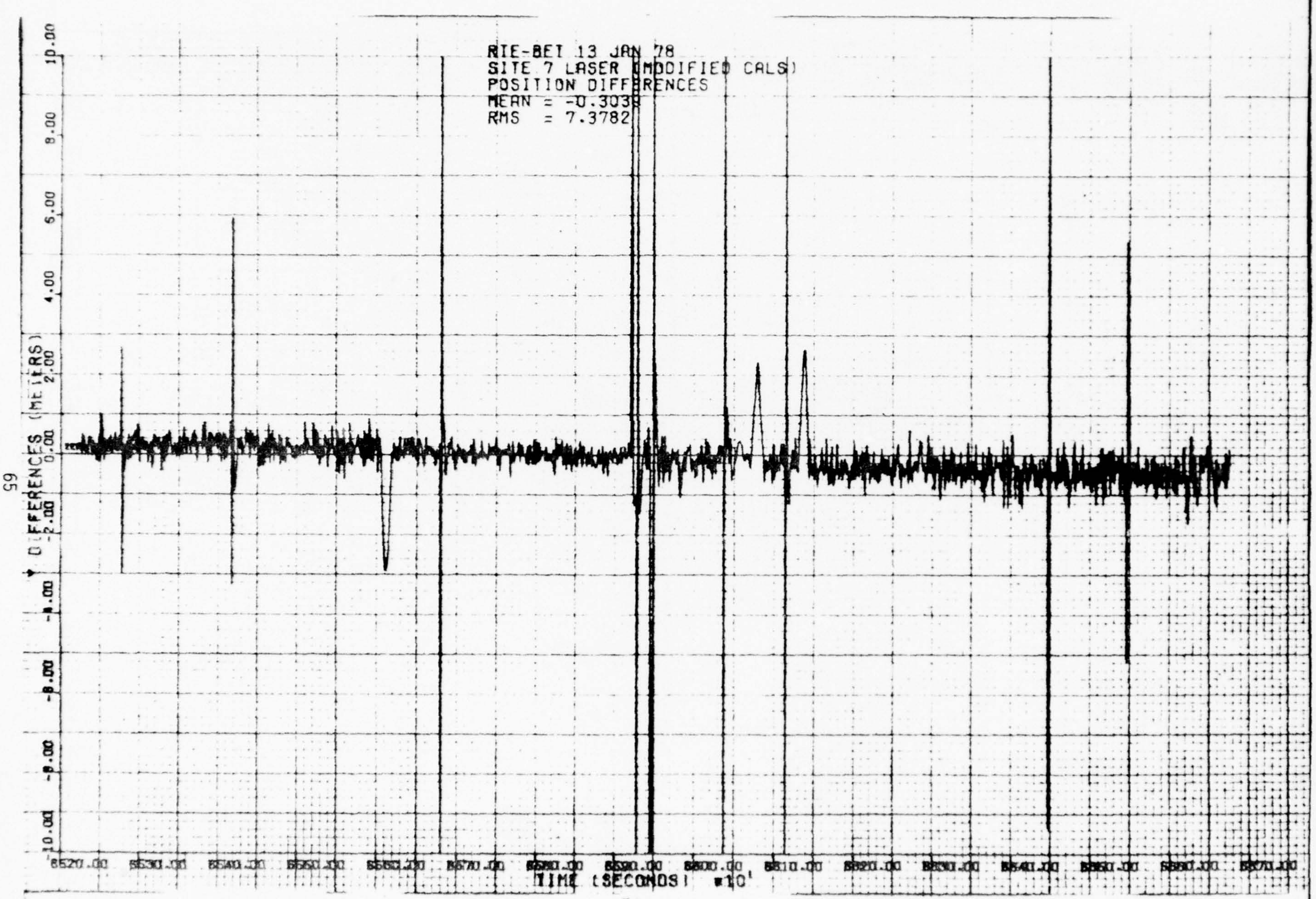

RIE-BET 13 JAN 78
SITE 7 LASER (MODIFIED CALS)
POSITION DIFFERENCES
MEAN = -0.3039
RMS = 7.3782
DIFFERENCES (METERS)
TIME (SECONDS) *10'

Figure 3

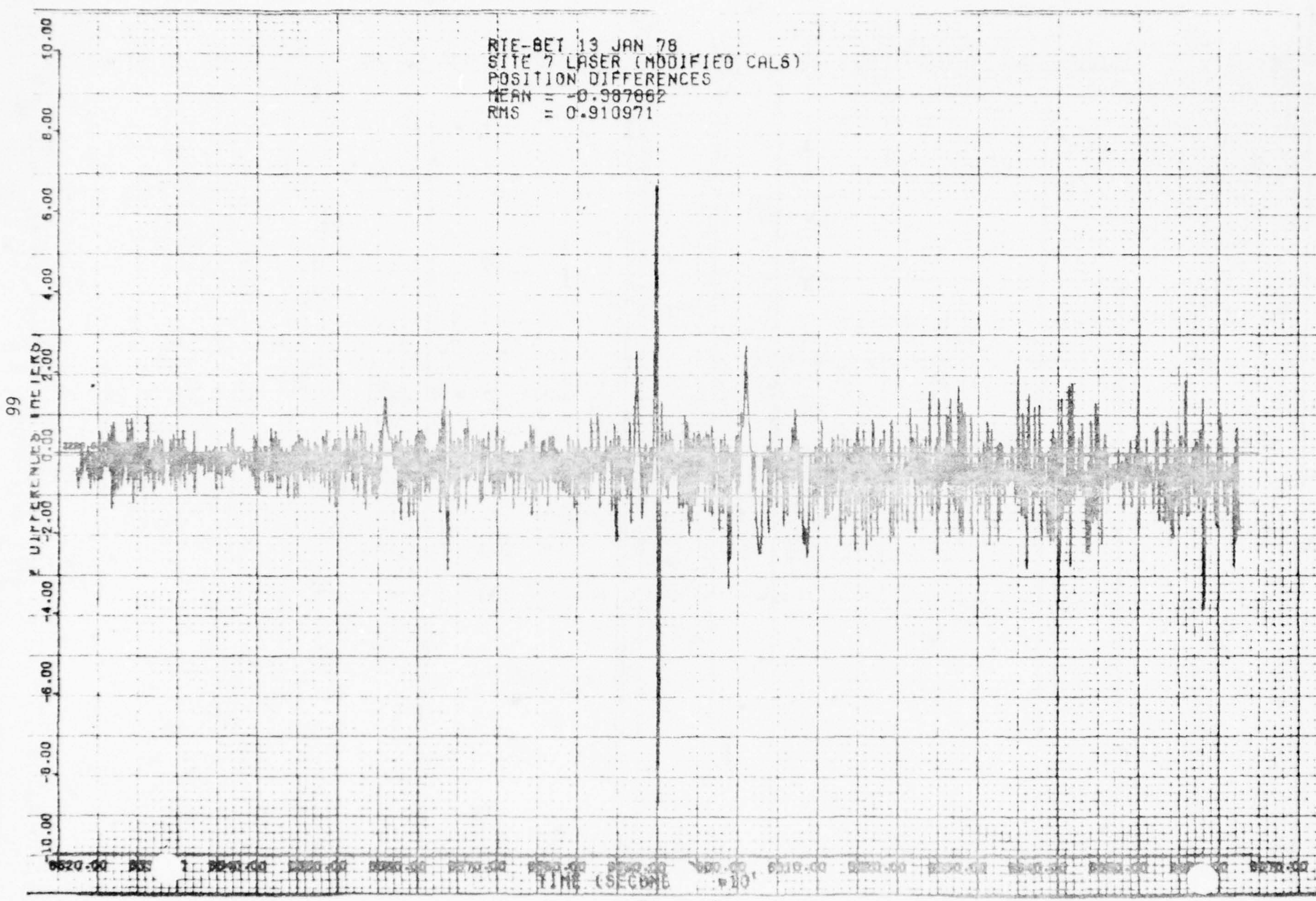

RTE-BET 13 JAN 78
SITE 7 LASER (MODIFIED CALS)
POSITION DIFFERENCES
MEAN = -0.987862
RMS = 0.910971
X DIFFERENCES (METERS)
TIME (SECONDS

Figure 4

of the differences appear in Figures 5-13. Figures 5, 6, 7 are the X, Y, Z differences between the Site 7 laser solution computed in real time and the three laser BET. Figures 8, 9, 10 are the X, Y, Z differences between the Site 9 laser solution computed in real time and the three laser BET. Figures 11, 12, 13 are the X, Y, Z differences between the Site 12 laser solution computed in real time and the three laser BET.

Contained on the output tape from the three laser BET run made on the 200-second segment of trajectory referred to above were the measurement residuals for the three lasers. Measurement residuals are computed by transforming the best estimate of X, Y, Z at each measurement time to range, azimuth, elevation from each laser and differencing them with the measurements. Figures 14-22 contain plots of the range, azimuth, elevation residuals for Sites 7, 9, 12. Residuals plots show what the magnitude of the measurement noise is and help to identify calibration problems. Notice that noise on the range measurements is about .5 meter and on azimuth and elevation is about .1 milliradian. The residuals plots in Figures 14-22 were generated to analyze a suspected accuracy problem and provided additional verification of a Site 12 azimuth encoder problem which was subsequently corrected.

8.2 COMPARISON WITH OTHER INSTRUMENTATION AND PROCESSING

GPSBET trajectory estimates have been compared with many other types of processing using the laser instrumentation and other YPG instrumentation such as cinetheodolites, PLS (Position Location System, which is an RMS II System manufactured by General Dynamics) and several radars. Also, comparisons were made with an airborne navigation solution generated by


SITE 7 RTE-S LASER BET.
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = -0.3560
RMS = 3.7199
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
DIFFERENCES (METERS)
TIME (SECONDS)

Figure 5

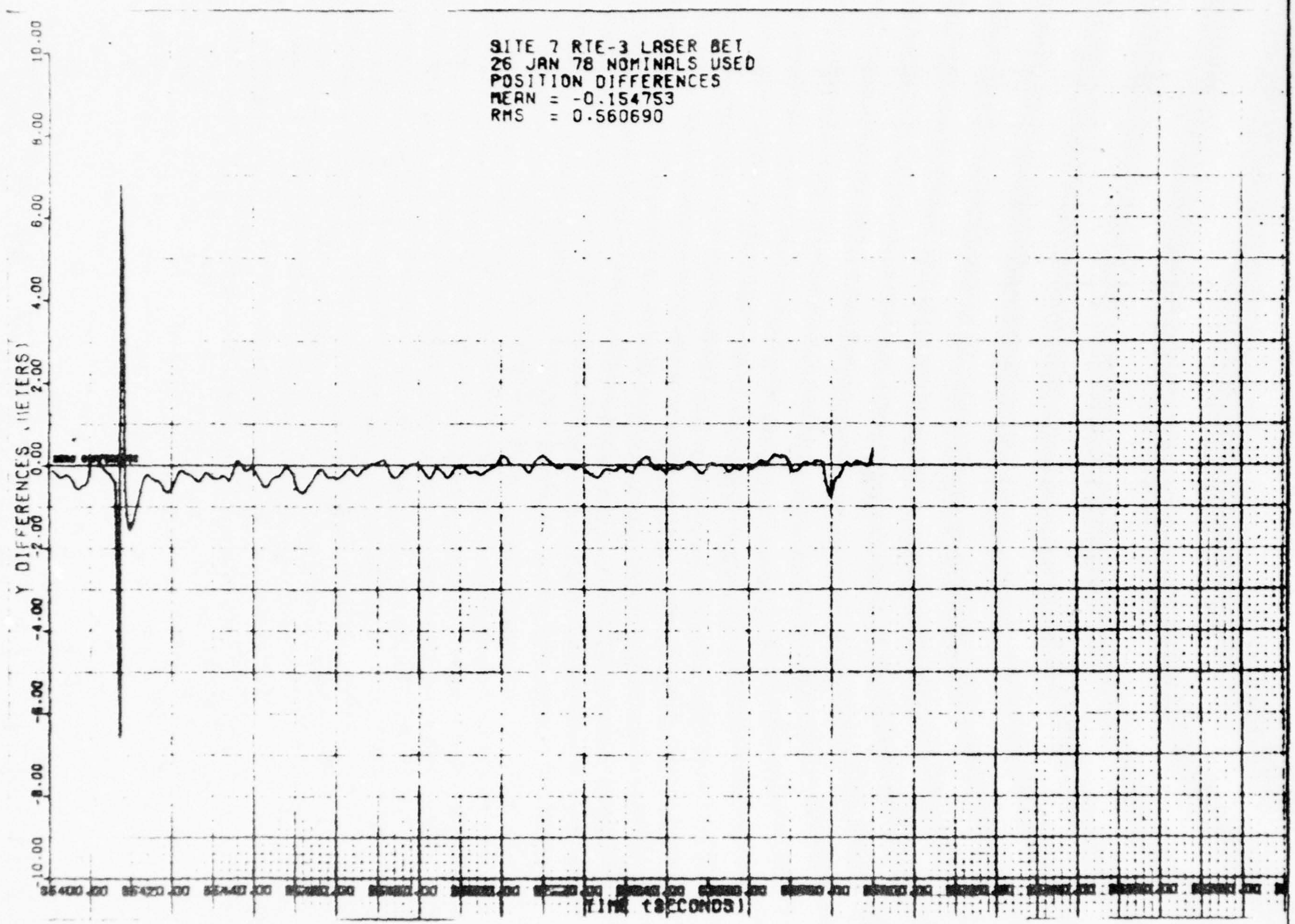

SITE 7 RTE-3 LASER BET
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = -0.154753
RMS = 0.560690
Y DIFFERENCES (METERS)
TIME (SECONDS)
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00

Figure 6

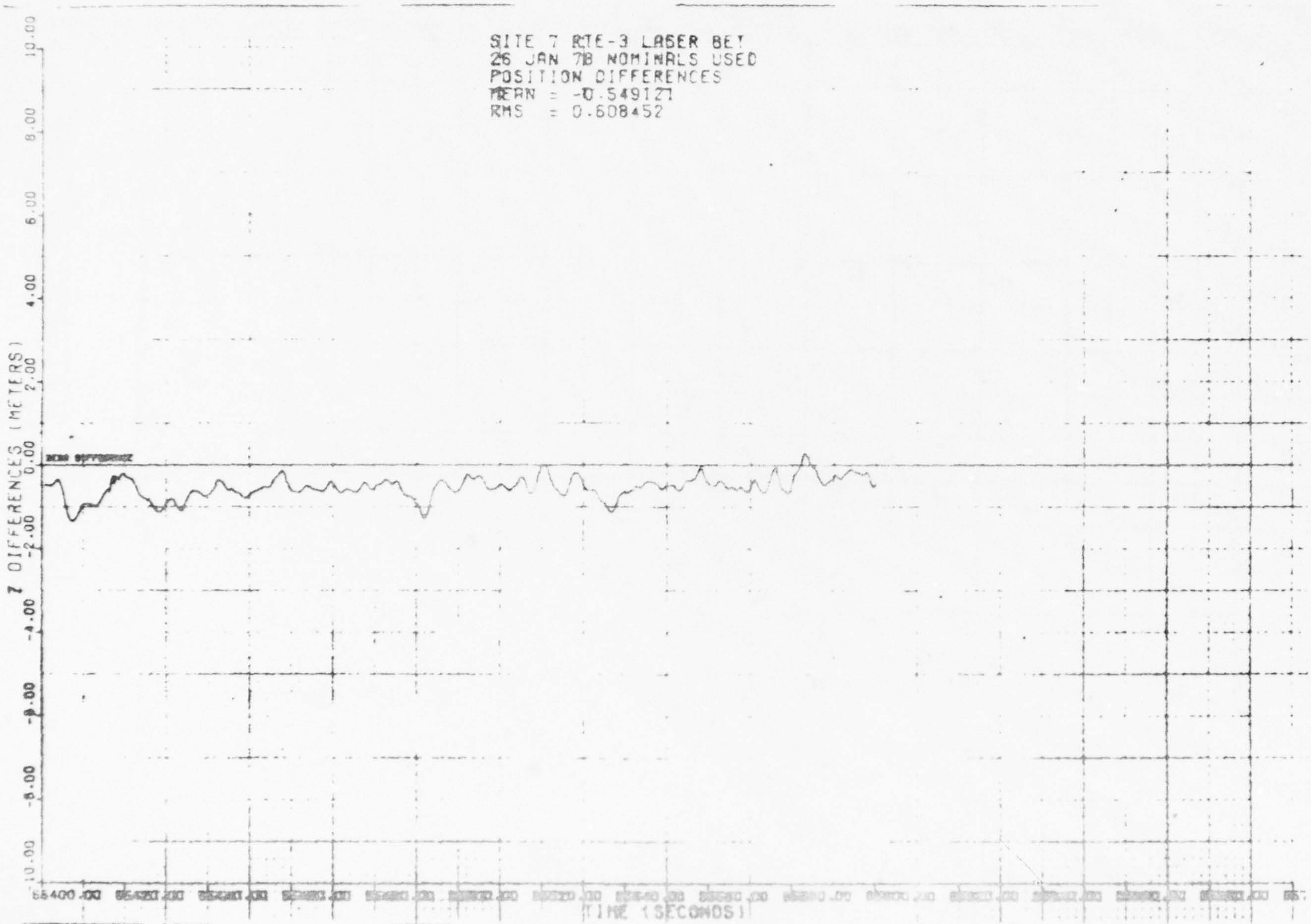

SITE 7 RTE-3 LASER BET
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = -0.549121
RMS = 0.608452
Z DIFFERENCES (METERS)
TIME (SECONDS)

Figure 7

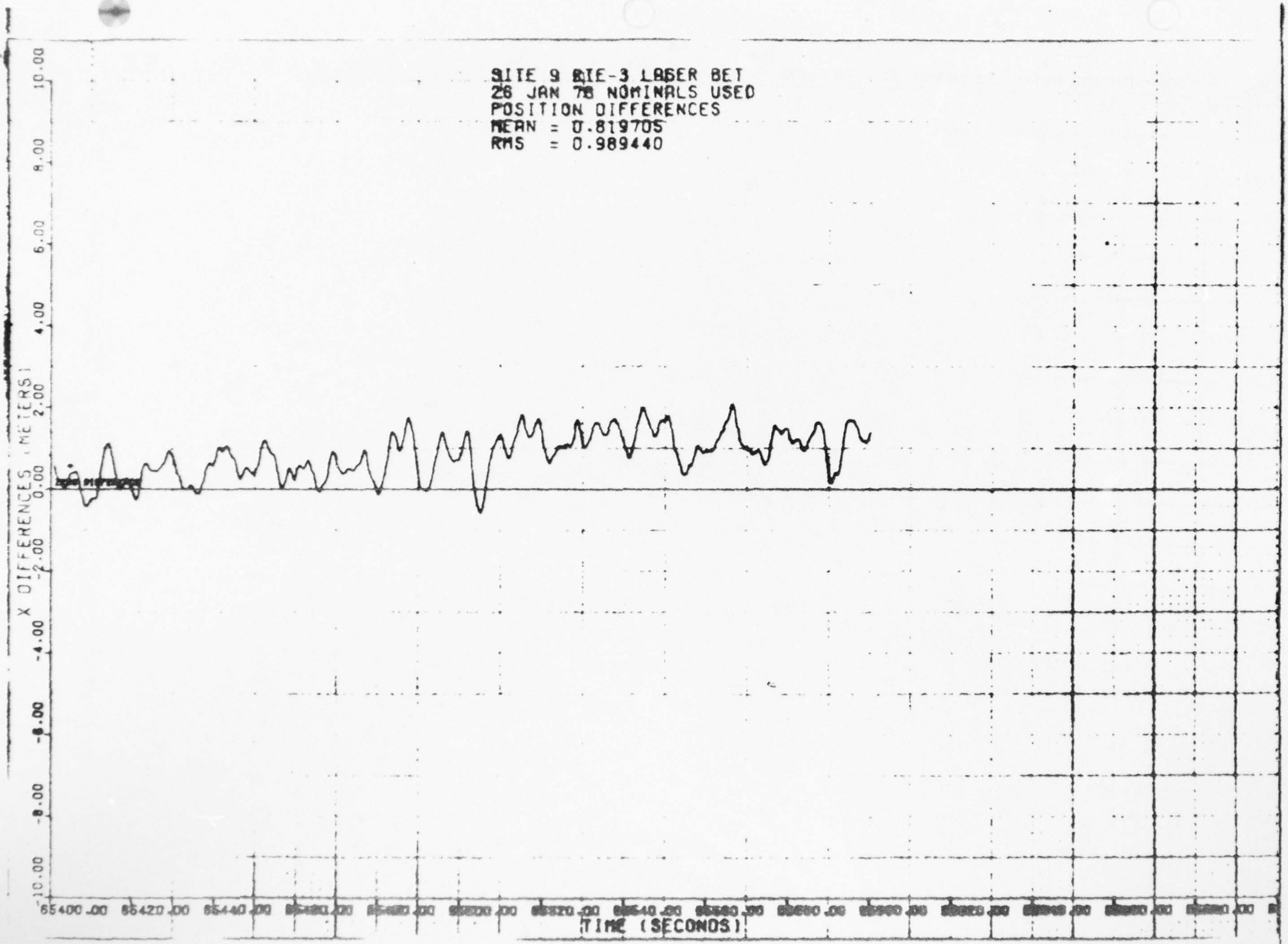
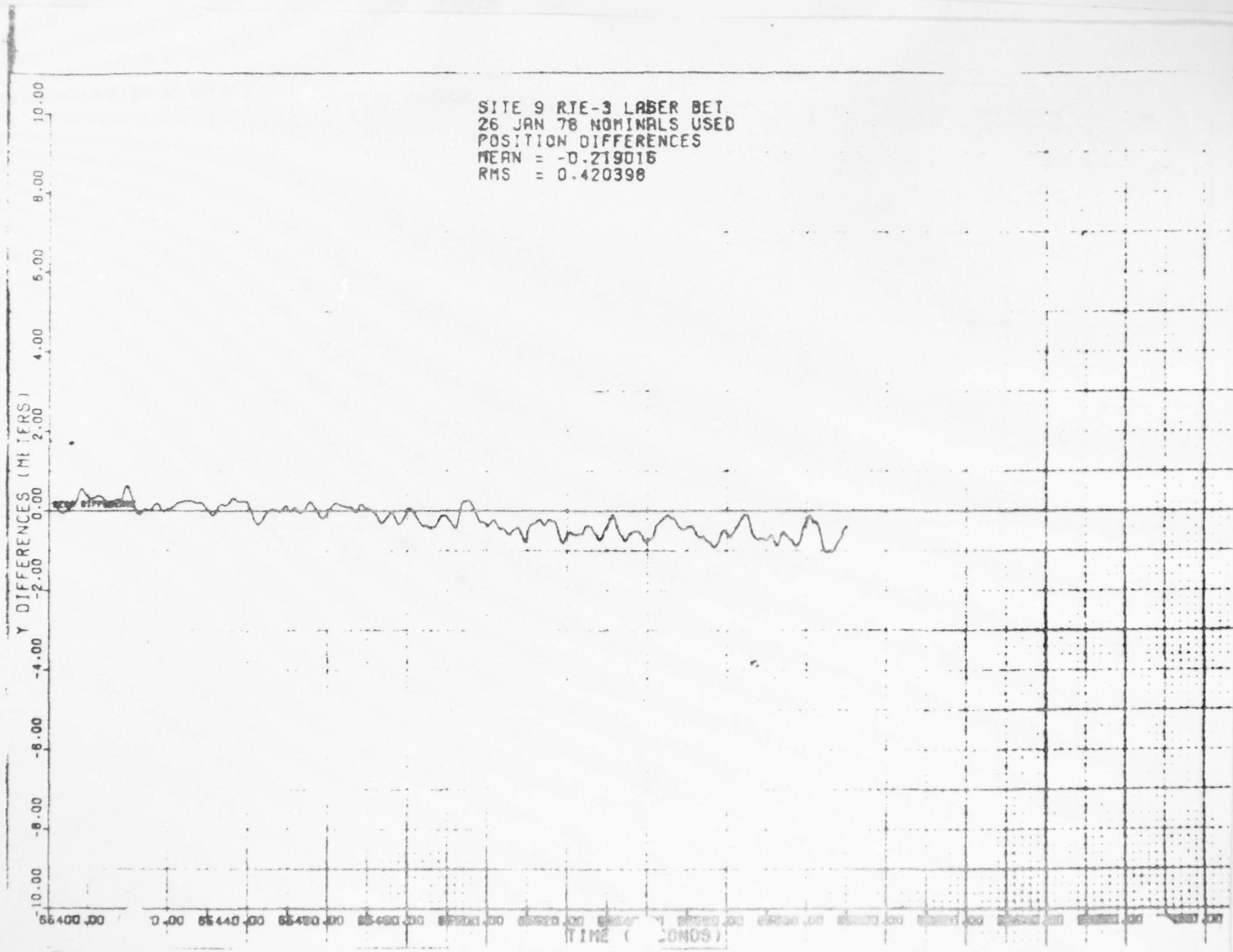

SITE 9 BIE-3 LASER BET
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = 0.819705
RMS = 0.989440
X DIFFERENCES (METERS)
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
65400.00
65420.00
65440.00
TIME (SECONDS)

Figure 8


SITE 9 RTE-3 LASER BET
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = -0.219016
RMS = 0.420398
Y DIFFERENCES (METERS)
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
65400.00
65440.00
65480.00
TIME (SECONDS)

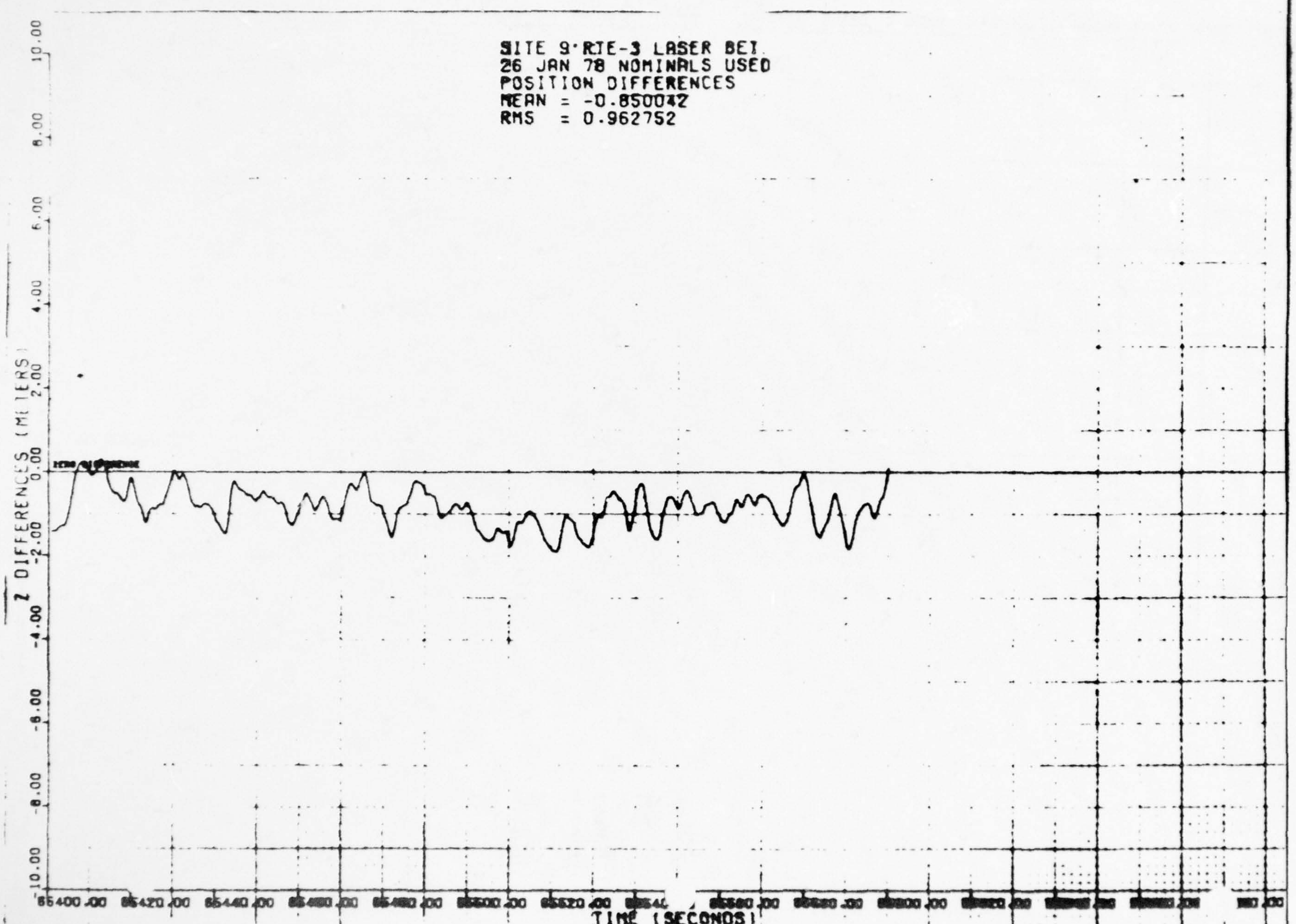

SITE 9 RIE-3 LASER BEI
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = -0.850042
RMS = 0.962752
Z DIFFERENCES (METERS)
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
65400.00
65420.00
65440.00
TIME (SECONDS)

Figure 10

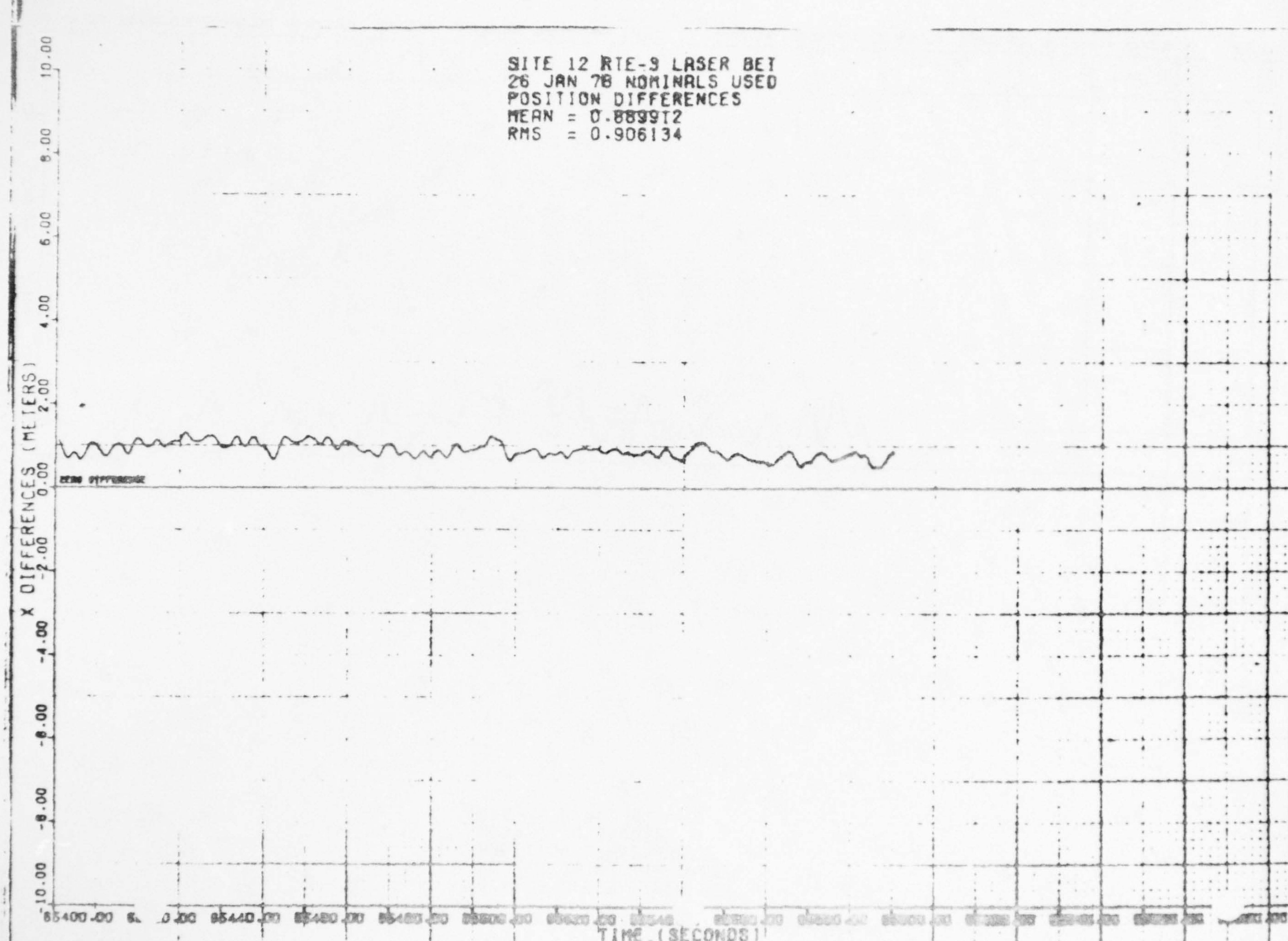

SITE 12 RTE-9 LASER BET
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = 0.889912
RMS = 0.906134
X DIFFERENCES (METERS)
TIME (SECONDS)

Figure 11

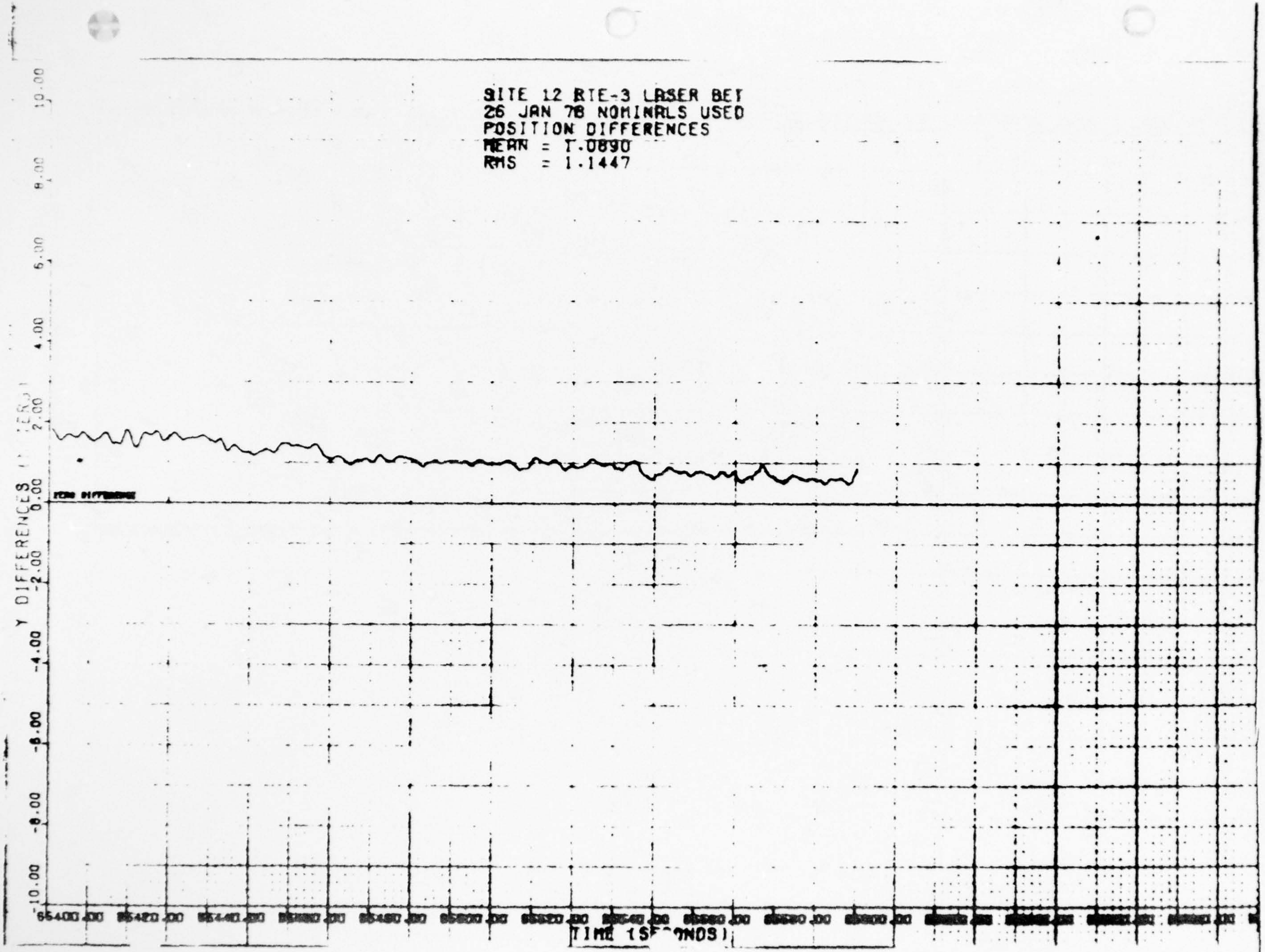

SITE 12 BTE-3 LASER BET
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = 1.0890
RMS = 1.1447
10.00
6.00
4.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
Y DIFFERENCES
TIME

Figure 12

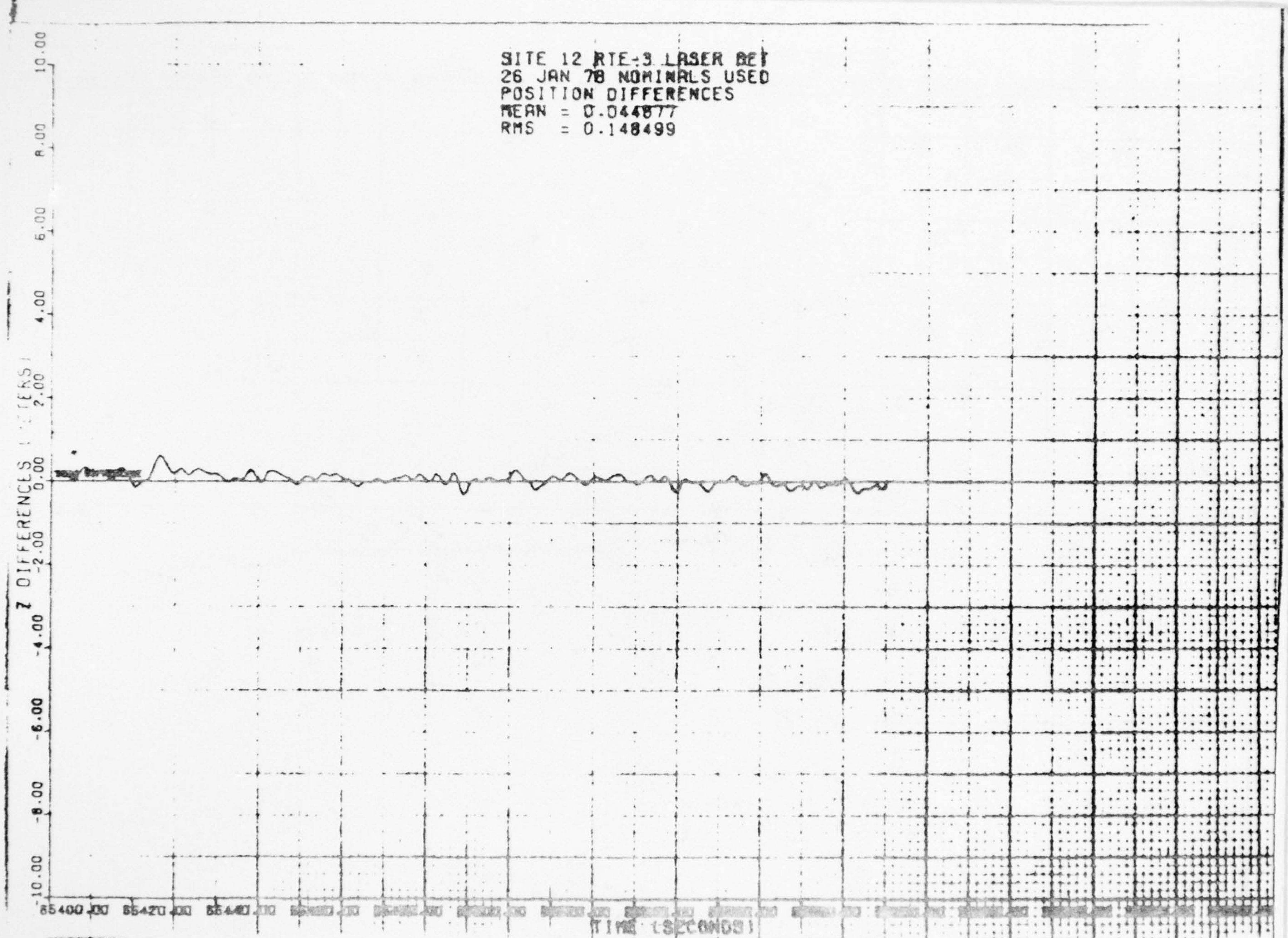

SITE 12 RTE-3 LASER BET
26 JAN 78 NOMINALS USED
POSITION DIFFERENCES
MEAN = 0.044877
RMS = 0.148499
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
Z DIFFERENCES (METERS)
85400.000
85420.000
85440.000
TIME (SECONDS)

Figure 13

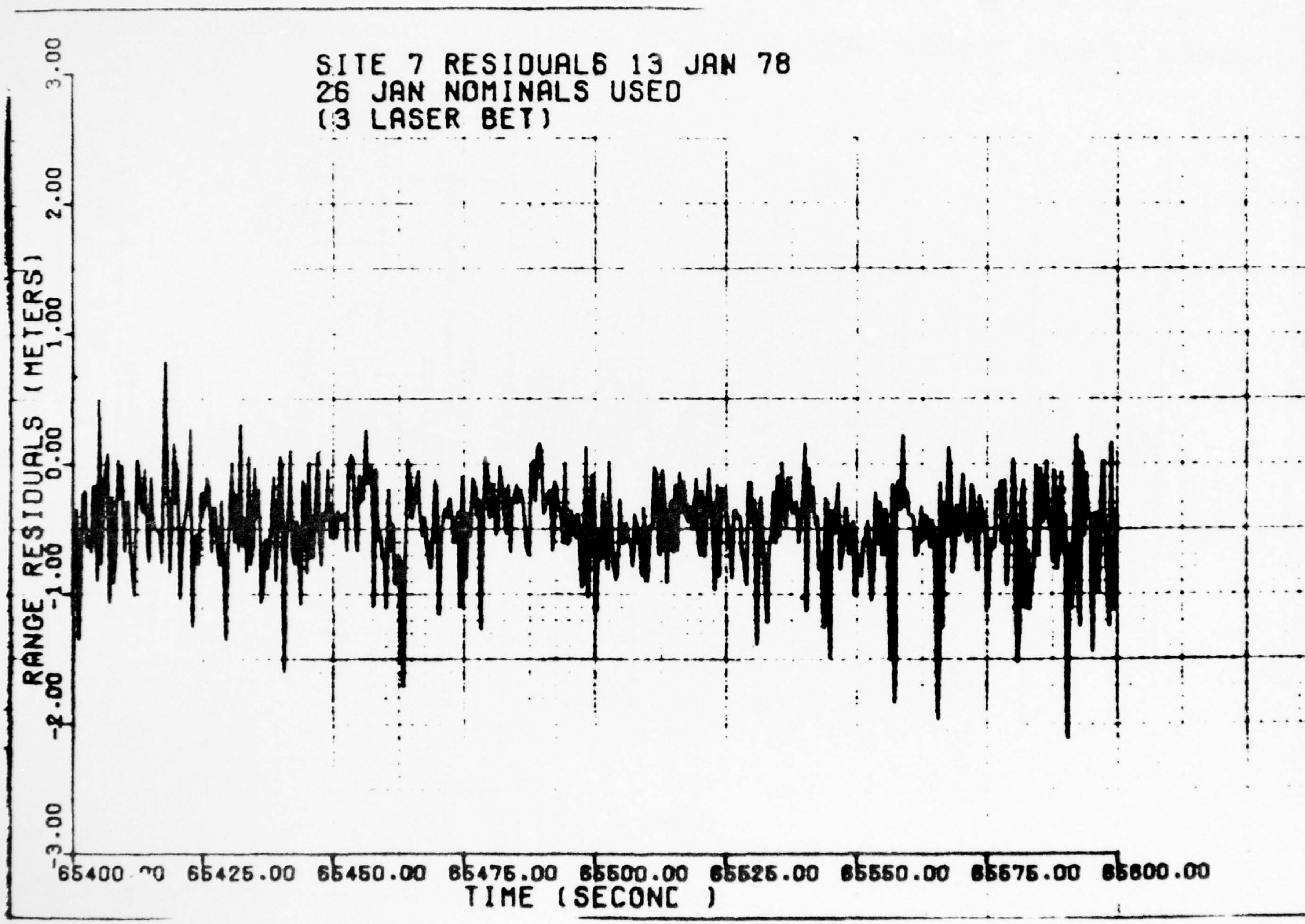

SITE 7 RESIDUALS 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
RANGE RESIDUALS (METERS)
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONC)

Figure 14

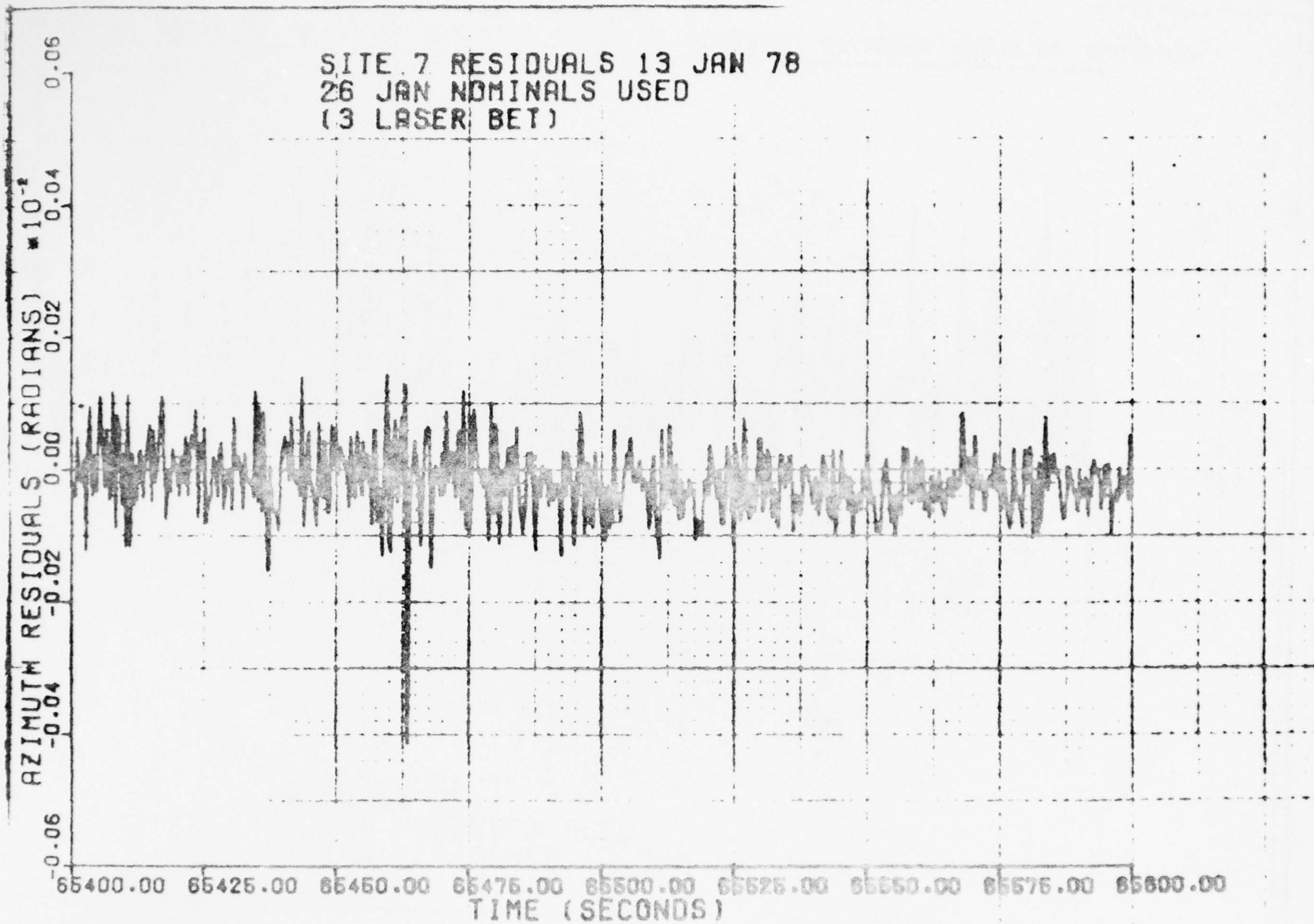

SITE 7 RESIDUALS 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
AZIMUTH RESIDUALS (RADIANS) *10⁻²
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONDS)

Figure 15

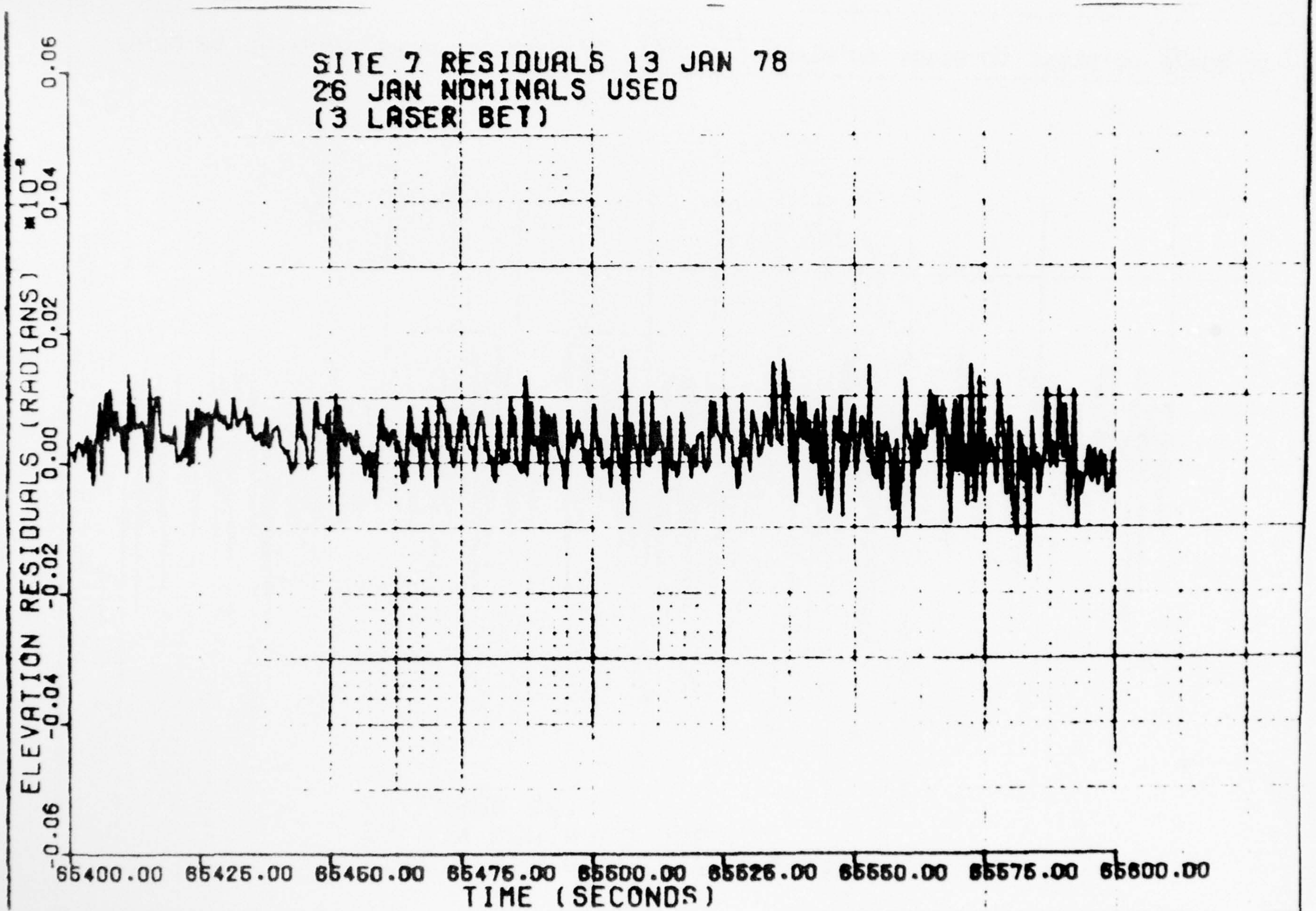

SITE 7 RESIDUALS 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
ELEVATION RESIDUALS (RADIANS) *10⁻²
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONDS)

Figure 16

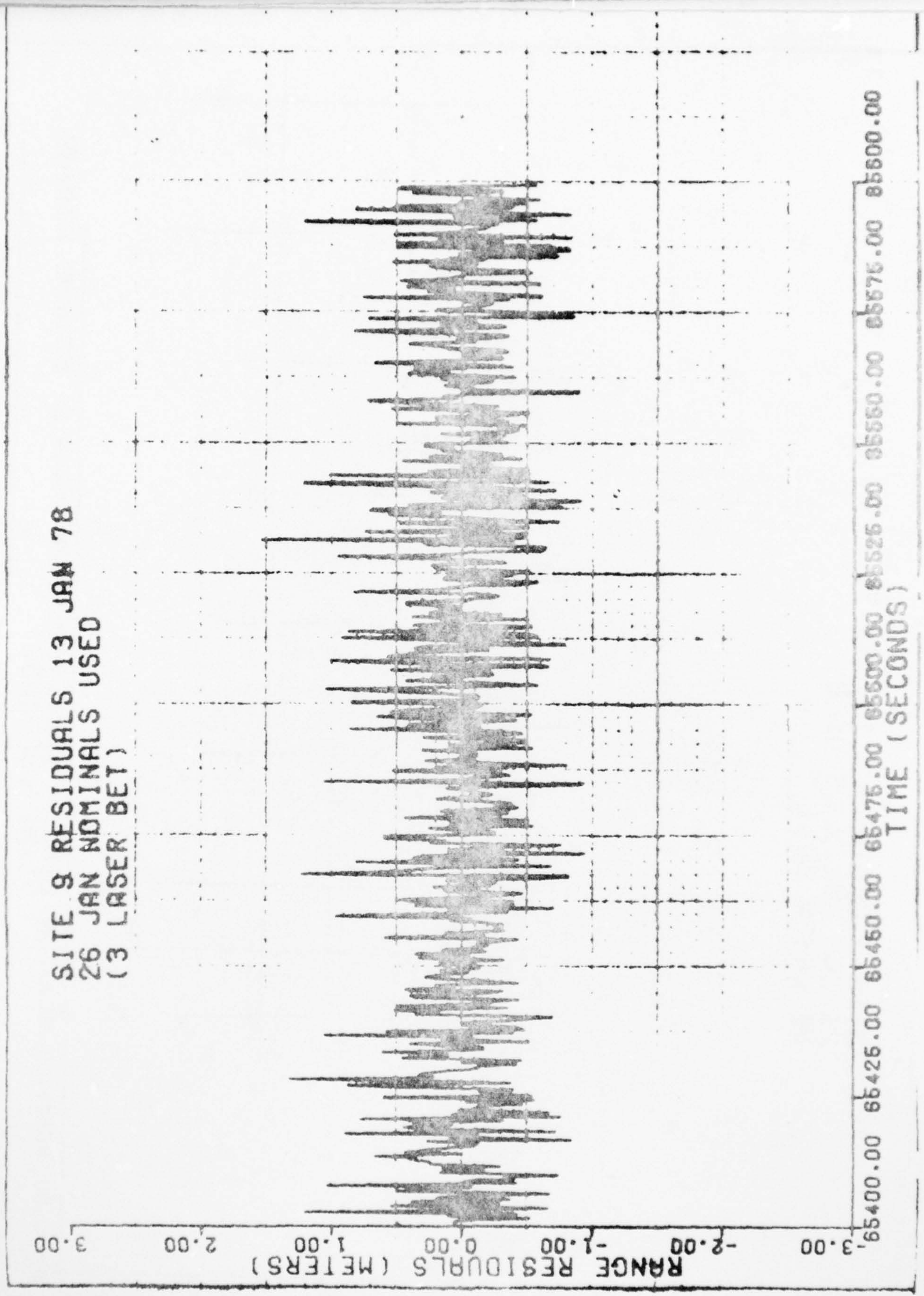

SITE 9 RESIDUALS 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONDS)
-3.00
-2.00
-1.00
0.00
1.00
2.00
3.00
RANGE RESIDUALS (METERS)

Figure 17

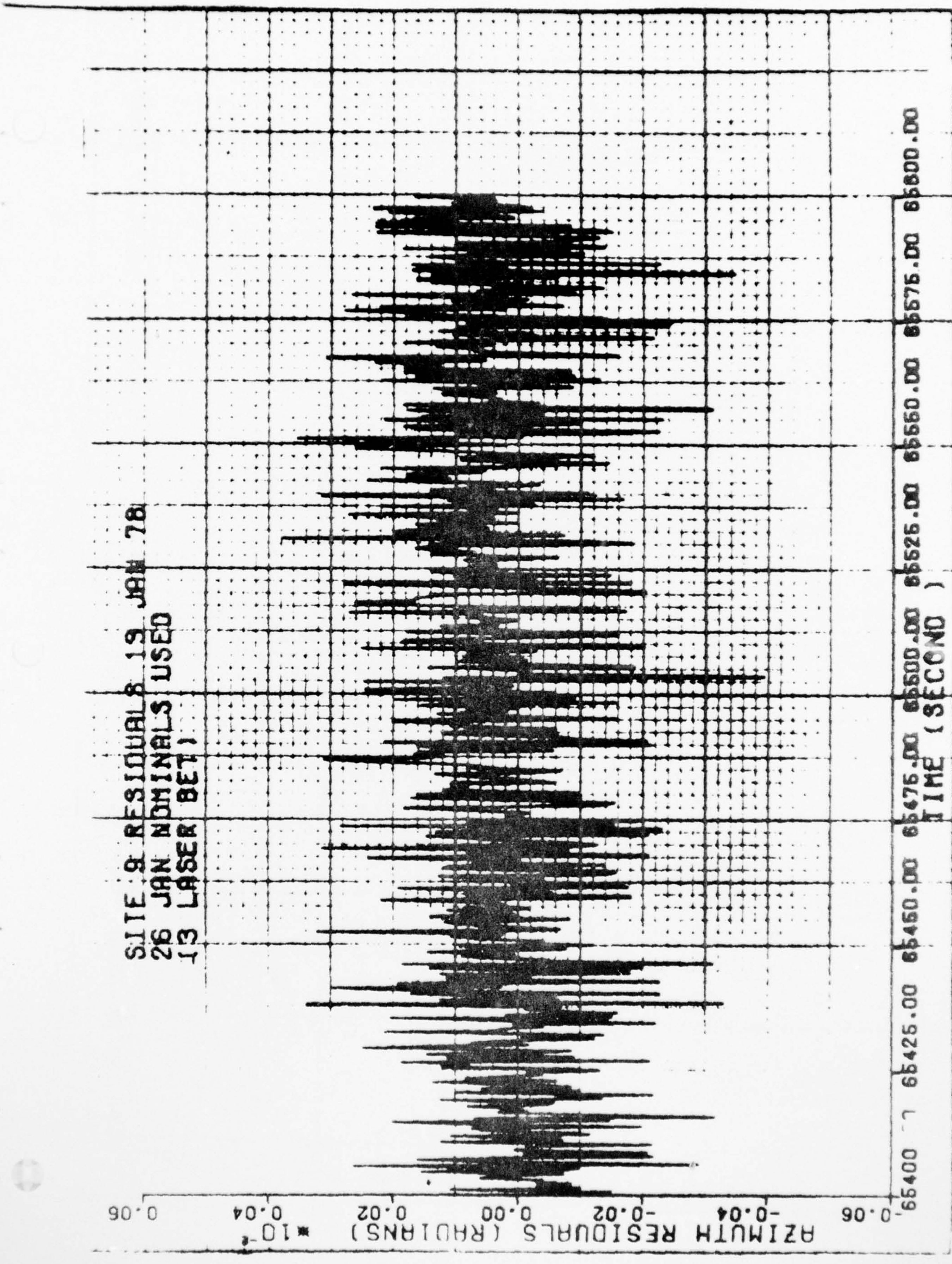

SITE 9 RESIDUALS 19 JAN 78
26 JAN NOMINALS USED
13 LASER BET)
AZIMUTH RESIDUALS (RADIANS) *10
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
65400
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECOND)

Figure 18

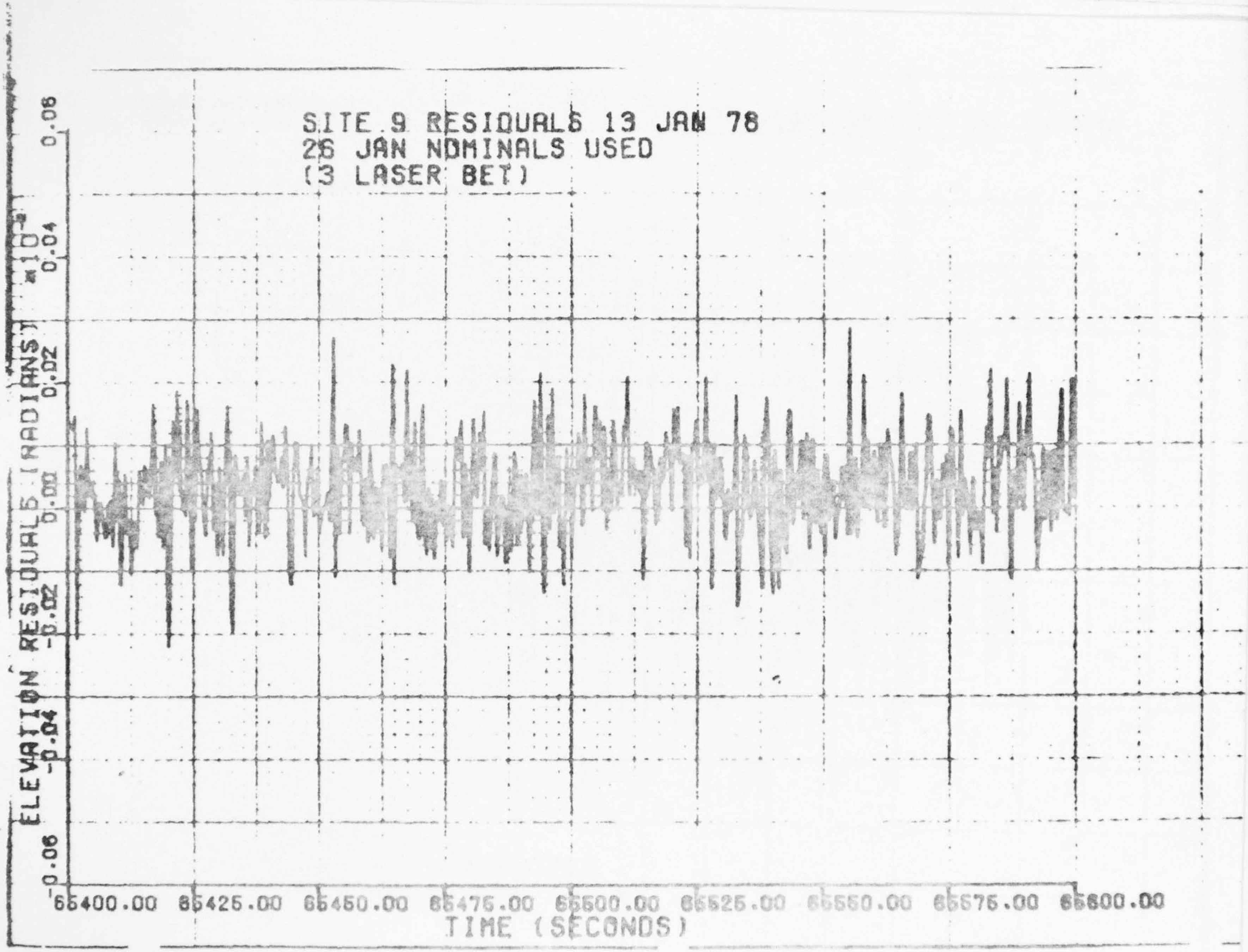

SITE 9 RESIDUALS 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
ELEVATION RESIDUALS (RADIANS) *10⁻³
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONDS)

Figure 19

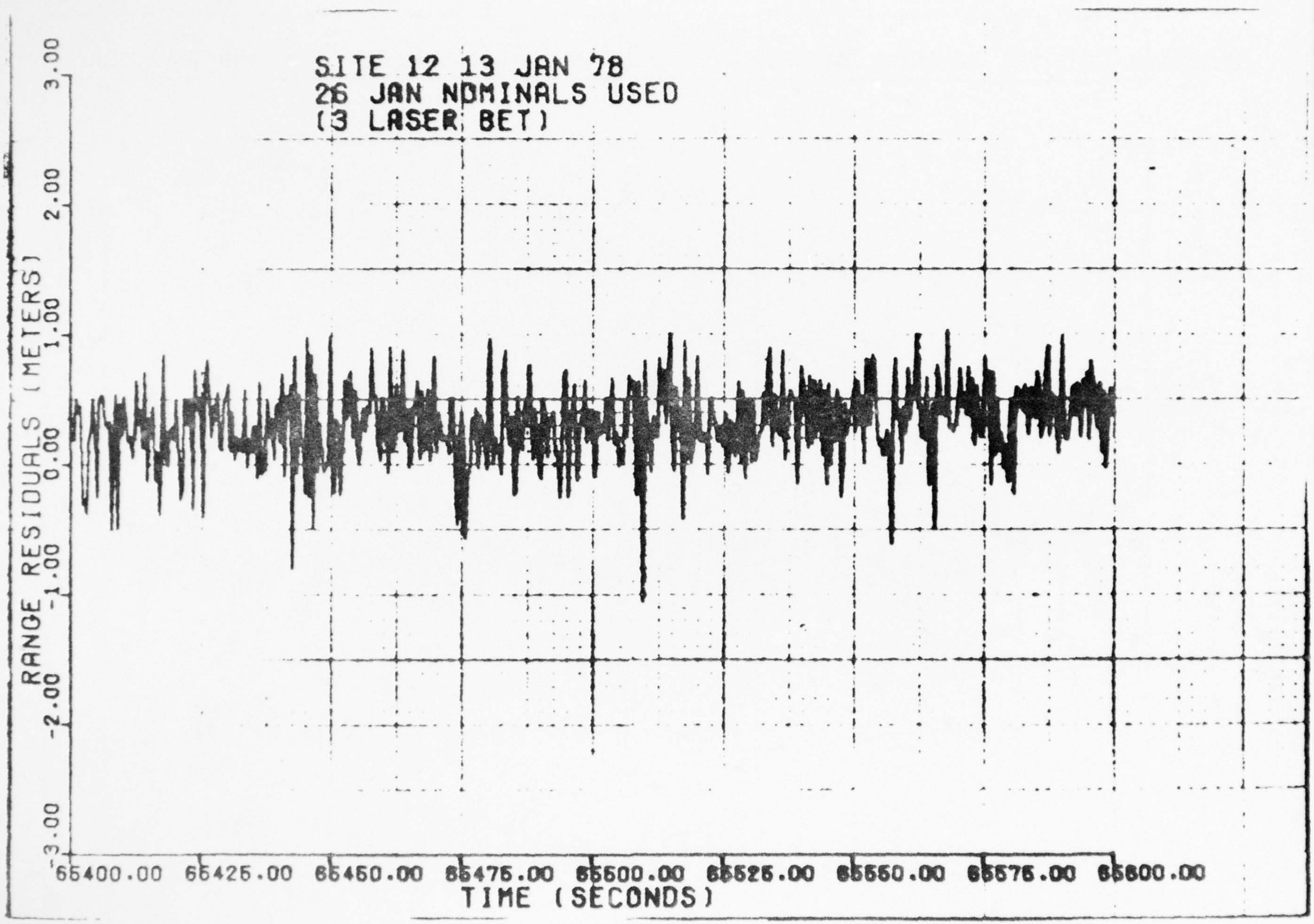

SITE 12 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
RANGE RESIDUALS (METERS)
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONDS)

Figure 20

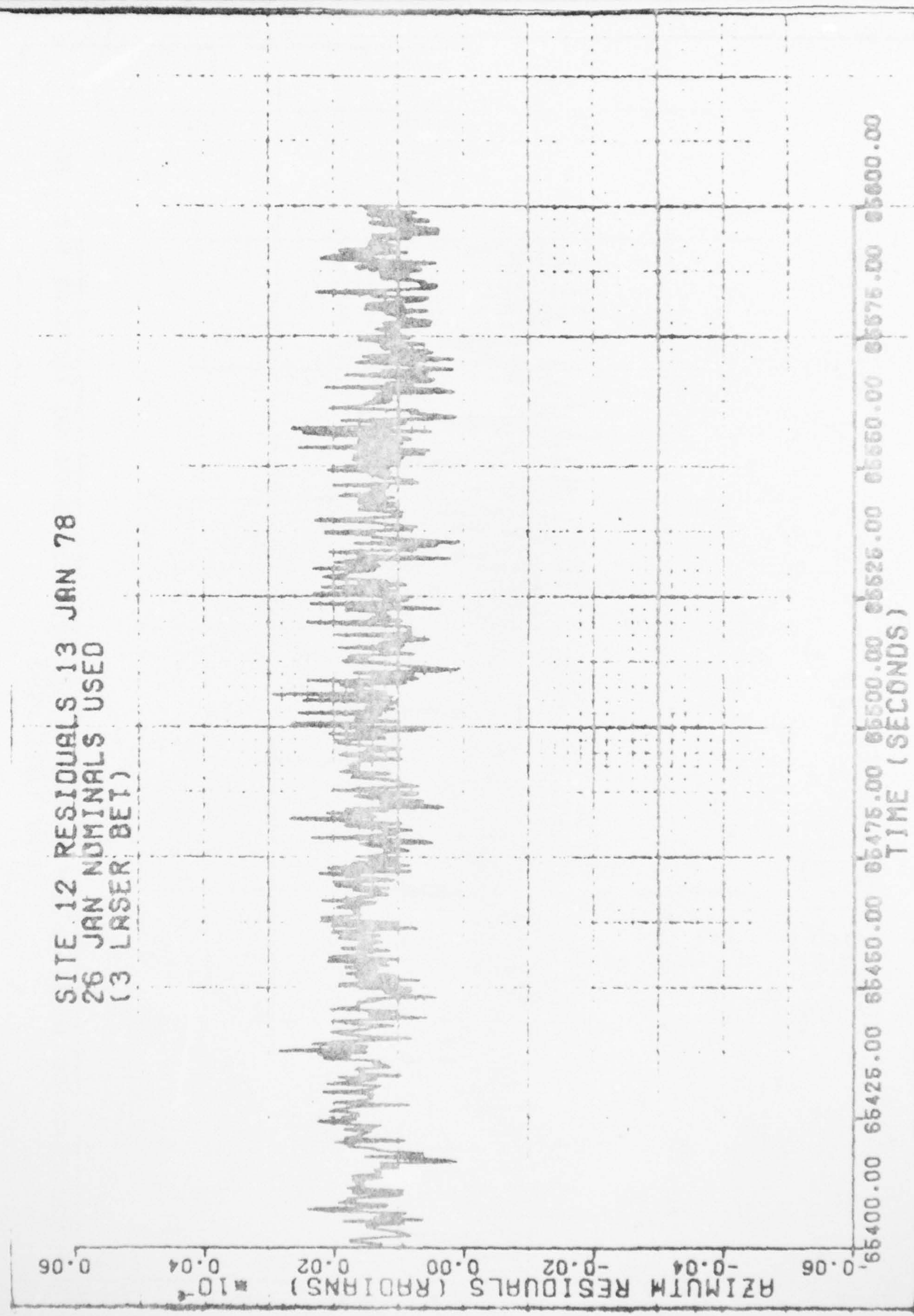

SITE 12 RESIDUALS 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
AZIMUTH RESIDUALS (RADIANS) *10⁻⁴
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONDS)

Figure 21

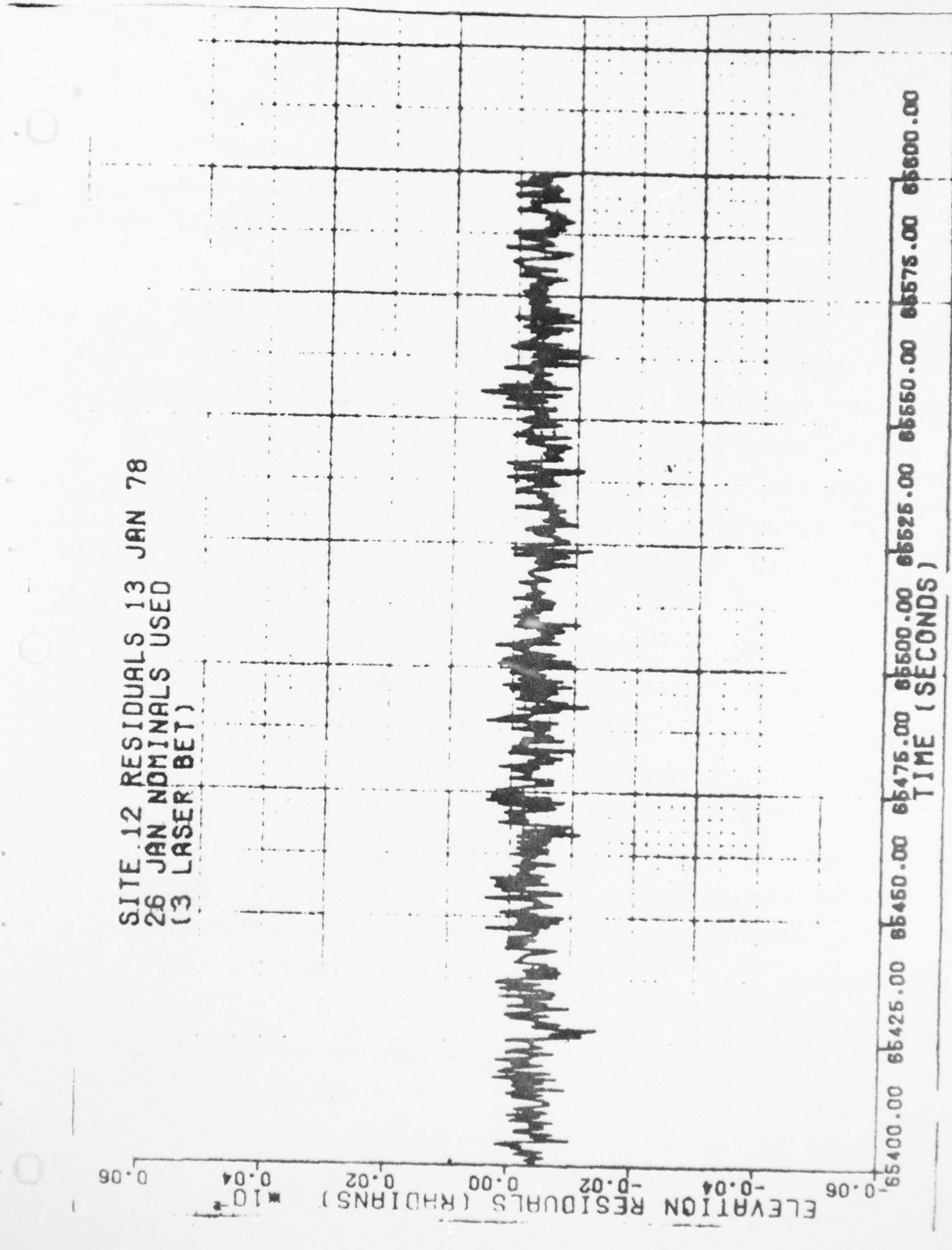

SITE 12 RESIDUALS 13 JAN 78
26 JAN NOMINALS USED
(3 LASER BET)
ELEVATION RESIDUALS (RADIANS) *10⁻²
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
65400.00
65425.00
65450.00
65475.00
65500.00
65525.00
65550.00
65575.00
65600.00
TIME (SECONDS)

Figure 22

AD-A077 170 RANGE COMMANDERS COUNCIL WHITE SANDS MISSILE RANGE N--ETC F/G 16/2
DR AND CG SEMINAR (5TH) BEST ESTIMATE OF TRAJECTORY TECHNIQUES.(U)
OCT 78

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2 OF 3
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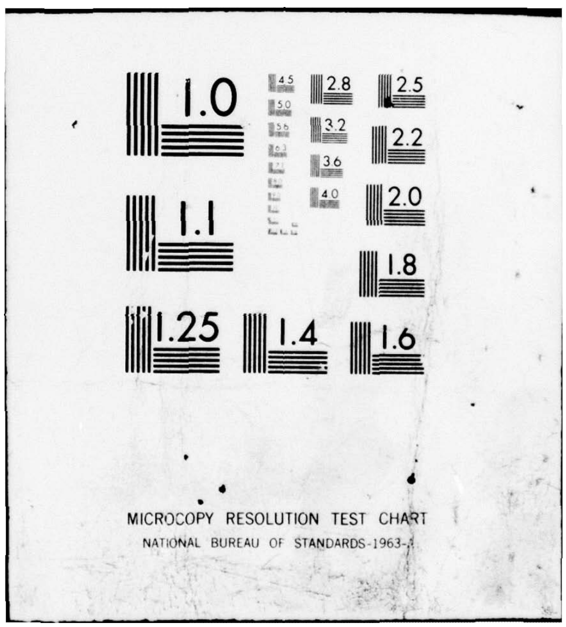

1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

CIRIS. The GPS project has repeatedly demonstrated agreement within 3. meters of the GPSBET solutions. In support of the GPS project, The Aerospace Corporation processes laser and INS data independently and consistently demonstrates 1 sigma differences within .25 meter of YPG's GPSBET trajectory estimates.

The most conclusive proof of .5 meter for the absolute accuracy (1 sigma) of GPSBET position estimates was derived from a Ballistic Camera Comparison Test. The test was conducted in January 1977. USATECOM funded White Sands Missile Range to set up four ballistic cameras on YPG's Cibola Range.

Data was collected for many passes of an F-4 jet through a volume of airspace providing good geometry for the ballistic cameras. Because of inclement weather and other data collection problems (a disadvantage of ballistic camera data collection), only eight passes of ballistic camera data could be reduced. Physical Sciences Laboratory (PSL) computed trajectory estimates using the ballistic camera data and provided their final output to YPG on magnetic tape. YPG processed the data from the three lasers through the GPSBET program. The GPSBET program made lever arm corrections so that the solution from GPSBET was for the position of the strobe light used in the ballistic camera reduction. Figures 23-26 are X vs. Y plots showing the ground track of the first four trajectories analyzed. The F-4 target aircraft was flying at a 15000 foot altitude for all passes.

The ballistic camera solution had a 1 sigma uncertainty of about .2 meter near the center of each pass. Near the ends, the solution degraded to a two-camera solution for some passes and the uncertainty increased to more than 1.0 meter.

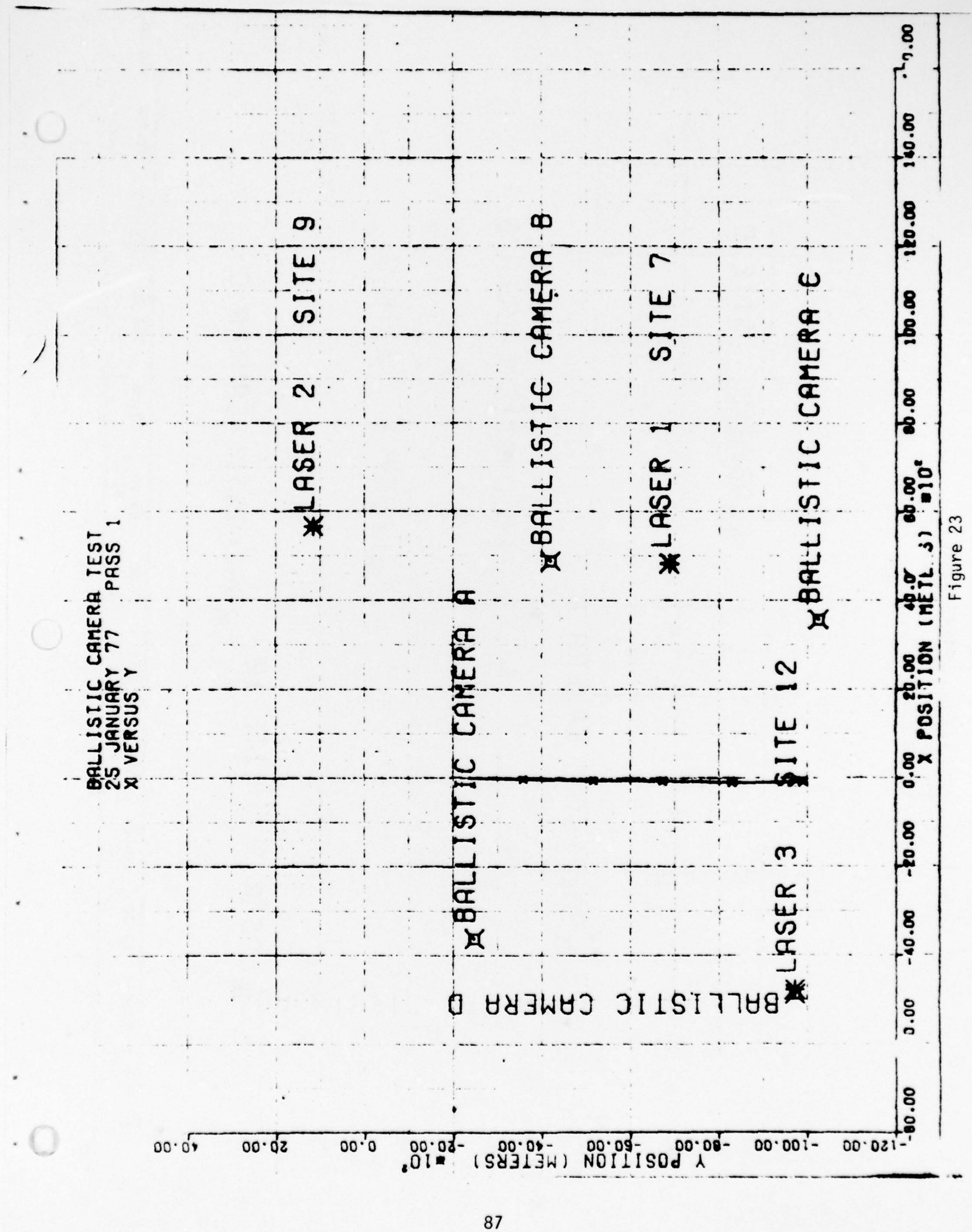

BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 1
X VERSUS Y
LASER 2
SITE 9
BALLISTIC CAMERA B
LASER 1
SITE 7
BALLISTIC CAMERA C
BALLISTIC CAMERA A
SITE 12
LASER 3
BALLISTIC CAMERA D
X POSITION
Y POSITION (METERS) *10²
-120.00
-100.00
-80.00
-60.00
-40.00
-20.00
0.00
20.00
40.00
-80.00
-40.00
-20.00
0.00
20.00
60.00
80.00
100.00
120.00
140.00

Figure 23

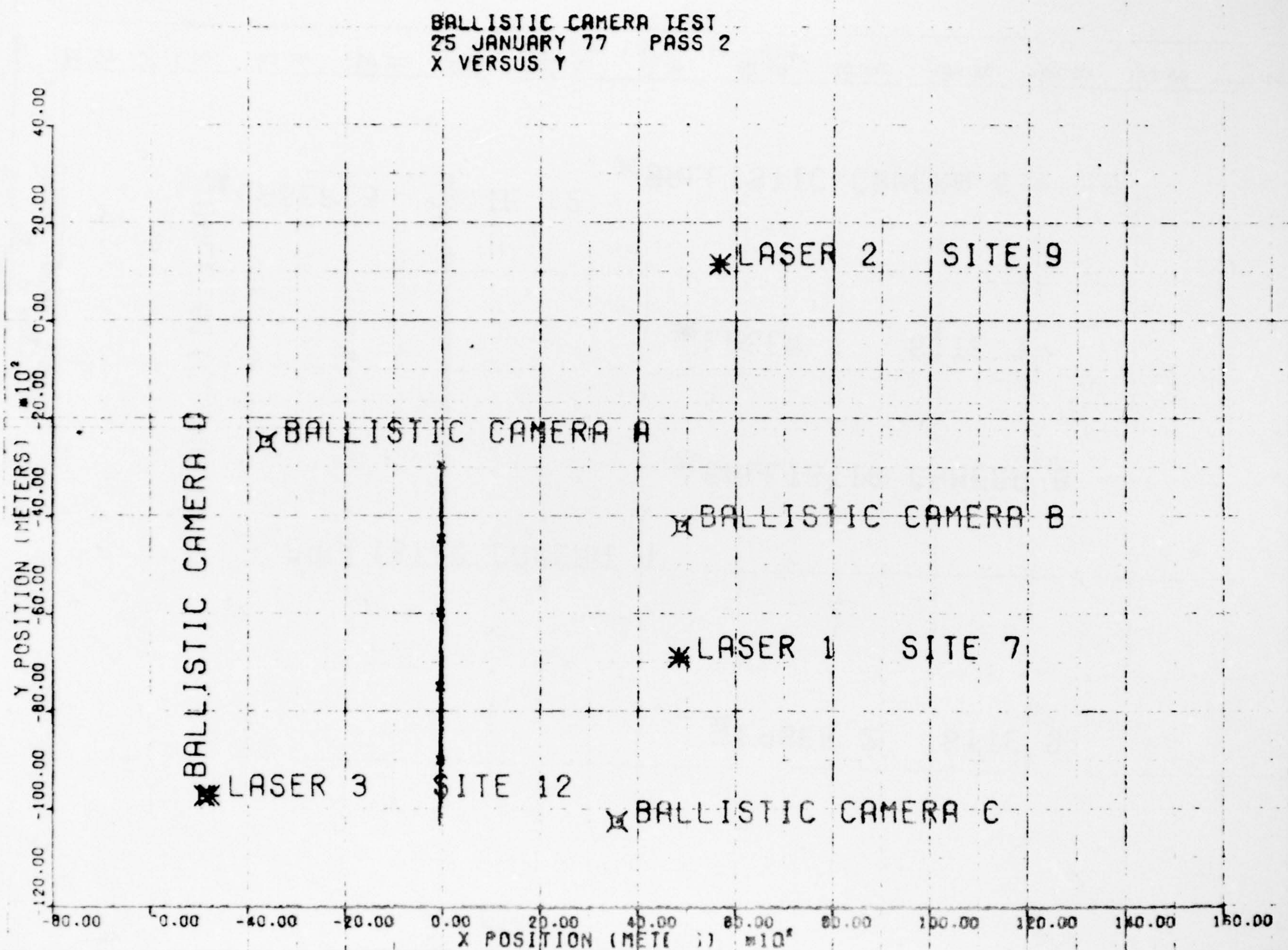
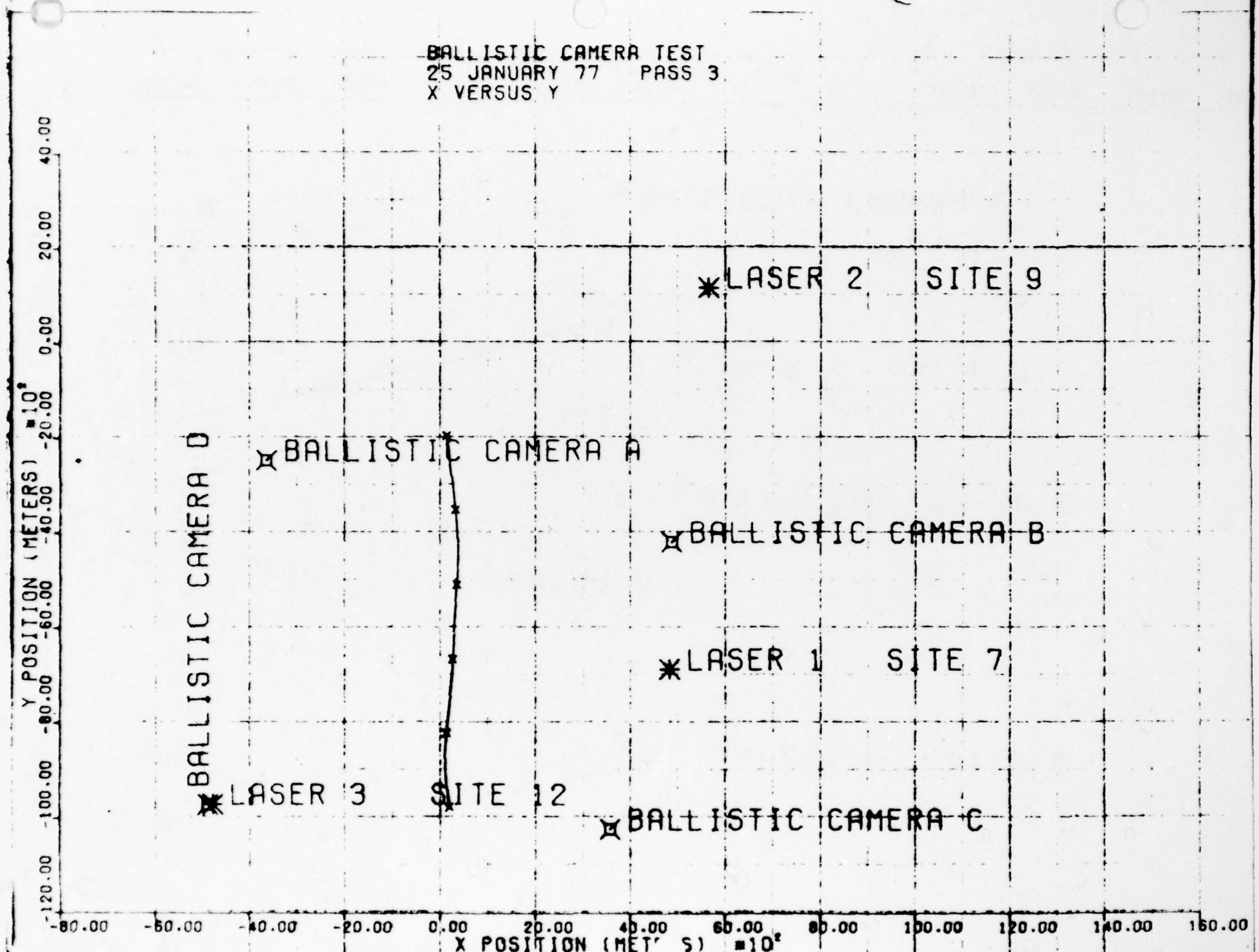
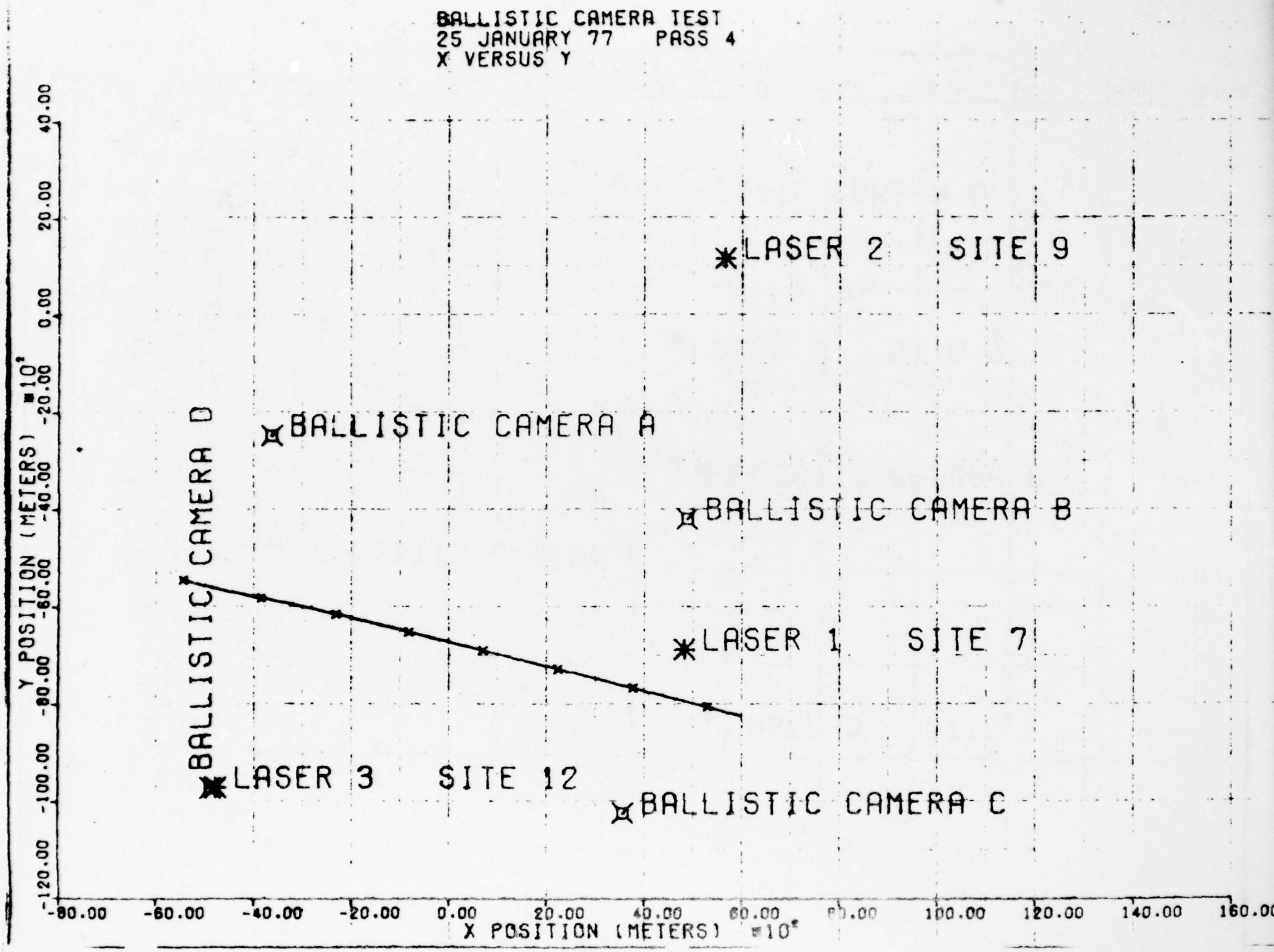

BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 2
X VERSUS Y
LASER 2 SITE 9
BALLISTIC CAMERA A
BALLISTIC CAMERA B
BALLISTIC CAMERA D
LASER 1 SITE 7
LASER 3 SITE 12
BALLISTIC CAMERA C
Y POSITION (METERS) *10²
X POSITION (METERS) *10²

Figure 24


BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 3
X VERSUS Y
LASER 2 SITE 9
BALLISTIC CAMERA A
BALLISTIC CAMERA B
BALLISTIC CAMERA D
LASER 1 SITE 7
LASER 3 SITE 12
BALLISTIC CAMERA C
Y POSITION (METERS) *10²
40.00
20.00
0.00
-20.00
-40.00
-60.00
-80.00
-100.00
-120.00
-80.00
-60.00
-40.00
-20.00
0.00
20.00
40.00
60.00
80.00
100.00
120.00
140.00
160.00
X POSITION (METERS) *10²
Figure 25



BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 4
X VERSUS Y
LASER 2 SITE 9
BALLISTIC CAMERA A
BALLISTIC CAMERA B
BALLISTIC CAMERA D
LASER 1 SITE 7
LASER 3 SITE 12
BALLISTIC CAMERA C
Y POSITION (METERS) *10²
X POSITION (METERS) *10²
-80.00 -60.00 -40.00 -20.00 0.00 20.00 40.00 60.00 100.00 120.00 140.00 160.00
40.00 20.00 0.00 -20.00 -40.00 -60.00 -80.00 -100.00 -120.00

Figure 26

Differences between the GPSBET solutions using three lasers and the ballistic camera solutions were about .5 meter. Table 3 contains the rms (root mean square) of the differences in X, Y, Z components for each of the eight passes for which ballistic camera data was available. Figures 27-38 are plots of the point-by-point differences between GPSBET and ballistic camera solutions for X, Y, Z components in passes 1-4.

8.3 COMPUTER TIME

The GPSBET program is implemented on an IBM 7094/7044 DCS computer. Timings were made using various types of instrumentation. Table 4 contains the results of the timings along with the approximate uncertainties estimated for the position and velocity components. The run time column represents multiples of the length of the trajectory being estimated. These times explain why the GPSBET program is not usually run over an entire mission but over smaller time segments determined from quick-look plots using RTE data.

TABLE 3.

RMS OF DIFFERENCES IN X, Y, Z
LASER CALIBRATIONS ADJUSTED FOR TEMPERATURE INVERSION
(meters)

	Pass 1 25 Jan	Pass 2 25 Jan	Pass 3 25 Jan	Pass 4 25 Jan	Pass 5 25 Jan	Pass 6 25 Jan	Pass 7 25 Jan	Pass 2 29 Jan
X	.16	.23	.34	.71	.28	.57	.13	.38
Y	.60	.15	.65	.39	.23	.59	.71	.27
Z	.32	.14	.64	.45	.30	.86	.41	.23

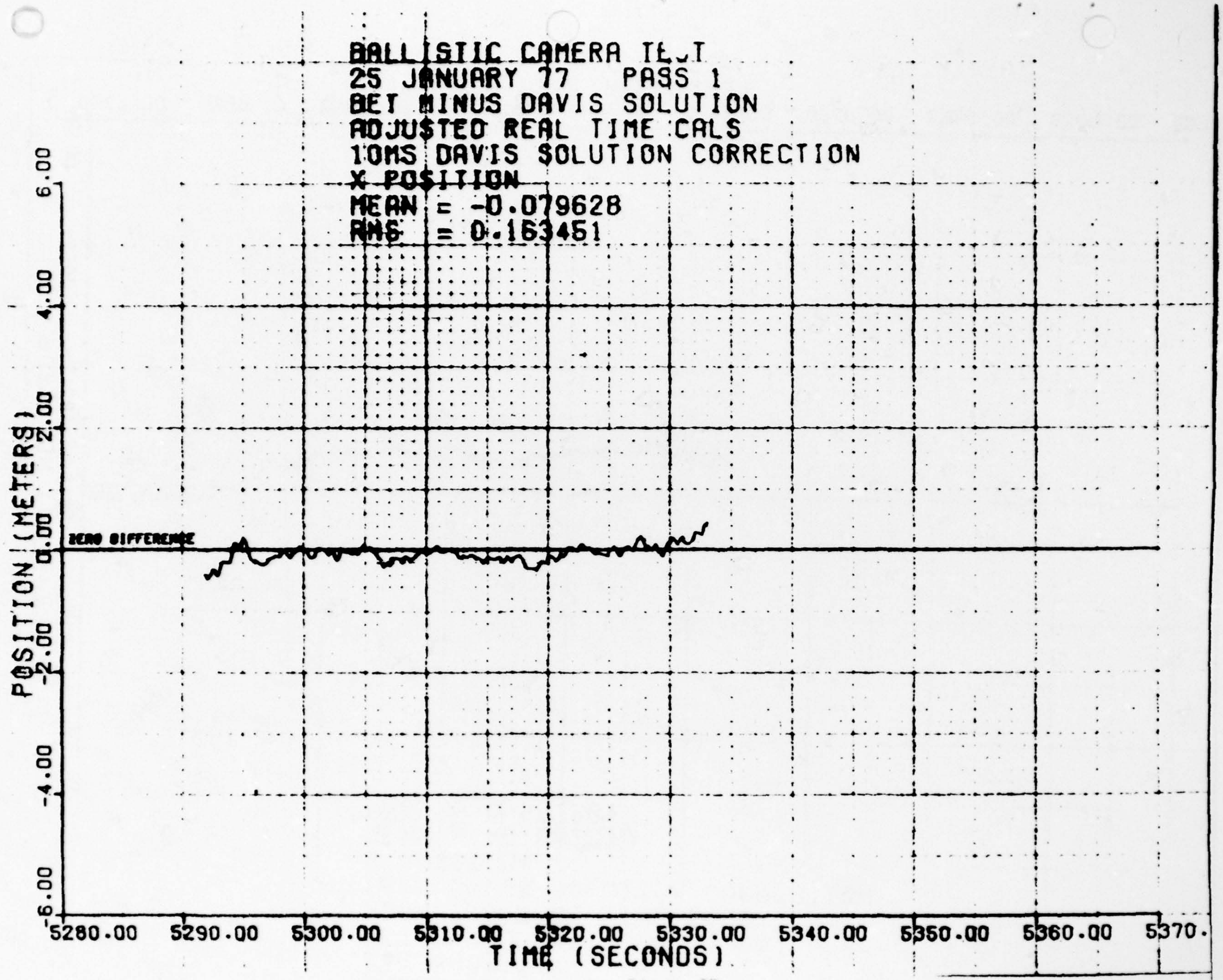

BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 1
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
X POSITION
MEAN = -0.079628
RMS = 0.163451
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
5280.00
5290.00
5300.00
5310.00
5320.00
5330.00
5340.00
5350.00
5360.00
5370.
TIME (SECONDS)

Figure 27

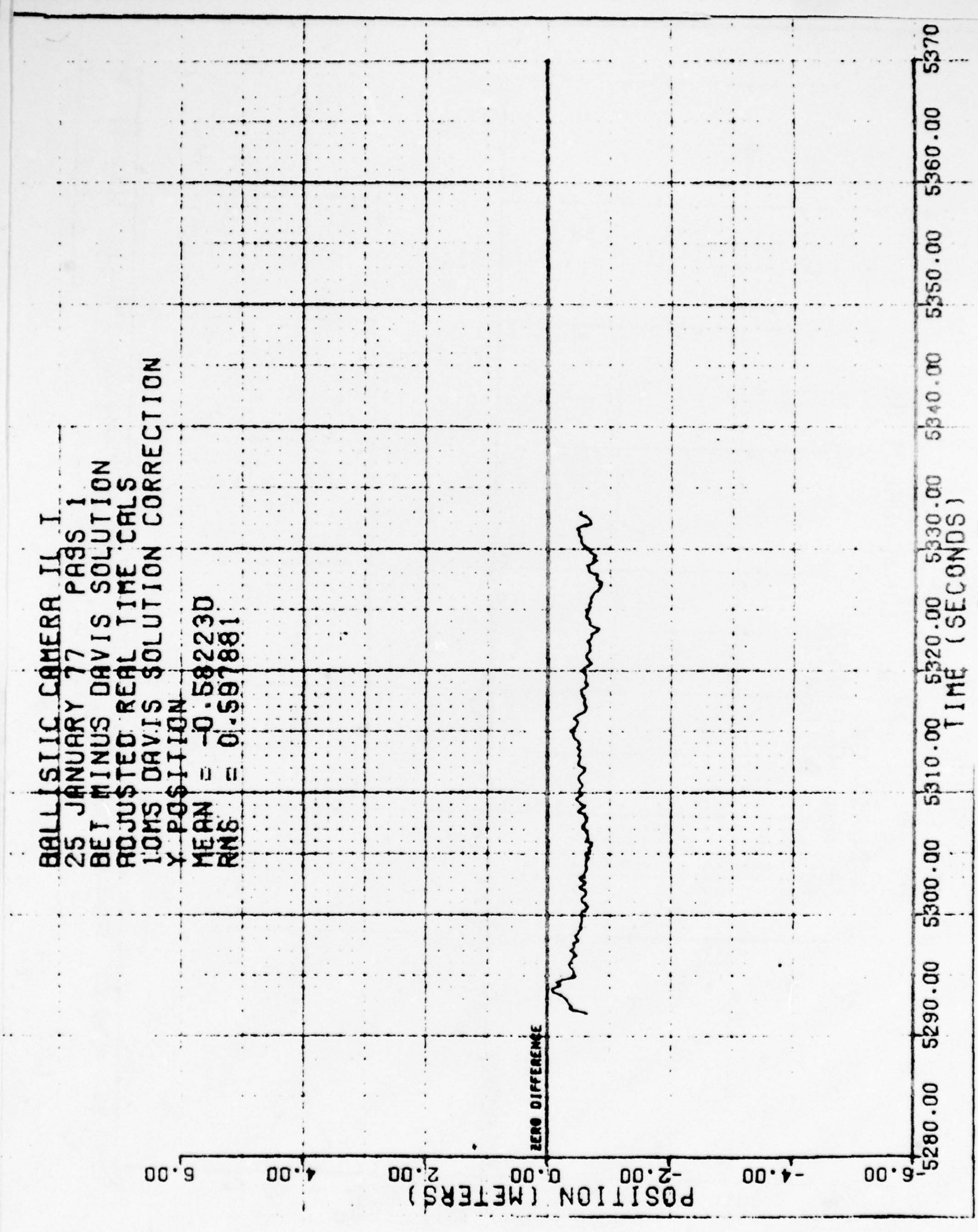
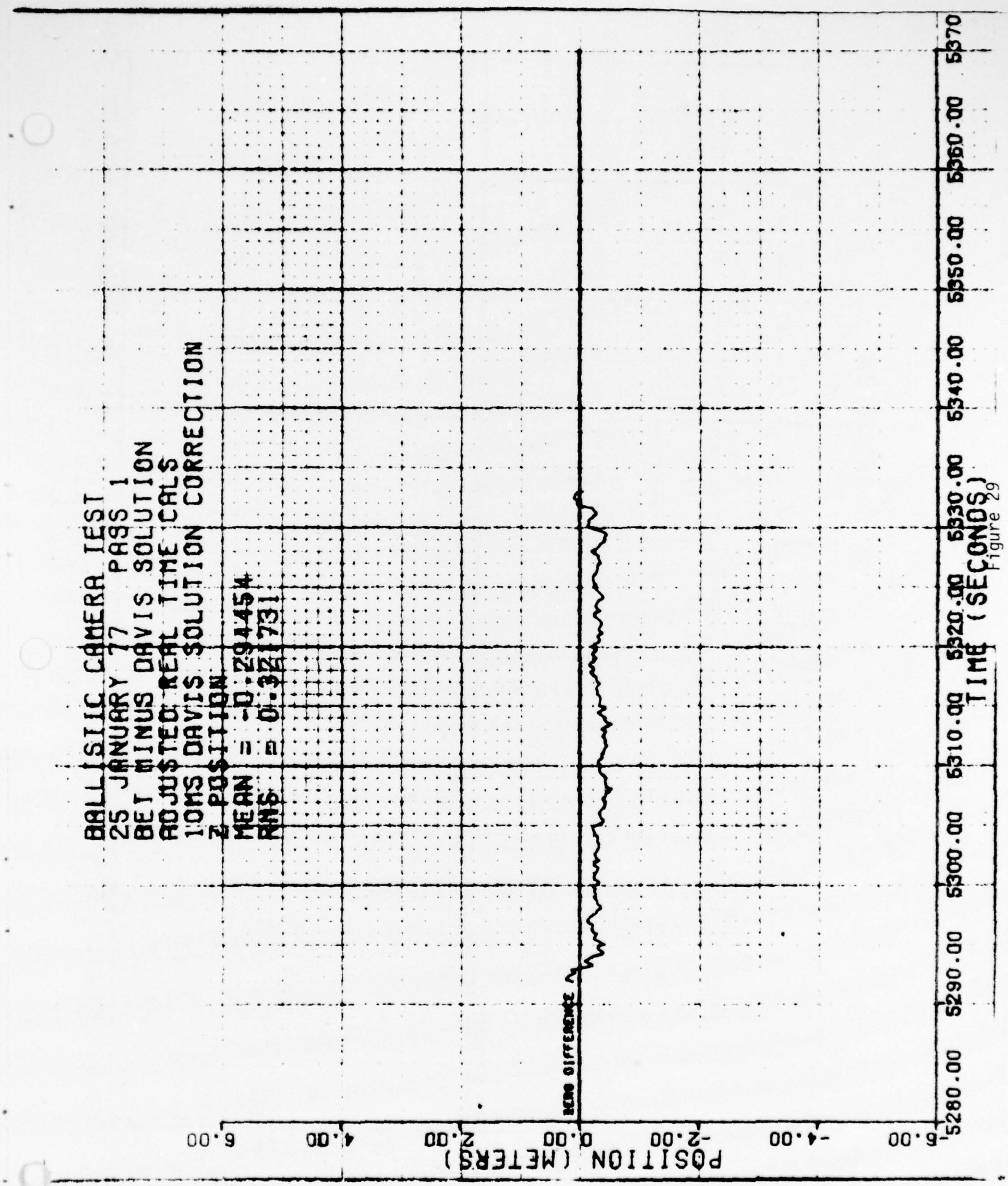

BALLISTIC CAMERA IL I
25 JANUARY 77 PASS 1
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Y POSITION
MEAN = -0.58223D
RMS = 0.597881
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
TIME (SECONDS)
5280.00
5290.00
5300.00
5310.00
5320.00
5330.00
5340.00
5350.00
5360.00
5370

Figure 28


BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 1
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Z POSITION
MEAN = -0.294454
RMS = 0.321731
ZERO DIFFERENCE
POSITION (METERS)
TIME (SECONDS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
5280.00
5290.00
5300.00
5310.00
5320.00
5330.00
5340.00
5350.00
5360.00
5370
Figure 29

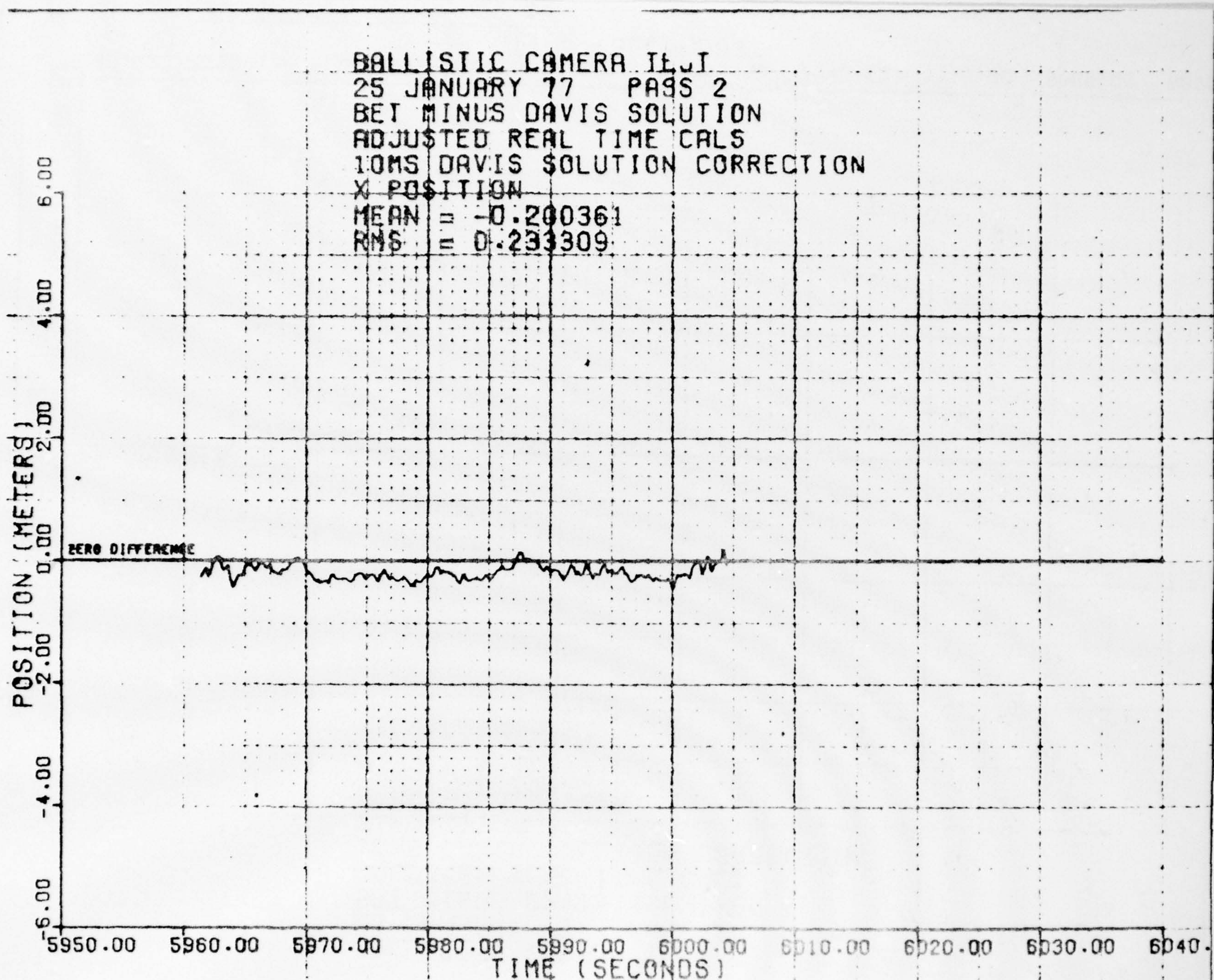

BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 2
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
X POSITION
MEAN = -0.200361
RMS = 0.233309
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
5950.00
5960.00
5970.00
5980.00
5990.00
6000.00
6010.00
6020.00
6030.00
6040.
TIME (SECONDS)

Figure 30

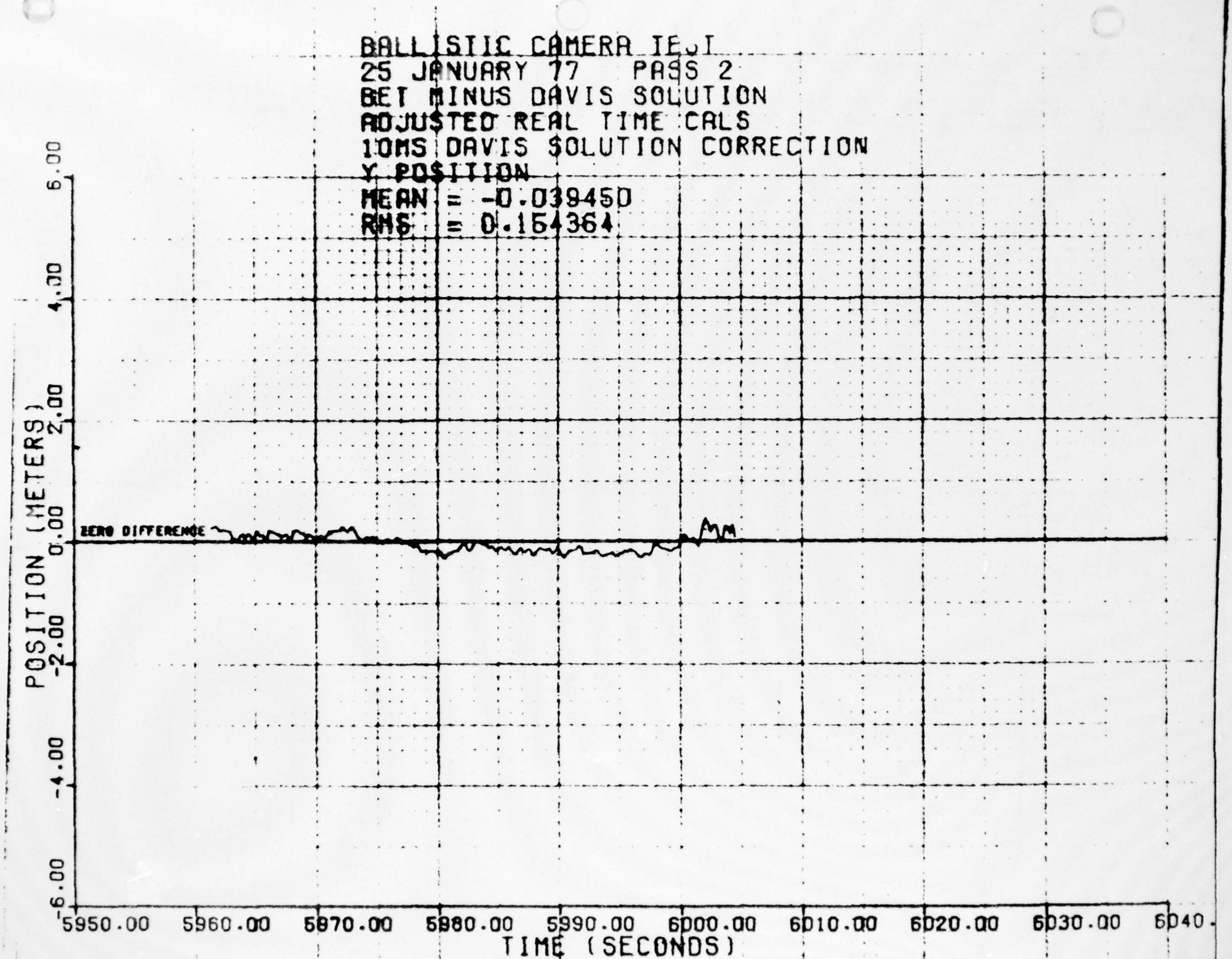
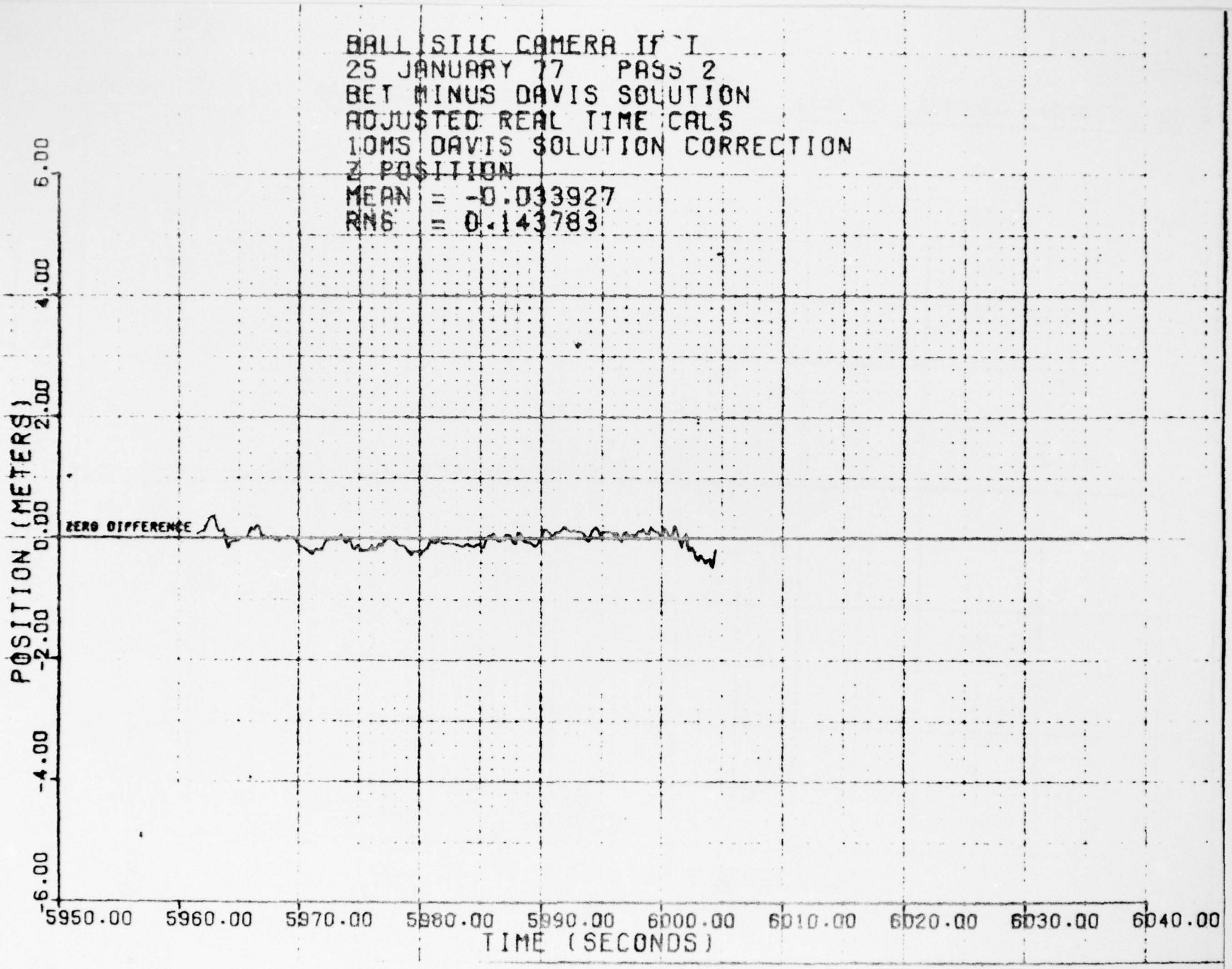

BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 2
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Y POSITION
MEAN = -0.039450
RMS = 0.154364
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
5950.00
5960.00
5970.00
5980.00
5990.00
6000.00
6010.00
6020.00
6030.00
6040.
TIME (SECONDS)

Figure 31



BALLISTIC CAMERA IF I
25 JANUARY 77 PASS 2
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Z POSITION
MEAN = -0.033927
RMS = 0.143783
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
5950.00
5960.00
5970.00
5980.00
5990.00
6000.00
6010.00
6020.00
6030.00
6040.00
TIME (SECONDS)

Figure 32

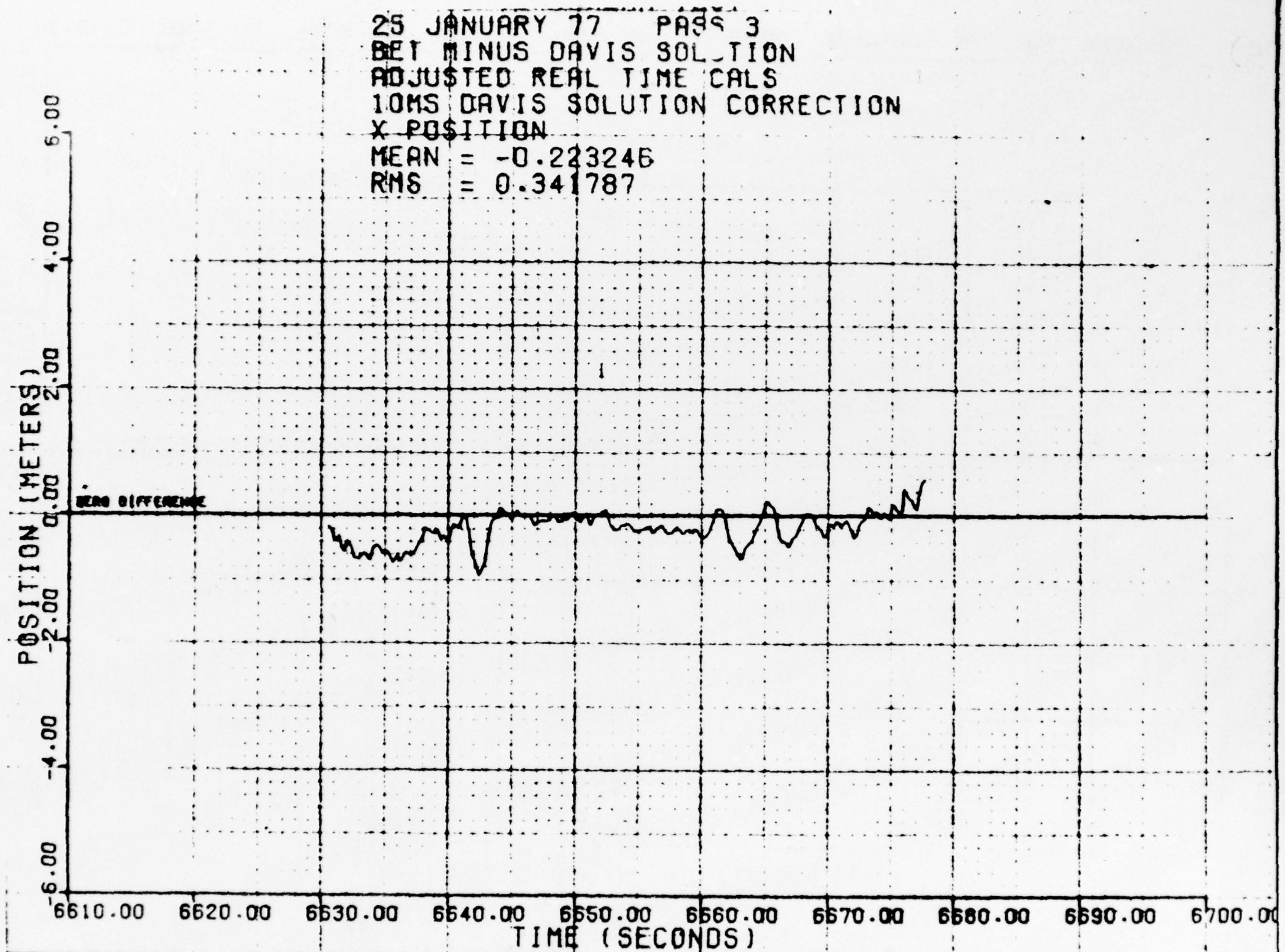

25 JANUARY 77 PASS 3
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
X POSITION
MEAN = -0.223246
RMS = 0.341787
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
6610.00
6620.00
6630.00
6640.00
6650.00
6660.00
6670.00
6680.00
6690.00
6700.00
TIME (SECONDS)

Figure 33

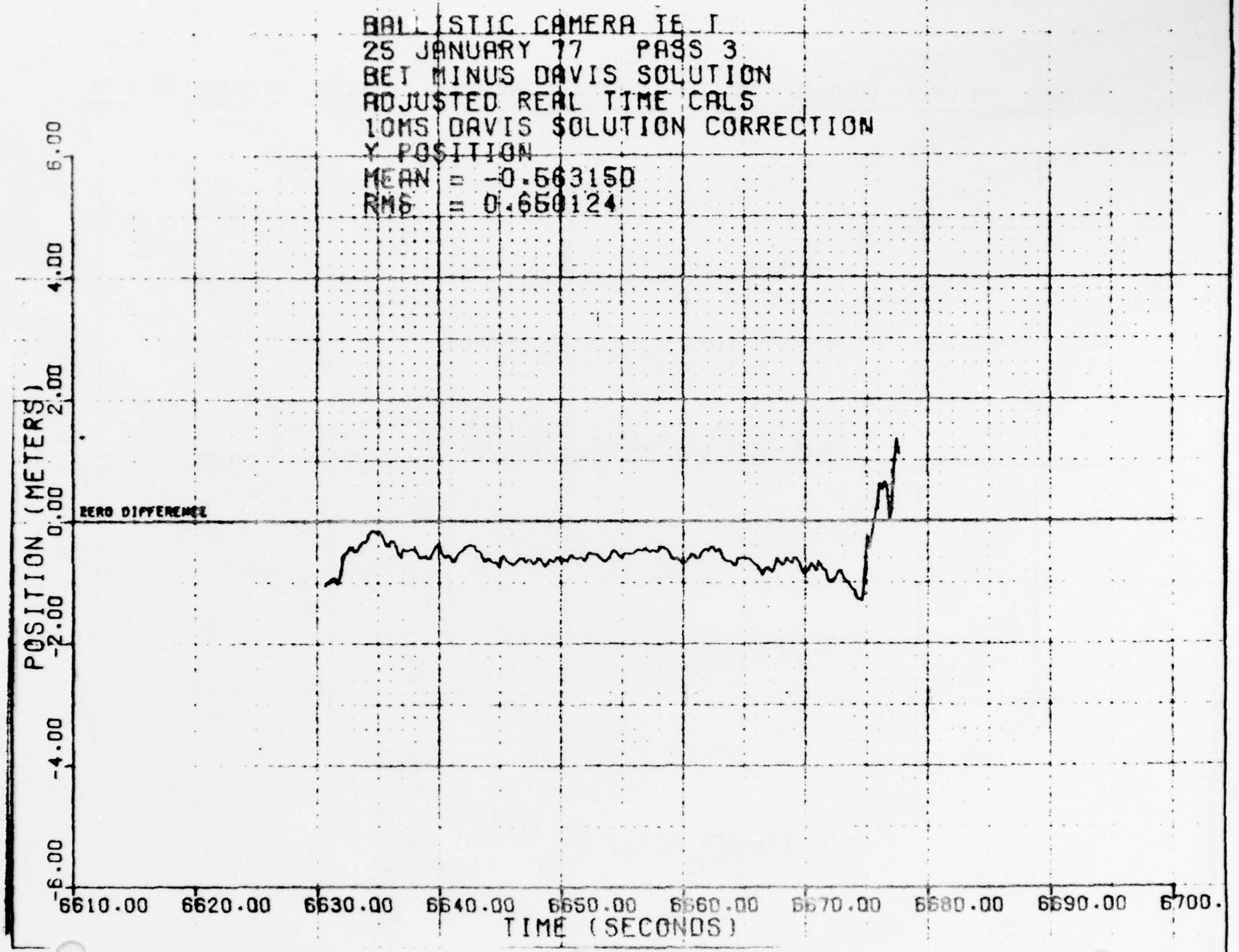

BALLISTIC CAMERA IE I
25 JANUARY 77 PASS 3
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Y POSITION
MEAN = -0.563150
RMS = 0.650124
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
6610.00
6620.00
6630.00
6640.00
6650.00
6660.00
6670.00
6680.00
6690.00
6700.
TIME (SECONDS)

Figure 34

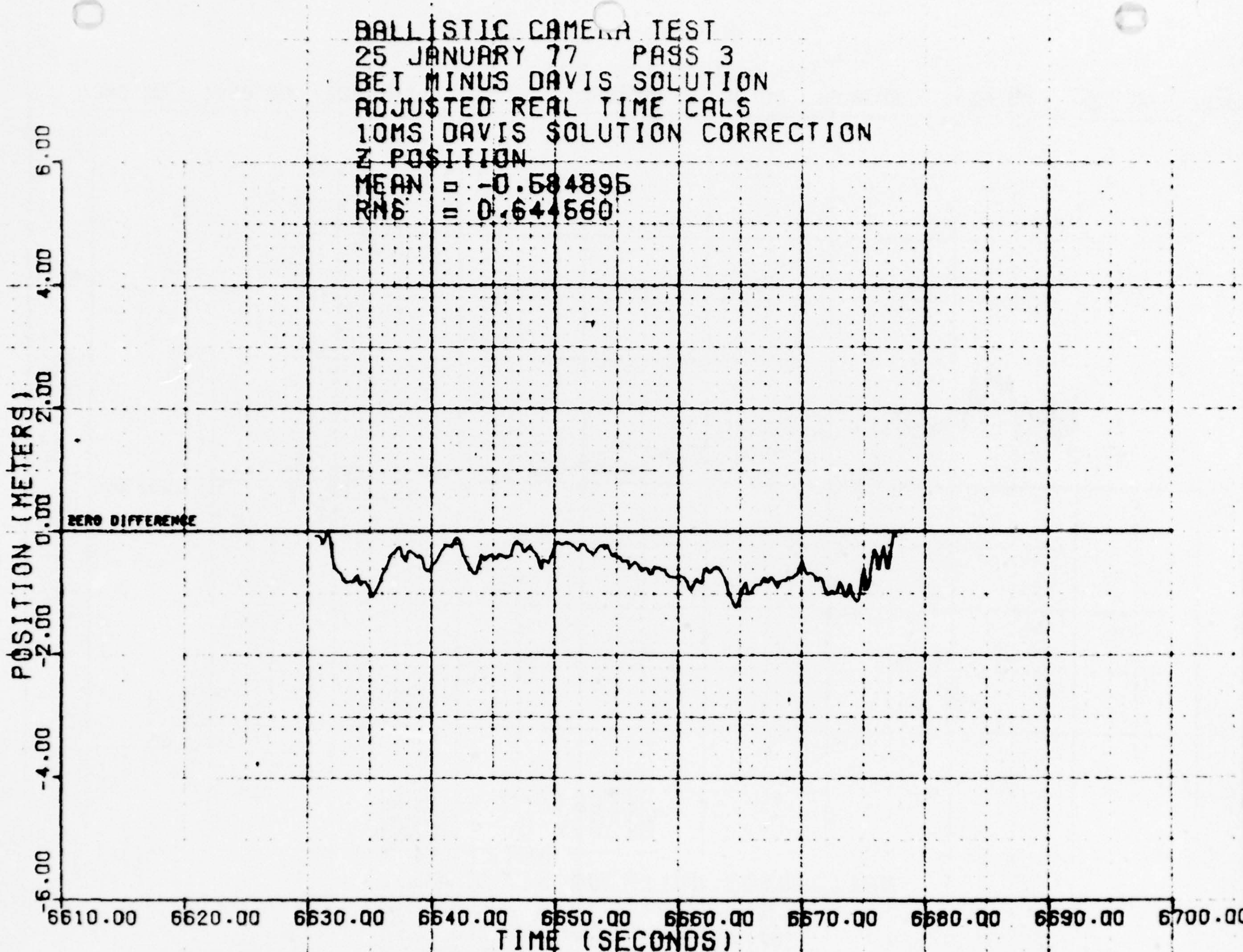
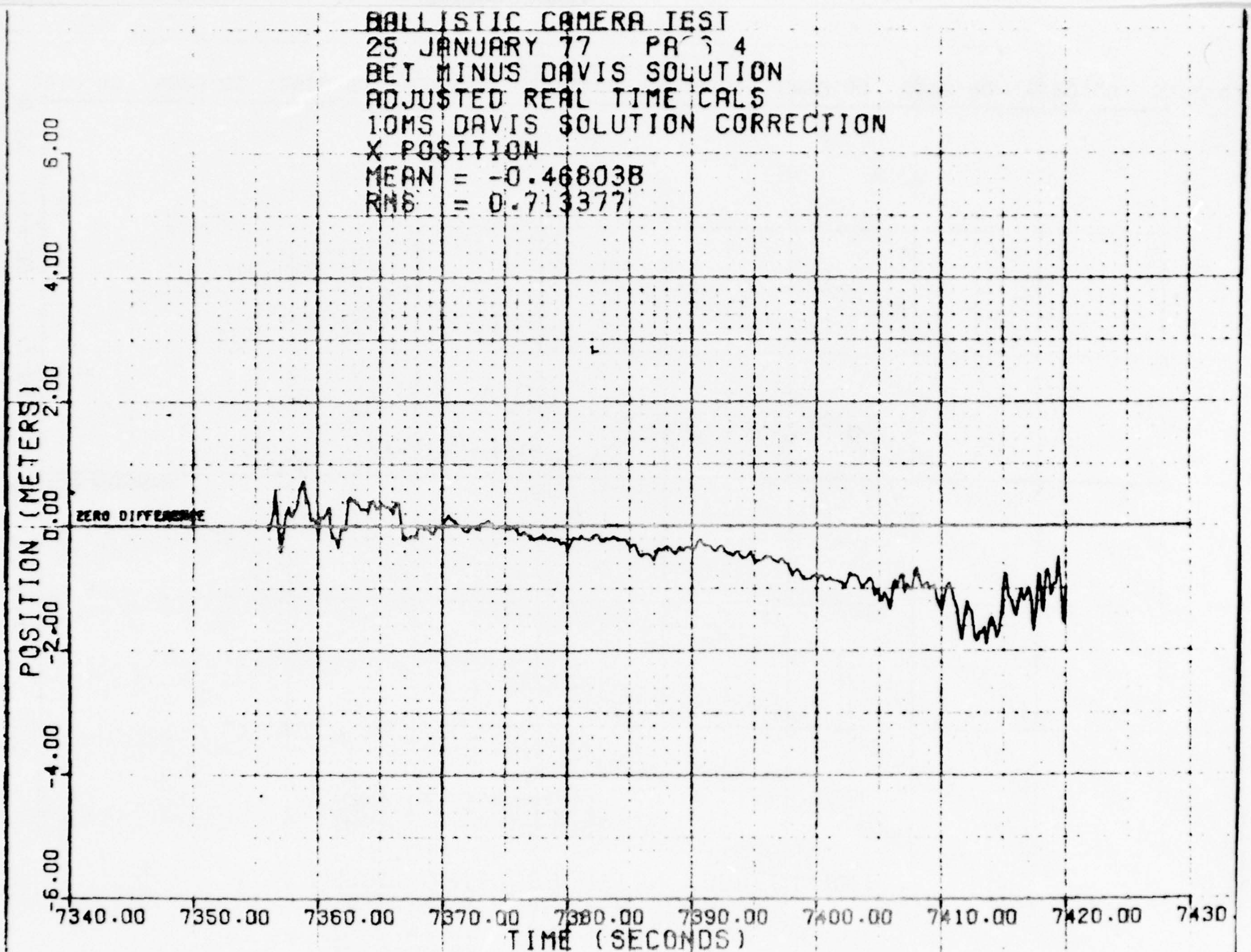

BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 3
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Z POSITION
MEAN = -0.584895
RMS = 0.644560
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
6610.00
6620.00
6630.00
6640.00
6650.00
6660.00
6670.00
6680.00
6690.00
6700.00
TIME (SECONDS)

Figure 35



BALLISTIC CAMERA TEST
25 JANUARY 77 PA 4
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
1.0MS DAVIS SOLUTION CORRECTION
X POSITION
MEAN = -0.468038
RMS = 0.713377
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
7340.00
7350.00
7360.00
7370.00
7380.00
7390.00
7400.00
7410.00
7420.00
7430
TIME (SECONDS)

Figure 36

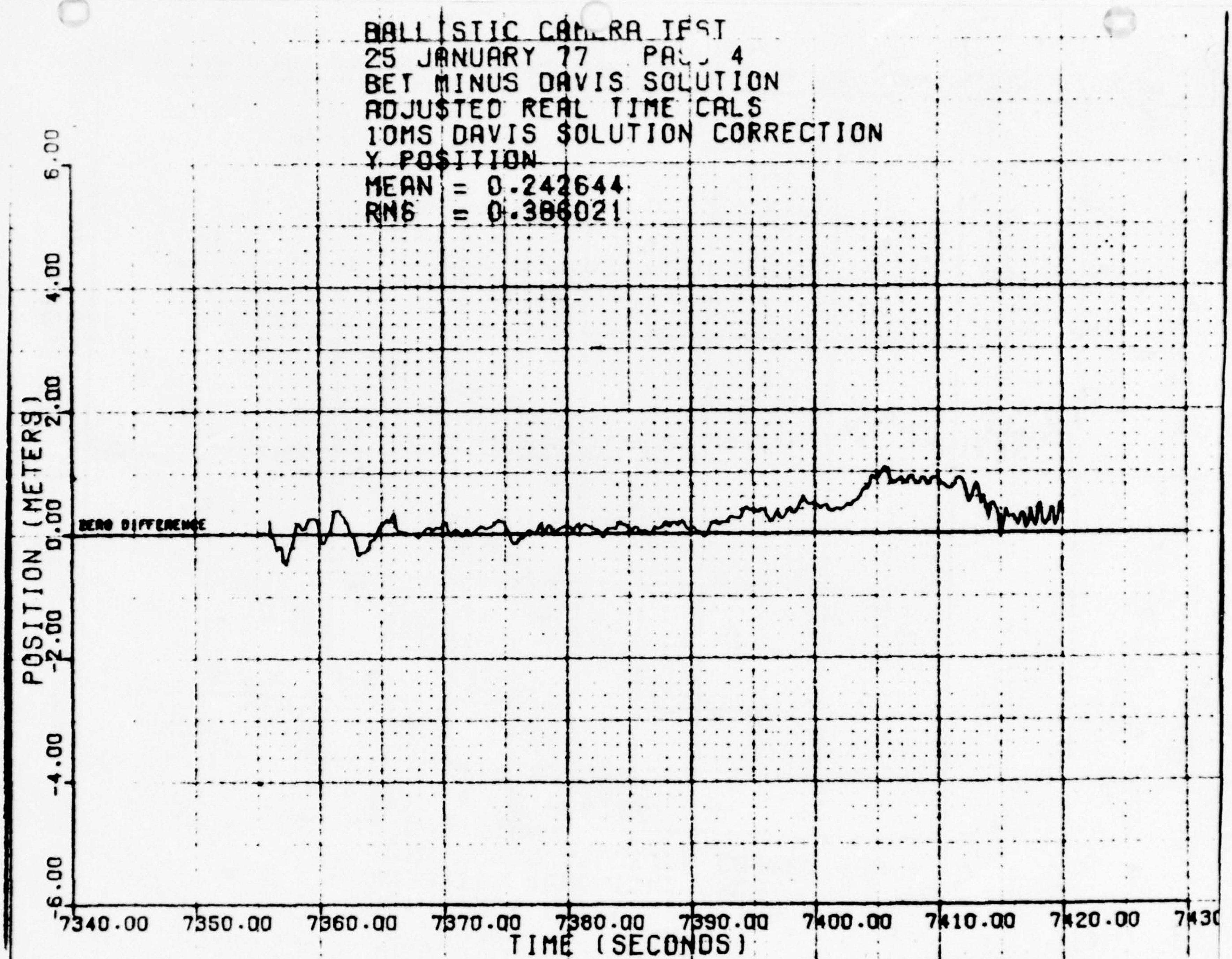

BALLISTIC CAMERA TEST
25 JANUARY 77 PAGE 4
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Y POSITION
MEAN = 0.242644
RMS = 0.386021
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
7340.00
7350.00
7360.00
7370.00
7380.00
7390.00
7400.00
7410.00
7420.00
TIME (SECONDS)

Figure 37

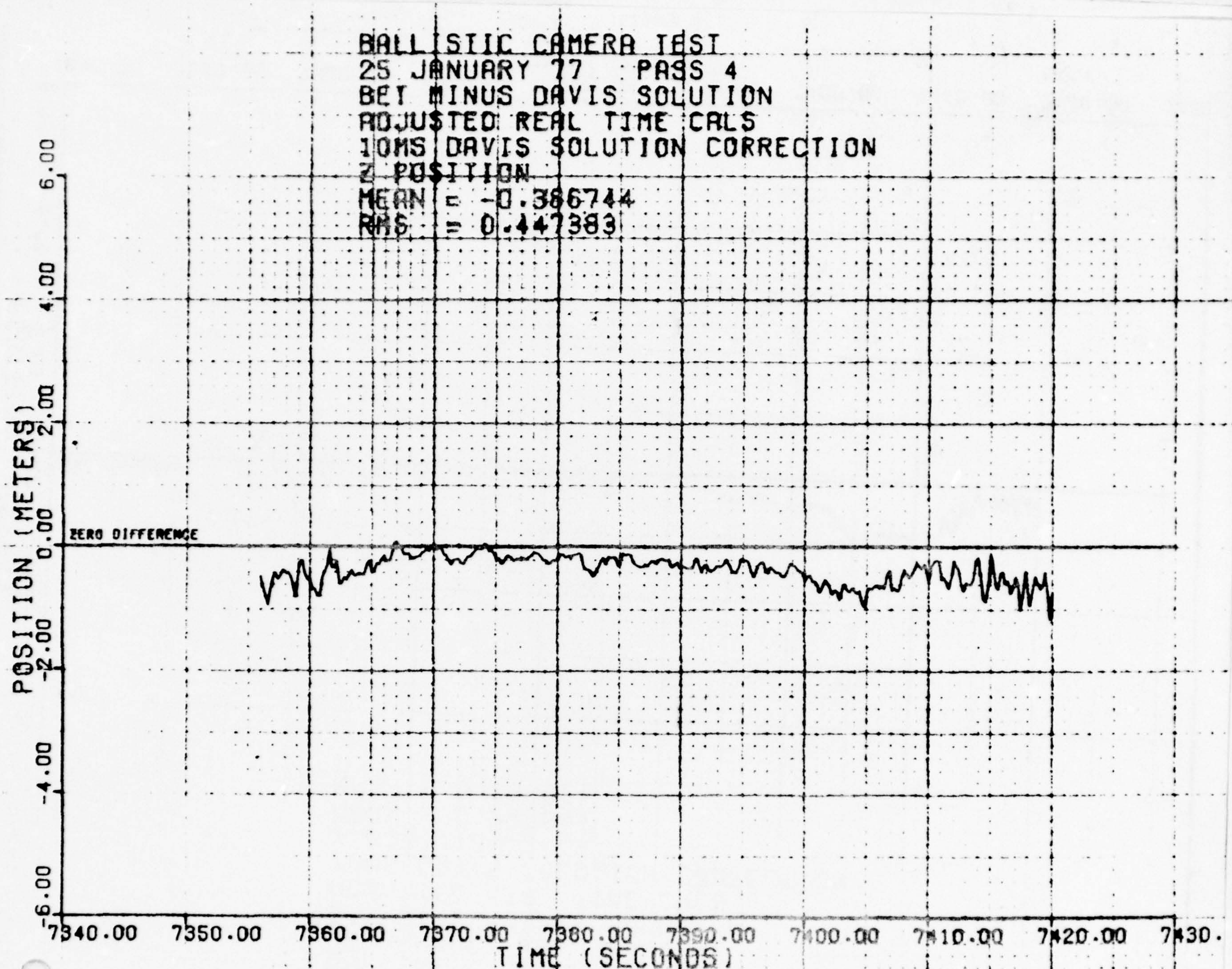

BALLISTIC CAMERA TEST
25 JANUARY 77 PASS 4
BET MINUS DAVIS SOLUTION
ADJUSTED REAL TIME CALS
10MS DAVIS SOLUTION CORRECTION
Z POSITION
MEAN = -0.386744
RMS = 0.447383
ZERO DIFFERENCE
POSITION (METERS)
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
7340.00
7350.00
7360.00
7370.00
7380.00
7390.00
7400.00
7410.00
7420.00
7430.
TIME (SECONDS)

Figure 38

TABLE 4.

GPSBET TIMINGS

Instruments Used	Number of States	Filter Only		Filter + Smooth	
		Accuracy	Run Time	Accuracy	Run Time
1 Laser	12		3.		5.
2 Lasers	15	1.00 m .40 m/sec	4.	.40 m .15 m/sec	8.
3 Lasers	18	.40 m .25 m/sec	5.	.20m .10 m/sec	10.
1 Laser 1 INS	18	.50 m .20 m/sec	4.	.40 m .10 m sec	9.
2 Lasers 1 INS	21	.50 m .15 m/sec	5.	.20 m .05 m/sec	12.
3 Lasers 1 INS	24	.40 m .15 m/sec	6.	.10 m .03 m/sec	16.
PLS	9 + (1 for each A-station used)	TBD	TBD	TBD	TBD
PLS + INS	15 + (1 for each A-station)	TBD	TBD	TBD	TBD

9. SUMMARY

USAYPG acquired three laser tracking instruments in 1973-1974. Accompanying this acquisition was the implicit assumption that USAYPG personnel would develop standard operating procedures, design hardware and design and develop software to take full advantage of these new tracking systems. Since it turned out that the lasers had much greater capabilities for accurate and timely data acquisition and reduction than any previous range instrumentation, this implicit assumption meant devising calibration procedures and trajectory estimation procedures at or beyond the state-of-the-art. From whence came the requirement for the GPSBET program.

As described in this paper, the GPSBET program computes optimal (or as nearly optimal as the truth of the assumptions allow) estimates of position, velocity and acceleration utilizing range, azimuth and elevation measurements from up to three laser trackers. Also, GPSBET accepts inertial platform measurements in the form of sensed velocity components and gimbal angles.

The GPSBET program accepts control information from cards and data from magnetic tape. It processes measurements through Kalman filter equations followed by an optional smoothing pass.

GPSBET is implemented on an IBM 7094/7044 Direct Couple System. On that computer, the Kalman filtering of data from three lasers takes about four times real time (i.e., a 15-minute segment of data takes 1 hour of computer time). Filtering plus smoothing takes about nine times real time.

The accuracy of position estimates made by GPSBET using data from three lasers has been demonstrated to be about .5 meter

Appendix A

BET INPUT CONTROL PARAMETERS

This appendix lists and describes all parameters that can be input to the BET program from cards as of 15 April 1976.

Parameter Name		Description
NJØB	- - - -	number of jobs to do
NTRAJ	- - - -	number of trajectories per job
GCØRSO	- - - -	geodetic coordinates of origin of reference coordinate system
GCØRS1	- - - -	geodetic coordinates of laser 1
GCØRS2	- - - -	geodetic coordinates of laser 2
GCØRS3	- - - -	geodetic coordinates of laser 3
RETLEV	- - - -	body coordinates of retroreflector
IMULEV	- - - -	body coordinates of IMU
GPSLEV	- - - -	body coordinates of GPS antenna
XS	- - - -	state vector
NI	- - - -	number of instruments
INSTYP	- - - -	type of each instrument 1 = laser 2 = cine 3 = range only 4 = sensed velocities 5 = gimbal angles
INUNIT	- - - -	logical unit number to read CDT
IØBET	- - - -	output unit for BET output tape
IMU	- - - -	option to use IMU data
ISMUTH	- - - -	option to make smoothing pass
ILEV	- - - -	option to make lever arm corrections

Parameter Name		Description
IREW	- - - -	option to rewind CDT after a run
IRDEØF	- - - -	option to read to an EØF on CDT after filtering a given section of data
INSTRU	- - - -	instrument to be forced as automatic initialization or reinitialization of the BET
IRATE	- - - -	array controlling modules for output of various types of data to line printer or tape
IXSB	- - - -	max points between state vector output during filtering
ISIG	- - - -	max points between sigma vector output during filtering
IRMS	- - - -	max points between printout and restart of rms and mean of residuals report during filtering
IRES	- - - -	max points between printout of actual residuals for a single time from each instrument
ICØV	- - - -	max points between printout of covariance matrix (actually has sigmas on diagonal and correlation coefficients in off diagonal elements)
MAXPTS	- - - -	maximum number of observations to filter on this trajectory
IFILES	- - - -	number of files to skip on the CDT prior to reading the data for this trajectory
IREF	- - - -	option on refraction corrections
XLAMDA	- - - -	wavelength of lasers
HSCL	- - - -	scale height parameter used in current refraction model
NIMU	- - - -	number of the receiver to use sensed velocities from (number refers to location on the CDT)
NGMBL	- - - -	number of the receiver to take gimbal angles from
M	- - - -	array containing options for prefiltering or skipping 0 = skip 1 = constant fit 2 = linear fit 3 = quadratic fit
N	- - - -	number of points to use in prefilter

Parameter Name		Description
PEDNSD	- - - -	number of multiples of residual standard deviation to use while editing prior to the pre-filter
RAWDT	- - - -	time increment between raw data for each instrument
FILDT	- - - -	time increment desired between measurements going through the Kalman filter
EDIT	- - - -	number of standard deviations of residuals to edit on during the Kalman filtering (after pre-filtering)
IEDLMT	- - - -	number of edit messages to print out during this run
SD	- - - -	standard deviations of noise on measurements of each tape and from each instrument
QI	- - - -	standard deviation used for deweighting the covariance matrix
PI	- - - -	initial values for square root of the diagonal of the covariance matrix (i.e., initial uncertainty in state vector)
TEMP	- - - -	temperature in degrees Kelvin at each laser site
PRES	- - - -	pressure in millibars at each laser site
RUNID	- - - -	72 character run identification
TESTI	- - - -	20 character test item identification
IWØN	- - - -	12 character work order number or other identification of project to be charged
PRØJ	- - - -	12 characters, for project engineer's name
DATE	- - - -	12 characters for date of test
RDATE	- - - -	12 characters for date of this report
NHI, NMI, NSI, MSI, NHF, NMF, NSF, MSF	- - - -	start and stop times for using measurements from various instruments in the filter

Parameter Name	Description
NHIBET NMIBET NSIBET MSIBET NHFBET NMFBET NSFBET MSFBET	- - - start and stop times for filtering on this - - - trajectory

Appendix B

BET OUTPUT TAPE

For each trajectory filtered, there will be a BCD file containing control information and a second file containing results. This appendix includes a description of the second file. The first file for each trajectory contains a copy of the control report, a sample of which is in Appendix C of this paper.

YUMA PROVING GROUND
DATA PROCESSING DEVELOPING WORKING GROUP

DPDWG REPORT NO. 2

BET OUTPUT RECORD FORMAT

18 December 1975

ABSTRACT

The agreed-upon record format in which the output from the Best Estimate of the Trajectory (BET) program will be written is presented; the information to be contained in the BET output is defined by this format.

Changes to this format must be documented in subsequent DPDWG reports.

BET OUTPUT RECORD FORMAT

The output from the BET (Best Estimate of the Trajectory) program will be written by the IBM 7044/7094 DCS in 36-bit words on 7-track magnetic tape. The output from each run will consist of two files:

File 1. The first file will be written in BCD, containing all of the information, and in identical form, of the introductory portion ("control report") of the BET printout. The first file will thus contain date and time of the test, identification and location of range sensors, and all input parameters.

File 2. The second file will contain the results. It will be written in binary words, with one record each 0.2 seconds, in order of increasing time, and one physical record per logical record.

If a physical BET output tape contains the output from several runs, there will be two files--the control report and the results file--for each run.

Table I summarizes the record format which will be employed in the second (results) file.

Representation of Words. A word in the results record will be written by the 7044/7094 DCS as a result of FORTRAN (binary) write commands, either as an integer word or as a floating point word.

Fixed Point (Integer) Word

36 35 1

The sign of the number is contained in bit 36. A "0" signifies a positive number and a "1" signifies a negative number. The magnitude of the number is in bits 35 (most significant) through 1 (least significant), with the binary point positioned to the right of bit 1.

Floating Point (Single Precision Real) Word

Characteristic Fraction

36 35 28 27 1

The sign of the number is contained in bit 36. In bits 35 through 28 is the binary exponent increased by 128 (decimal) = 200 (octal) = 10000000 (binary). In bits 27 through 1 is the fraction (mantissa), with the binary point positioned between bits 28 and 27.

TABLE I

BET OUTPUT RECORD FORMAT

Word	Variable	Units	Representation
1	Time, from previous midnight	seconds	Real
2	Source	N/A	Integer
3-5	Position vector of BET reference point	meter	Real
6-8	Velocity vector of BET reference point	meter/sec	Real
9-11	Acceleration vector of BET reference	meter/sec^2	Real
12	Estimated Range Bias, Laser 1	meter	Real
13	Estimated Azimuth Bias, Laser 1	radian	Real
14	Estimated Elevation Bias, Laser 1	radian	Real
15-17	Estimated Bias States, Laser 2	---	Real
18-20	Estimated Bias States, Laser 3	---	Real
21	Estimated INS orientation bias about INS east axis	radian	Real
22	Estimated INS orientation bias about INS north axis	radian	Real
23	Estimated INS orientation bias about INS vertical axis	radian	Real
24	Estimated INS velocity bias in INS east direction	meter/sec	Real
25	Estimated INS velocity bias in INS north direction	meter/sec	Real
26	Estimated INS velocity bias in INS vertical direction	meter/sec	Real
27-50	Standard deviations of state vector elements	---	Real
51-59	Aircraft attitude matrix	N/A	Real

TABLE I

BET OUTPUT RECORD FORMAT (CONT'D)

Word	Variable	Units	Representation
60-65	First six elements of covariance matrix, in packed form	---	Real
66-89	State vector from filter solution	---	Real
90-113	Standard deviation of state vector elements from filter solution	---	Real
114	Measurement Residual, Laser 1 Range	meter	Real
115	Measurement Residual, Laser 1 Azimuth	radian	Real
116	Measurement Residual, Laser 1 Elevation	radian	Real
117-119	Measurement Residuals, Laser 2	---	Real
120-122	Measurement Residuals, Laser 3	---	Real
123	Measurement Residual: INS velocity in INS east direction	meter/sec	Real
124	Measurement Residual: INS velocity in INS north direction	meter/sec	Real
125	Measurement Residual: INS velocity in INS vertical direction	meter/sec	Real

Time (Word 1)

Word 1 gives the time, in seconds, of UT ("Universal time," or GMT). Since local time at Yuma Proving Ground is Mountain Standard Time (MST), the time in word 1 will be greater by 7 hr = 25200 sec than local time.*

Source (Word 2)

Word 2 indicates:

(1) Whether the BET is the result of a filter pass only, or the result of a smoother pass.

(2) Which sensors contributed to the solution in the filter (not smoother) pass. A sensor will be indicated as contributing if it had meaningful data at the time its last measurement set was due and at least two of three of the measurements were accepted (not edited in the filter).

Table II gives the various values which word 2 (source) can take on.

BET State Vector (Words 3-26)

The BET, based upon measurements from one to three laser trackers and, on some flights, on inertial measurement unit, is generated by a Kalman filter with an optional smoother. The program develops estimates of the host vehicle position, velocity, and acceleration, of the range, azimuth, and elevation biases of each laser tracker, and of the orientation and velocity biases from the inertial navigation system (INS). These quantities are referred to as the "state vector," which will contain from 12 to 24 elements, depending upon the number of sensors employed. The program also computes the covariance matrix containing the expected errors of the state vector parameters and the expected correlations among the errors.

Normally, the BET will be the result of a smoother pass, and the state vector (words 3-26) will be generated by the smoother pass. Should

*It is anticipated that YPG range time will be expressed in UT for GPS testing. Should range time still be expressed in MST, the correction will be performed by software.

TABLE II

SOURCE IDENTIFICATION FOR BET (FORMAT WORD 2)

SOURCE	VALUE	
	Filter Pass	Smoother Pass
GPS (Telemetered Data)	-999999	999999
No data (extrapolated)	-99999	99999
Laser 1	-1	1
Laser 2	-2	2
Laser 3	-3	3
PLS	-4	4
Lasers 1 & 2	-12	12
Lasers 1 & 3	-13	13
Lasers 2 & 3	-23	23
Lasers 1 & 2 & 3	-123	123
IMU	-1000	1000
IMU + Laser 1	-1001	1001
IMU + Laser 2	-1002	1002
IMU + Laser 3	-1003	1003
IMU + PLS	-1004	1004
IMU + Lasers 1 & 2	-1012	1012
IMU + Lasers 1 & 3	-1013	1013
IMU + Lasers 2 & 3	-1023	1023
IMU + Lasers 1 & 2 & 3	-1123	1123

the BET be generated by the filter pass only, the state vector, of course, will come from that pass.

Trajectory

Reference Point. Words 3-11 contain the position, velocity, and acceleration of the point for which the BET is calculated. If the BET is based entirely upon laser data, the calculation point will be the laser retroreflector. If the BET is based upon INS/IMU plus laser data or upon INS/IMU plus PLS data, the calculation point will be the gimbal center of the IMU.

Coordinate System. The trajectory will be referred to a cartesian coordinate system ("IRCC coordinate system") which will be centered at the IRCC with the x, y, z axis directed in the local east, north, vertical directions, respectively.

Position. Words 3, 4, and 5 contain x, y, and z, the coordinates of the reference point (retroreflector or IMU); position coordinates will be expressed in meters.

Velocity. Words 6, 7, and 8 contain $\dot{x}$, $\dot{y}$, and $\dot{z}$, the velocity components of the reference point, expressed in meters/sec.

Acceleration. Words 9, 10, 11 contain $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, the acceleration components of the reference point, expressed in meter/sec^2.

Laser Bias States

Laser 1 — Word 12 contains the filter estimate of the range bias of Laser 1, expressed in meters.

Word 13 contains the filter estimate of the azimuth bias of Laser, expressed in radians.

Word 14 contains the filter estimate of the elevation bias of Laser 1, expressed in radians.

Laser 2 — Words 15-17 contain the filter estimates of the range, azimuth, and elevation biases of Laser 2, expressed, respectively, in meters, radians, and radians. Should only one laser have been employed, words 15-17 will contain zero.

Laser 3 Words 18-20 contain the filter estimates of the range, azimuth and elevation biases of Laser 3, expressed, respectively, in meters, radians, and radians. Should only one or two lasers have been employed, words 18-20 will contain zero.

INS Bias States

Words 21-26 contain the filter estimates of the INS bias state. Should no IMU have been employed, words 21-26 will contain zero.

Words 21-23 contain the estimated INS orientation biases about the INS east, north, and vertical axes, expressed radians.

Words 24-26 contain the estimated INS velocity biases in the INS east, north, and vertical directions, expressed in meter/sec.

Standard Deviations of State Vector Elements (Words 27-50)

Words 27-50 contain the uncertainty (standard deviations) in the estimated values of the state variables, computed by taking the square roots of the diagonal elements of the covariance matrix. The uncertainties are in the same order as the state vector elements, and expressed in the same units. Thus word 27 contains the standard deviation of the x position coordinate (word 3) and is expressed in meters; similarly, word 41 contains the standard deviation of the elevation bias of Laser 2 (word 17) and is expressed in radians.

When a bias state word contains zero because the sensor was not employed, the corresponding standard deviation will likewise contain zero.

Aircraft Attitude (Words 51-59)

In order to compare the GPS solution with the BET, it is necessary to transform the BET position to the position of the GPS reference location on the aircraft. This transformation is

$$\vec{X}_r = \vec{X}_c + B\,\delta\vec{X}_b$$

in which B is a tranformation (rotation) matrix and

$\dot{X}_r$ = position of GPS reference location, in IRCC coordinates.

$\dot{X}_c$ = position of point for which BET was calculated, in IRCC coordinates.

$\delta\dot{X}_b$ = relative location of GPS reference location to BET calculation point, in aircraft body coordinates.

The 3 x 3 matrix B(I, J) is contained in words 51 through 59. The first and second indices denote row and column, respectively.

Word	Matrix Element
51	B(1, 1)
52	B(2, 1)
53	B(3, 1)
54	B(1, 2)
55	B(2, 2)
56	B(3, 2)
57	B(1, 3)
58	B(2, 3)
59	B(3, 3)

Covariance Matrix (Words 60-65)

In words 60-65 are recorded, in packed form, the covariance elements corresponding to position states. The matrix is written out by half columns

Word	Covariance Element
60	$P(1, 1) = \sigma_x^2$
61	$P(1, 2) = P(2, 1) = \rho_{12}\,\sigma_x\,\sigma_y$
62	$P(2, 2) = \sigma_y^2$
63	$P(1, 3) = P(3, 1) = \rho_{13}\,\sigma_x\,\sigma_z$
64	$P(2, 3) = P(3, 2) = \rho_{23}\,\sigma_y\,\sigma_z$
65	$P(3, 3) = \sigma_z^2$

with the ρ_{ij} being correlation coefficients.

State Vector from Filter Solution (Words 66-89)

Words 66-89, containing the estimated values of the state variables which were computed during the filter pass, correspond in order, units, and representation to words 3-26. Should the BET be based upon the filter pass alone, words 66-89 will all contain zero.

Standard Deviation of State Vector Elements from Filter Solution (Words 90-113)

Words 90-113, containing the standard deviations of the values of the state variables estimated during the filter pass, correspond in order, units, and representation to words 27-50. Should the BET be based upon the filter pass alone, words 90-113 will all contain zero.

Table III summarizes the location in the BET output record of the values of the state vector elements and their uncertainties, depending upon whether the parameter was determined in the smoother pass or in the filter pass.

Measurement Residuals (Words 114-125)

The capability to calculate, by means of a measurement model, the value which a measurement would give in the absence of noise, is inherent in a Kalman filter. (The measurement model calculation is based upon the value of the state vector elements.) The measurement residuals contained in words 114-125 are the result of applying the measurement model to the BET state vector and forming the difference:

(Residual) = ("Measurement") - (Measurement Model)

For the laser measurements, the "measurement" is the value produced by pre-editing laser data and pre-filtering from 20 samples per second to 5 samples per second. A separate laser residual is calculated for each BET output record. If a laser was not employed, its measurement residuals will contain zero.

The data interval for IMU/INS data is expected to be variable, within a pass, but to be greater than 0.2 sec. Consequently, the INS measurement residuals will be included if an INS measurement set occurred within the previous 0.2 sec and will be zero otherwise. If an IMU was not employed, all INS measurement residuals will be zero.

TABLE III

LOCATION OF STATE VECTOR ELEMENTS AND UNCERTAINTIES

Variable	Units	State Vector (Smoother) Element	State Vector (Smoother) Uncertainty	State Vector (Filter) Element	State Vector (Filter) Uncertainty
x	meter	3	27	66	90
y	meter	4	28	67	91
z	meter	5	29	68	92
$\dot{x}$	meter/sec	6	30	69	93
$\dot{y}$	meter/sec	7	31	70	94
$\dot{z}$	meter/sec	8	32	71	95
$\ddot{x}$	meter/sec^2	9	33	72	96
$\ddot{y}$	meter/sec^2	10	34	73	97
$\ddot{z}$	meter/sec^2	11	35	74	98
Laser 1 range bias	meter	12	36	75	99
Laser 1 azimuth bias	radian	13	37	76	100
Laser 1 elevation bias	radian	14	38	77	101
Laser 2 range bias	meter	15	39	78	102
Laser 2 azimuth bias	radian	16	40	79	103
Laser 2 elevation bias	radian	17	41	80	104
Laser 3 range bias	meter	18	42	81	105
Laser 3 azimuth bias	radian	19	43	82	106
Laser 3 elevation bias	radian	20	44	83	107
INS east axis orientation bias	radian	21	45	84	108
INS north axis orientation bias	radian	22	46	85	109
INS vertical axis orientation bias	radian	23	47	86	110
INS east velocity bias	meter/sec	24	48	87	111
INS north velocity bias	meter/sec	25	49	88	112
INS vertical velocity bias	meter/sec	26	50	89	113

APPENDIX C

This appendix contains samples of the types of data available as line printer output from the GPSBET program. The first four pages are an example of the Kalman filter control report. This report lists input control parameters and a priori information for the Kalman filter. Next are the first three pages of a typical run of the Kalman filter accepting data from three lasers and INS (24 components in the state vector). The last two pages of this appendix are copies of the first two pages of the Kalman smoothing report. Note that output from Kalman smoothing is in reverse chronological order (time backs up) since smoothing is accomplished by a backwards recursion on the filtered output.

YUMA PROVING GROUND - DATA ACQUISITION AND REDUCTION BRANCH

CONTROL REPORT FOR KALMAN FILTER

TEST ITEM	WORK ORDER NUMBER	PROJECT ENGINEER	DATE OF TEST	DATE OF REPORT
END AROUND CHECK	G P S	MAJ ABBEY	24 MAY 76	24 MAY 76

CASE 2 OF THE END AROUND CHECK

THIS IS TRAJECTORY NUMBER 1 OF JOB NUMBER 1
THERE ARE 1 TRAJECTORIES IN THIS JOB
THERE ARE 1 JOBS IN THIS COMPUTER RUN

OPTIONS SELECTED FOR THIS TRAJECTORY ARE

SMOOTHING PASS	IMU DATA	LEVER ARM CORRECTIONS	REFRACTION CORRECTIONS	REWIND INPUT TAPE	READ TO AN EOF
YES	YES	YES	NO	NO	YES

TIME LIMITS FOR BET CALCULATION

	HOUR	MIN	SEC	MSEC -----	SECONDS
START TIME......	12	30	0	0 -----	45000.000
STOP TIME......	24	0	0	0 -----	86400.000

MAXIMUM NUMBER OF MEASUREMENT SETS TO FILTER =40000

NUMBER OF INSTRUMENTS USED = 4

NUMBER OF STATE VECTOR COMPONENTS = 24

INSTRUMENT AND DATA RATES

ORDER FOR PROCESSING	INSTRUMENT TYPE	SYMBOLIC INPUT UNIT NUMBER	RAW DATA SAMPLING INC	PREFILTERED SAMPLING INC	FILES TO BE SKIPPED
1	LASER	15	0.05	0.20	0
2	LASER	15	0.05	0.20	0
3	LASER	15	0.05	0.20	0
4	INS	92	0.50	0.50	0

GEODETIC COORDINATES OF ORIGIN OF COORDINATE SYSTEM AND OF EACH INSTRUMENT

	LATITUDE DEG	MIN	SECONDS	LONGITUDE DEG	MIN	SECONDS	ALTITUDE METERS
ORIGIN	33.	1.	4.800	-114.	-23.	-50.000	146.304
1	33.	1.	24.995	-114.	-22.	-14.983	188.476

CARTESIAN COORDINATES OF LASERS WITH RESPECT TO THE ORIGIN

	XI	YI	ZI
1	2465.853	622.504	42.165
2	3888.358	11313.637	121.334
3	-5864.986	1178.471	-42.427

START AND STOP TIMES FOR USING DATA FROM EACH INSTRUMENT

	HOUR	MIN	SEC	MSEC	-----	SECONDS
START TIME.....	12	30	0	0	-----	45000.000
STOP TIME......	24	-0	-0	-0	-----	86400.000
START TIME.....	12	30	0	0	-----	45000.000
STOP TIME......	24	-0	-0	-0	-----	86400.000
START TIME.....	12	30	0	0	-----	45000.000
STOP TIME......	24	-0	-0	-0	-----	86400.000
START TIME.....	12	30	0	0	-----	45000.000
STOP TIME......	24	0	0	0	-----	86400.000

LINE PRINTER OUTPUT CONTROL FOR INTERMEDIATE REPORTS DURING FILTERING

	MAXIMUM NUMBER OF POINTS BETWEEN PRINTOUTS
STATE VECTOR	500 POINTS
SIGMA VECTOR	500 POINTS
RMS OF RESIDUALS	500 POINTS
RESIDUALS REPORT	40000 POINTS
COVARIANCE MATRIX	170 POINTS

MODULUS IN MILLISECONDS FOR ADDITIONAL OUTPUT CONTROL

STATE VECTOR (FILTER)	1000
SIGMA VECTOR (FILTER)	10000
COVARIANCE (FILTER)	11111
RMS OF RESIDUALS (FILTER)	10000
STATES, SIGMAS, DIFFERENCES TO LP (FILTER AND SMOOTH)	1000
SET OUTPUT TO TAPE	200

PREFILTER CONTROL INFORMATION

INSTRUMENT NUMBER	PREFILTER OR SKIP	ORDER OF PREFILTER	NO. POINTS TO PREF OR SKIP
1	PREFILTER	QUADRATIC	5
2	PREFILTER	QUADRATIC	5
3	PREFILTER	QUADRATIC	5
4	SKIP	NA	1

LEVER ARMS IN BODY COORDINATES

	X METERS	Y METERS	Z METERS
RETRO	-3.348	-0.762	1.214
IMU	0.	0.	0.
GPS	0.	0.	0.

RECEIVERS SELECTED FOR IMU/INS DATA

RECEIVER NUMBER

CALIBRATION COEFFICIENTS

INSTRUMENT NUMBER	AZ BIAS (DEG)	EL BIAS (DEG)	COLLIMATION (DEG)	NORTH TILT (DEG)	EAST TILT (DEG)	RANGE BIAS (FEET)	RANGE SCALE FACTOR
1	-0.0040	0.0220	0.	-0.0160	0.0050	-4.00	0.
2	0.0020	-0.0150	0.	0.0210	0.0030	1.00	0.
3	0.	0.	0.	0.	0.	0.	0.

METEOROLOGICAL DATA

INSTRUMENT NUMBER	TEMPERATURE (DEG KELVIN)	PRESSURE (MILLIBARS)
1	302.71	1000.00
2	303.68	990.00
3	303.15	1010.00

EDITING CRITERIA - NUMBER OF STANDARD DEVIATIONS

PRE EDIT (ALL INSTRUMENTS)

RANGE (OR VE)	AZIMUTH (OR VN)	ELEVATION (OR VZ)
10.00	10.00	10.00

AFTER PREFILTER (ALL INSTRUMENTS)

RANGE (OR VE)	AZIMUTH (OR VN)	ELEVATION (OR VZ)
5.00	5.00	5.00

EXPECTED MEASUREMENT NOISE

INSTRUMENT NUMBER	MEASUREMENT NUMBER	STANDARD DEVIATIONS
1	1	0.500000E-03
	2	0.200000E-03
	3	0.200000E-03
2	1	0.500000E-03
	2	0.200000E-03
	3	0.200000E-03
3	1	0.500000E-03
	2	0.200000E-03
	3	0.200000E-03
4	1	0.100000E-01
	2	0.100000E-01
	3	0.100000E-01

A PRIORI INFORMATION ABOUT EXPECTED STATE VECTOR PERTURBATIONS PER SECOND

	X	Y	Z	XVEL	YVEL	ZVEL	XACC	YACC	ZACC
	0.	0.	0.	0.	0.	0.	4.400	4.400	4.400
BIASES	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.110000E-05	0.110000E-05	0.110000E-05	0.220000E-03	0.220000E-03	0.220000E-03			

INITIAL VALUES FOR STATE VECTOR UNCERTAINTY

COMPONENT NO.	1	2	3	4	5	6	7	8	9
STAN DEVS	30.000	30.000	30.000	50.000	50.000	50.000	20.000	20.000	20.000

10	11	12	13	14	15	16	17	18
0.500000E 00	0.100000E-03	0.100000E-03	0.500000E 00	0.100000E-03	0.100000E-03	0.500000E 00	0.100000E-03	0.100000E-03

19	20	21	22	23	24
0.300000E-03	0.300000E-03	0.100000E-02	0.200000E 01	0.200000E 01	0.200000E 01

INITIAL VALUES READ IN FOR STATE VECTOR

COMPONENT NO.	1	2	3	4	5	6	7	8	9
STAN DEVS	0.	-4137.770	2444.510	0.	-231.650	-0.330	0.	0.	0.

10	11	12	13	14	15	16	17	18
-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19

19	20	21	22	23	24
-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19	-0.000000E-19

YUMA PROVING GROUND - DATA ACQUISITION AND REDUCTION BRANCH

OUTPUT REPORTS FROM KALMAN FILTER

TEST ITEM	WORK ORDER NUMBER	PROJECT ENGINEER	DATE OF TEST	DATE OF REPORT
FNL AFFUNE CHECK	G P S	MAJ ABBEY	24 MAY 76	24 MAY 76

CASE 2 F OF THE END AROUND CHECK
IL IS 30

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STATE AT TIME  45010.000 SECOND     -64.170 -11404.258   2431.648    -65.636   -231.688     -0.614    -39.318      9.776     -0.063
 -0.203596F 00  0.296306F-04  0.108125F-03 -0.346385E 00  0.146359E-04  0.546433E-04  0.477660E 00  0.154625F-03 -0.162880F-03
  0.149103F-05  0.172666E-05  0.108274E-03  0.489256E 00  0.606712F 03  0.339831E 00
SIGMA AT TIME  45010.000 SECOND       0.687      0.334      0.683      0.472      0.461      0.476      2.349      2.267      2.358
  0.327021F 00  0.558765F-04  0.698940F-04  0.320359E 00  0.312116F-04  0.381543E-04  0.387356F 00  0.461353F-04  0.614847F-04
  0.299830E-03  0.299872E-03  0.943705F-03  0.220328E 00  0.287749E-01  0.849709E-01
```

RESIDUALS REPORT AT TIME 45010.000
RESIDUALS ARE PREDICTED MINUS MEASUREMENTS

INSTRUMENT IDENTIFICATION	MEASUREMENT IDENTIFICATION	MEAN OF RESIDUALS	RMS OF RESIDUALS	NUMBER OF POINTS
LASER 75601	1	-0.282619E-02	0.958452F-01	46
LASER 75601	2	-0.828310E-05	0.184739F-04	46
LASER 75601	3	-0.789855E-05	0.974363F-05	46
LASER 15456	1	0.955333E-03	0.634676F-01	46
LASER 15456	2	-0.532619E-05	0.107265F-04	46
LASER 15456	3	-0.549304E-05	0.676323F-05	46
LASER 16238	1	0.447149E-01	0.138558F 00	46
LASER 16238	2	-0.111314E-04	0.157761F-04	46
LASER 16238	3	0.153942E-04	0.190225F-04	46
INS 58457	1	0.683440E-04	0.128338F-03	18
INS 58457	2	-0.153648E-04	0.826550F-04	18
INS 58457	3	0.204917E-06	0.592257F-05	18

IL IS 30
IL IS 30

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STATE AT TIME  45020.000 SECOND   -1931.183 -11893.593   2433.140   -160.985    166.144      1.075     31.745     36.130      0.151
 -0.190153E 00  0.262716E-04  0.138869F-03 -0.363516F 00  0.133756E-04  0.693106F-04  0.507926F 00  0.148431F-03 -0.157109F-03
 -0.102956F-03 -0.847112F-04  0.116487F-03  0.508492F 00  0.598571F 00  0.372727E 00
SIGMA AT TIME  45020.000 SECOND       0.676      0.334      0.909      0.475      0.454      0.480      2.350      2.264      2.358
  0.323133F 00  0.510723F-04  0.602250F-04  0.313737E 00  0.285530F-04  0.365426E-04  0.370010F 00  0.437511F-04  0.600044F-04
  0.240872F-03  0.246110F-03  0.150348F-03  0.308741F-01  0.235795F-01  0.584307E-01
```

RESIDUALS REPORT AT TIME 45020.000
RESIDUALS ARE PREDICTED MINUS MEASUREMENTS

INSTRUMENT IDENTIFICATION	MEASUREMENT IDENTIFICATION	MEAN OF RESIDUALS	RMS OF RESIDUALS	NUMBER OF POINTS
LASER 75601	1	0.715736F-01	0.126692F 00	50
LASER 75601	2	0.650823F-05	0.151731F-04	50
LASER 75601	3	-0.558075E-05	0.699450E-05	50
LASER 15456	1	0.480378E-01	0.792804E-01	50
LASER 15456	2	0.310659E-05	0.876553E-05	50
LASER 15456	3	-0.645636E-05	0.853188E-05	50

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        LASER 16238          3
          INS  58497         1                     0.138341E-05        0.188641F-03        20
          INS  58497         2                    -0.422277E-04        0.126109E-03        20
          INS  58497         3                    -0.252409E-05        0.785319F-05        20
 IT IS    30

STATE AT TIME  45030.000 SEC=01   -2024.991   -9703.373   2594.449    32.797    216.834    46.696     0.199    -1.205     4.203
  -0.179067F 00  0.210489E-04  0.142827F-03 -0.366368F 00  0.104854E-04  0.762805E-04  0.569129E 00  0.142050F-03 -0.153070F-03
  -0.123007F-03 -0.593456F-04  0.567183F-04  0.497924F 00  0.597543F 00  0.379568E 00
SIGMA AT TIME  45030.000 SEC=01      2.495       0.328       0.729       0.470       0.454      0.474      2.344      2.266      2.355
   0.314512F 00  0.423710F-04  0.624309E-04  0.305307F 00  0.230123E-04  0.343729E-04  0.337949E 00  0.380358F-04  0.580074F-04
   0.110466F-03  0.156215F-03  0.832346F-04  0.159055E-01  0.159620E-01  0.247304E-01

                                  RESIDUALS REPORT AT TIME  45030.000
                             RESIDUALS ARE PREDICTED MINUS  MEASUREMENTS

               INSTRUMENT          MEASUREMENT         MEAN OF             RMS OF              NUMBER OF
               IDENTIFICATION      IDENTIFICATION      RESIDUALS           RESIDUALS           POINTS

               LASER 75601              1             -0.658325E-01        0.154735E 00           50
               LASER 75601              2             -0.447590E-05        0.280167F-04           50
               LASER 75601              3             -0.499174E-05        0.896691F-05           50
               LASER 15456              1             -0.174609E-01        0.673369F-01           50
               LASER 15456              2             -0.190794E-05        0.147921F-04           50
               LASER 15456              3             -0.395447E-05        0.678142E-05           50
               LASER 16238              1             -0.237544E-01        0.164764F 00           50
               LASER 16238              2              0.281930E-06        0.181302E-04           50
               LASER 16238              3              0.992831E-05        0.138271F-04           50
                 INS  58497             1              0.132894E-04        0.287738F-03           20
                 INS  58497             2              0.261508E-04        0.178441F-03           20
                 INS  58497             3             -0.114938E-04        0.391555F-04           20
 IT IS    30

          CORRELATION COEFFICIENTS AND STANDARD DEVIATIONS FOR STATE VECTOR COMPONENTS AT TIME  45034.200
                THIS MATRIX WAS COMPUTED AFTER FILTERING A DATA POINT FROM  INS  79601

  0.442   -0.053    0.035   -0.165    0.040   -0.015   -0.344   -0.000   -0.000    0.452    0.322   -0.062
     0.357    0.891   -0.037   -0.568    0.923    0.021   -0.001    0.058    0.351   -0.076    0.012
     0.004
 -0.053    0.321    0.307    0.015    0.110   -0.133   -0.000   -0.062    0.000    0.700   -0.258   -0.390
     0.445   -0.198   -0.353    0.717    0.158   -0.393   -0.227   -0.030    0.024   -0.077   -0.055
    -0.077
  0.035    0.307    0.452   -0.355    0.140   -0.197    0.000   -0.000   -0.030   -0.204    0.004   -0.950
     0.021    0.017   -0.535   -0.233    0.109   -0.547   -0.395   -0.059    0.141    0.009   -0.034
    -0.095
 -0.164    0.015   -0.055    0.024   -0.250   -0.156    0.011    0.000    0.000   -0.083   -0.266    0.026
    -0.057   -0.256    0.020    0.183   -0.249    0.003    0.025    0.266   -0.615   -0.522    0.179
     0.140
  0.040    0.110    0.140   -0.230    0.019   -0.102    0.001    0.019   -0.001    0.030    0.056   -0.144
     0.058    0.052   -0.140   -0.040    0.085   -0.139   -0.223   -0.337    0.334   -0.021   -0.792
    -0.056
 -0.015   -0.133   -0.157   -0.156   -0.102    0.032    0.000   -0.001    0.008    0.010    0.037    0.349
    -0.079    0.027    0.342    0.002   -0.035    0.350    0.555   -0.502   -0.003   -0.049   -0.067
    -0.056
 -0.344   -0.000    0.000    0.011    0.001    0.000    0.007   -0.004    0.001   -0.000   -0.000    0.000
    -0.000   -0.000    0.000   -0.000   -0.000    0.000   -0.000   -0.001    0.001    0.001   -0.001
    -0.000
 -0.000   -0.062   -0.000    0.000    0.019   -0.001   -0.004    0.495    0.007    0.000   -0.000    0.000
     0.000   -0.000    0.000    0.000   -0.000    0.000   -0.001    0.001    0.000   -0.000   -0.001
     0.001
 -0.000    0.000   -0.030    0.000   -0.001    0.008    0.001    0.007    0.908   -0.000    0.000    0.010
    -0.000    0.000    0.000   -0.000   -0.000    0.000   -0.001    0.001   -0.000    0.000    0.001
     0.001
  0.452    0.700   -0.204   -0.083    0.030    0.010   -0.000    0.000   -0.000    0.309    0.266    0.103
     0.960    0.257    0.142    0.502    0.594    0.144    0.016    0.007    0.237   -0.092   -0.007
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 0.922   -0.256    0.004   -0.266    0.056    0.037   -0.000   -0.000    0.000    0.266    0.000    0.000
    0.146    0.919    0.018   -0.677    0.868    0.082    0.040    0.027    0.403    0.004   -0.014
            -0.022
 0.362   -0.350   -0.550    0.026   -0.144    0.349    0.000    0.000    0.000    0.103    0.000    0.000
   -0.133   -0.019    0.564    0.157   -0.153    0.979    0.468   -0.019   -0.045   -0.004    0.004
            -0.015
 0.357    0.845    0.021   -0.057    0.058   -0.079   -0.000    0.000   -0.000    0.960    0.146   -0.133
    0.300    0.169   -0.090    0.564    0.532   -0.100   -0.106    0.008    0.171   -0.093   -0.007
             0.306
 0.491   -0.158    0.017   -0.256    0.058    0.027   -0.000   -0.000    0.000    0.297    0.919   -0.019
    0.169    0.000   -0.001   -0.016    0.850    0.060    0.027    0.026    0.341   -0.001   -0.014
            -0.020
-0.037   -0.253   -0.535    0.020   -0.140    0.342    0.000    0.000    0.000    0.142    0.016    0.964
   -0.090   -0.001    0.000    0.170   -0.120    0.962    0.474   -0.018   -0.033   -0.003    0.006
            -0.014
-0.508    0.717   -0.233    0.183   -0.040    0.002   -0.000    0.000   -0.000    0.502   -0.677    0.157
    0.564   -0.016    0.170    0.329   -0.362    0.113    0.015   -0.021   -0.217   -0.072    0.008
             0.015
 0.923    0.156    0.104   -0.245    0.085   -0.035   -0.000   -0.000   -0.000    0.594    0.868   -0.153
    0.532    0.650   -0.120   -0.362    0.000   -0.070   -0.057    0.027    0.411   -0.034   -0.015
            -0.015
 0.021   -0.353   -0.547    0.003   -0.135    0.350    0.000    0.000    0.000    0.144    0.082    0.979
   -0.100    0.060    0.562    0.113   -0.070    0.000    0.483   -0.017   -0.033   -0.006    0.005
            -0.016
-0.301   -0.227   -0.295    0.025   -0.223    0.556   -0.000   -0.001   -0.001    0.016    0.140    0.488
   -0.106    0.027    0.478    0.015   -0.057    0.468    0.000    0.155    0.038    0.012   -0.058
             0.162
 0.054   -0.030   -0.059    0.266   -0.037   -0.502   -0.001    0.001    0.001    0.007    0.027   -0.019
    0.008    0.026   -0.018   -0.021    0.027   -0.017    0.155    0.000    0.059    0.000    0.007
             0.047
 0.350    0.024    0.041   -0.615    0.338   -0.003    0.001    0.000   -0.000    0.207    0.403   -0.045
    0.171    0.591   -0.033   -0.217    0.411   -0.008    0.038    0.064    0.000   -0.064   -0.271
             0.025
-0.076   -0.077    0.009   -0.522   -0.021   -0.049    0.001   -0.000    0.000   -0.092    0.004   -0.004
   -0.093   -0.051   -0.003   -0.072   -0.034   -0.006    0.012    0.000   -0.034    0.015    0.012
             0.056
 0.012   -0.055   -0.034    0.175   -0.792   -0.057   -0.001   -0.001    0.001   -0.007   -0.014   -0.006
   -0.007   -0.014    0.006    0.008   -0.015    0.095   -0.053    0.007   -0.271    0.012    0.014
             0.038
 0.004   -0.027   -0.059    0.150   -0.668   -0.656   -0.000    0.001    0.001   -0.010   -0.022   -0.015
    0.006   -0.020   -0.014    0.019   -0.015   -0.016    0.162    0.047    0.025    0.058    0.035
             0.020
IT IS  30

STATE AT TIME  45040.000 SECONDS   -1675.892  -7514.205   3065.759     37.126    221.064     45.620     0.355     0.387    -0.852
 -0.241614E 00  0.175467E-03  0.113052E-03 -0.386035E 00  0.682113E-06  0.551513E-04  0.652437E 00  0.132583E-03 -0.181769E-03
 -0.151152E-03 -0.457955E-04  0.219035E-04  0.441571E 00  0.595306E 00  0.381023E 00
SIGMA AT TIME  45040.000 SECONDS      0.383      0.299      0.542      0.653      0.455      0.471      2.348      2.272      2.349
  0.247261E 00  0.176064E-04  0.556364E-04  0.264542E 00  0.209632E-04  0.301747E-04  0.516411E 00  0.353311E-04  0.514933E-04
  0.244612E-04  0.166658E-03  0.606579E-04  0.138180E-01  0.131343E-01  0.179070E-01
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RESIDUALS REPORT AT TIME 45140.000
RESIDUALS ARE PREDICTED MINUS MEASUREMENTS

INSTRUMENT IDENTIFICATION	MEASUREMENT IDENTIFICATION	MEAN OF RESIDUALS	RMS OF RESIDUALS	NUMBER OF POINTS
LASER 75601	1	0.151641E-01	0.343444E-01	50
LASER 75601	2	-0.374190E-05	0.606543E-05	50
LASER 75601	3	0.245043E-05	0.647655E-05	50
LASER 15456	1	0.475049E-01	0.626741E-01	50
LASER 15456	2	0.914700E-06	0.414841E-05	50
LASER 15456	3	0.727147E-05	0.918082E-05	50
LASER 16238	1	-0.357358E-01	0.524870E-01	50
LASER 16238	2	-0.114334E-04	0.147411E-04	50

YUMA PROVING GROUND - DATA ACQUISITION AND REDUCTION BRANCH

BET SMOOTH OUTPUT REPORT

TEST ITEM	WORK ORDER NUMBER	PROJECT ENGINEER	DATE OF TEST	DATE OF REPORT
END ARGUND CHECK	G P S	MAJ ABBEY	24 MAY 76	24 MAY 76

CASE 2 F OF THE END ARGUND CHECK

SMOOTHING STARTS AT TIME 45240.200

RANGE TIME 45260.000

	FILTERED		SMOOTHED		DIFFERENCE	F RATIO
	STATE VECTOR	SIGMA	STATE VECTOR	SIGMA	STATE VECTOR	VARIANCES
1	-0.6747888E 02	0.1707655E 00	-0.6748135E 02	0.1303128E 00	-0.2463109E-02	0.1310427E 01
2	-0.8990157E 04	0.1201320E 00	-0.8990174E 04	0.7754751E-01	0.2355957E-01	0.1549140E 01
3	0.2370686E 04	0.2050507E 00	0.2370720E 04	0.1862730E 00	0.3421021E-01	0.1122781E 01
4	0.1076931E 02	0.5483558E 00	0.1077410E 02	0.6891120E 00	-0.1120508E-01	0.1376264E 01
5	-0.2265423E 03	0.7411750E 00	-0.2263971E 03	0.4025162E 00	0.1451683E 00	0.1840907E 01
6	0.9650368E 01	0.9920788E 00	0.3815761E 01	0.8634543E 00	0.1591937E 00	0.1148965E 01
7	-0.1756580E 00	0.2889570E 01	-0.2077428E 00	0.2192334E 01	-0.3208478E-01	0.1318033E 01
8	0.1251947E 01	0.2460965E 01	0.1681763E 01	0.1507021E 01	0.4238158E 00	0.1593186E 01
9	-0.1244066E 01	0.2996224E 01	-0.7605019E 00	0.2645325E 01	0.4835038E 00	0.1132650E 01
10	0.3648159E 00	0.3033760E-01	0.3647705E 00	0.3033540E-01	-0.4453641E-04	0.1000070E 01
11	0.5363108E-04	0.3494064E-05	0.5533211E-04	0.3492307E-05	-0.2396741E-07	0.1000503E 01
12	0.1262555E-03	0.4545007E-05	0.1264693E-03	0.4544474E-05	0.1713743E-06	0.1000117E 01
13	-0.1470842E 00	0.2172643E-01	-0.1470510E 00	0.2672122E-01	0.3518723E-04	0.1000181E 01
14	0.4211553E-04	0.3682457E-05	0.4206779E-04	0.3683214E-05	-0.5173433E-07	0.1000621E 01
15	0.7582104E-04	0.3389437E-05	0.7589369E-04	0.3388017E-05	0.7259951E-07	0.1000356E 01
16	0.1157251E 01	0.2739690E-01	0.1157251E 01	0.2739415E-01	-0.4533558E-04	0.1000101E 01
17	0.2500824E-03	0.3681817E-05	0.2499773E-03	0.3680096E-05	-0.1050103E-06	0.1000468E 01
18	0.3155348E-03	0.4302850E-05	0.3153180E-03	0.4302172E-05	-0.2167944E-06	0.1000158E 01
19	-0.2373251E-03	0.2380634E-04	-0.2375527E-03	0.2380020E-04	-0.2235793E-06	0.1000258E 01
20	0.4650734E-05	0.7446470E-04	0.4419840E-05	0.7446322E-04	-0.2308936E-06	0.1000087E 01
21	-0.2043585E-04	0.2517827E-04	-0.2049104E-04	0.2517380E-04	-0.5323750E-07	0.1000178E 01
22	0.5173464E 00	0.5423057E-02	0.5173798E 00	0.5422305E-02	0.3412366E-05	0.1000139E 01
23	0.3394034E 00	0.5273662E-02	0.3393892E 00	0.5273553E-02	-0.1426414E-04	0.1000021E 01
24	0.3055178E 00	0.5449392E-02	0.3056659E 00	0.5448039E-02	-0.4788861E-04	0.1000248E 01

RANGE TIME 45280.000

	FILTERED		SMOOTHED		DIFFERENCE	F RATIO
	STATE VECTOR	SIGMA	STATE VECTOR	SIGMA	STATE VECTOR	VARIANCES
1	-0.7831658E 02	0.1100019E 00	-0.7532164E 02	0.9424110E-01	-0.5063057E-02	0.1167238E 01
2	-0.8763640E 04	0.7317857E-01	-0.8763516E 04	0.6270482E-01	0.1302490E 00	0.1167352E 01
3	0.2360450E 04	0.1556875E 00	0.2360491E 04	0.1410307E 00	0.4100491E-01	0.1107525E 01
4	0.1064847E 02	0.1274393E-01	0.1064644E 02	0.1270438E-01	-0.3015995E-04	0.1003137E 01
5	-0.2267451E 03	0.1131561E-01	-0.2267425E 03	0.1129005E-01	0.2611160E-02	0.1002264E 01
6	0.1071745E 02	0.1252099E-01	0.1071766E 02	0.1249086E-01	0.1780071E-03	0.1002412E 01
7	0.1256654E-01	0.9066980E 00	0.1675086E-01	0.8470344E 00	0.4174320E-02	0.1069341E 01
8	-0.5081278E 00	0.8947343E 00	-0.5239815E 00	0.8342242E 00	-0.1558537E 00	0.1071707E 01
9	-0.8504673E 00	0.9080735E 00	-0.8591512E 00	0.8502543E 00	-0.8393941E-02	0.1068002E 01
10	0.3656676E 00	0.3034844E-01	0.3647704E 00	0.3033546E-01	-0.2971590E-03	0.1000428E 01

14	0.4238355E-04	0.3693987E-05	0.4206779E-04	0.3690214E-05	-0.3152004E-06	0.1003763E 01
15	0.7542154E-04	0.3496634E-05	[illegible]	0.3983017E-05	0.4717431E-06	[illegible]
16	0.1157324E 01	0.2741107E-01	[illegible]	0.2739415E-01	-0.1132935E-03	[illegible]
17	0.2505239E-03	0.3690454E-05	0.2499773E-03	0.3680096E-05	-0.5456537E-06	0.1002815E 01
18	0.3165852E-03	0.4306302E-05	0.3153180E-03	0.4302172E-05	-0.1267272E-05	0.1000960E 01
19	[illegible]	0.2382623E-04	-0.2375527E-03	0.2377474E-04	-0.4635496E-06	0.1002164E 01
20	[illegible]	0.7451627E-04	0.4419845E-05	0.7445537E-04	-0.4545495E-06	0.1001390E 01
21	[illegible]	0.2521564E-04	-0.2049311E-04	0.2514978E-04	0.3164246E-07	[illegible]
22	[illegible]	[illegible]	0.5173798E 00	0.5417845E-02	0.3425777E-06	[illegible]
23	[illegible]	[illegible]	[illegible]	0.5264106E-02	-0.1254863E-03	[illegible]
24	[illegible]	[illegible]	[illegible]	0.5443598E-02	-0.1326653E-03	[illegible]

RANGE TIME 49258.000

	FILTERED		SMOOTHED		DIFFERENCE	F RATIO
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2	[illegible]	0.7331203E-01	-0.4537179E 04	0.6124034E-01	0.1735840E 00	[illegible]
3	[illegible]	0.1533752E 00	0.2349374E 04	0.1305124E 00	0.7458496E-01	[illegible]
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6	[illegible]	0.1253229E-01	0.1145201E 02	0.1247459E-01	[illegible]	[illegible]
7	[illegible]	[illegible]	0.2398598E-01	0.8459393E 00	0.6144634E-04	[illegible]
8	[illegible]	[illegible]	-0.5028700E 00	0.8341713E 00	-0.1002822E 00	[illegible]
9	[illegible]	[illegible]	-0.4534493E 00	0.8473632E 00	-0.4710204E-02	[illegible]
10	[illegible]	[illegible]	0.3647704E 00	0.3033546E-01	[illegible]	[illegible]
11	[illegible]	[illegible]	0.5633211E-04	0.3492307E-05	[illegible]	[illegible]
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14	[illegible]	[illegible]	0.4206779E-04	0.3683214E-05	[illegible]	[illegible]
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17	[illegible]	0.3699136E-05	0.2499773E-03	0.3680096E-05	[illegible]	[illegible]
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RANGE TIME 49257.000

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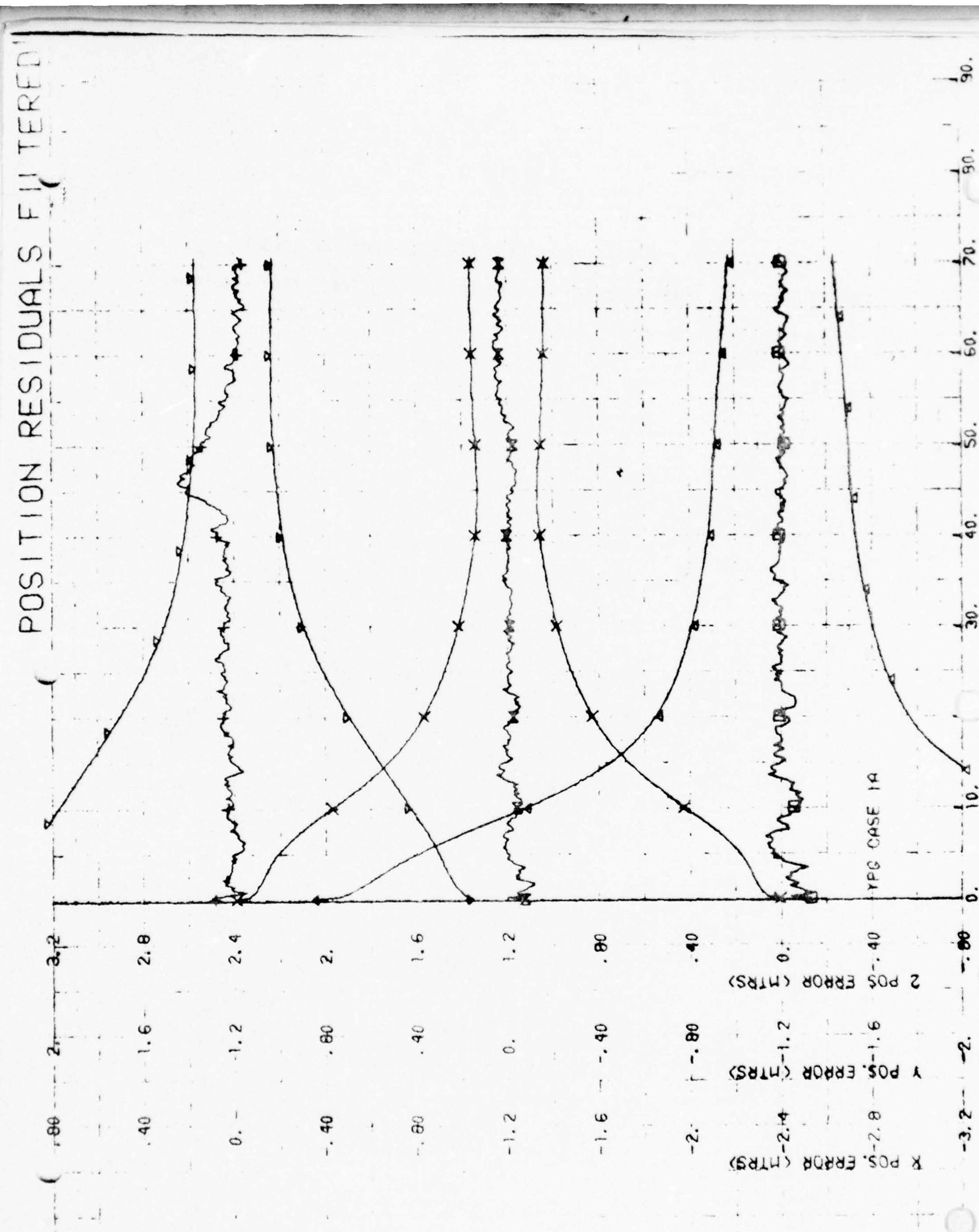
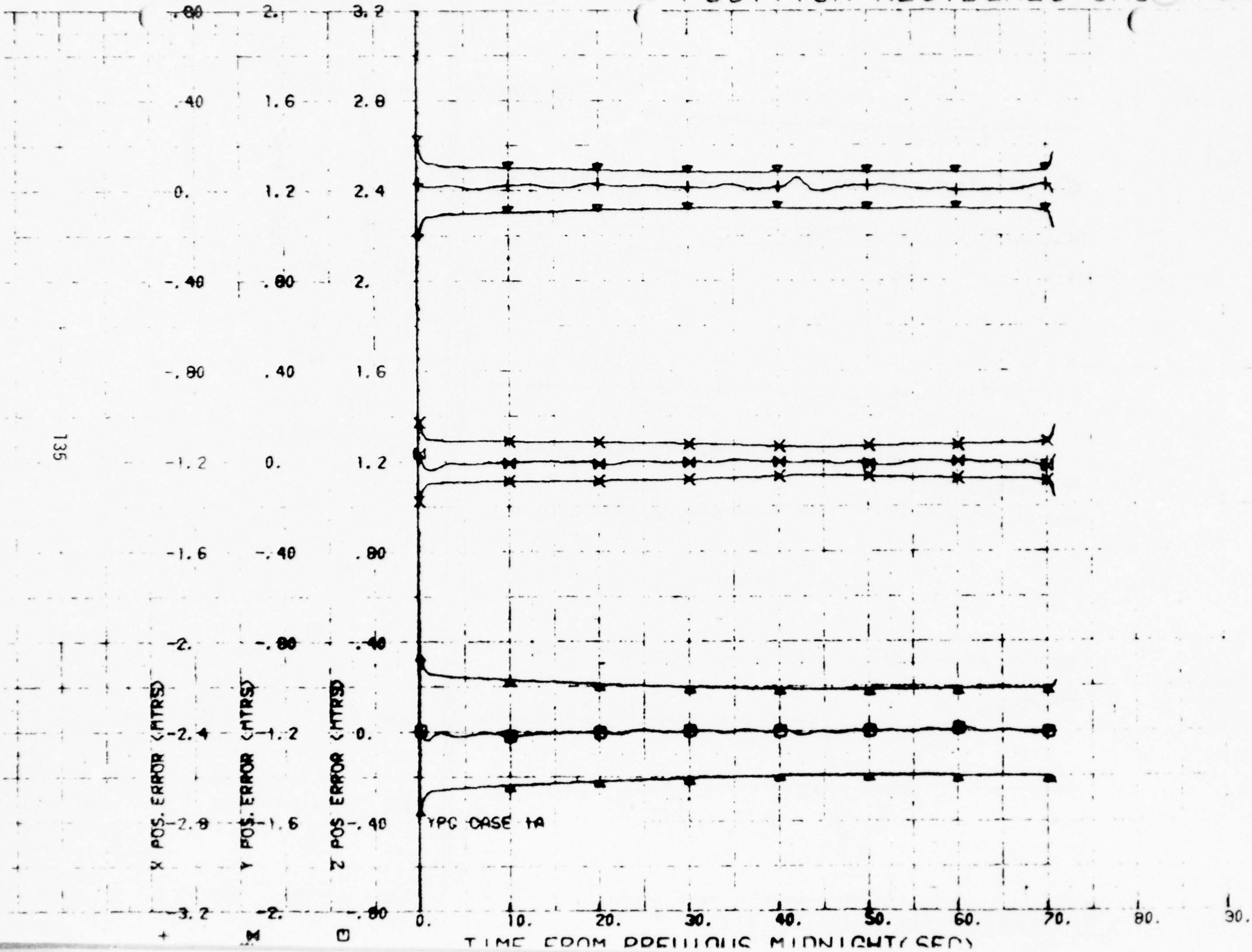
3	[illegible]	0.1516540E 00	0.2337787E 04	0.1240096E 00	0.1015859E 00	0.1222927E 01
4	[illegible]	0.1279294E-01	0.1082905E 02	0.1268713E-01	-0.1254082E-03	0.1008340E 01
5	[illegible]	0.1132299E-01	-0.2260030E 03	0.1127748E-01	0.5939175E-02	0.1004036E 01
6	[illegible]	0.1254248E-01	0.1169110E 02	0.1246162E-01	[illegible]	0.1006529E 01
7	[illegible]	0.9065116E 00	0.1168265E-01	0.8453463E 00	-0.4267920E-02	0.1017721E 01
8	[illegible]	[illegible]	-0.5343492E 00	0.8342375E 00	-0.7442778E-01	0.1012772E 01
9	[illegible]	0.9274808E 00	0.2082199E 00	0.8473188E 00	-0.2643355E-01	0.1071599E 01
10	[illegible]	0.3037038E-01	0.3647704E 00	0.3033546E-01	-0.3447543E-03	0.1011151E 01
11	[illegible]	0.3523585E-05	0.5633211E-04	0.3492307E-05	-0.5690999E-06	0.1008097E 01
12	[illegible]	0.4553221E-05	0.1264593E-03	0.4544474E-05	0.2842017E-05	0.1001925E 01
13	[illegible]	0.2880569E-01	-0.1470510E 00	0.2872122E-01	0.2249178E-02	0.1002941E 01
14	[illegible]	0.3717324E-05	0.4206779E-04	0.3683214E-05	[illegible]	[illegible]

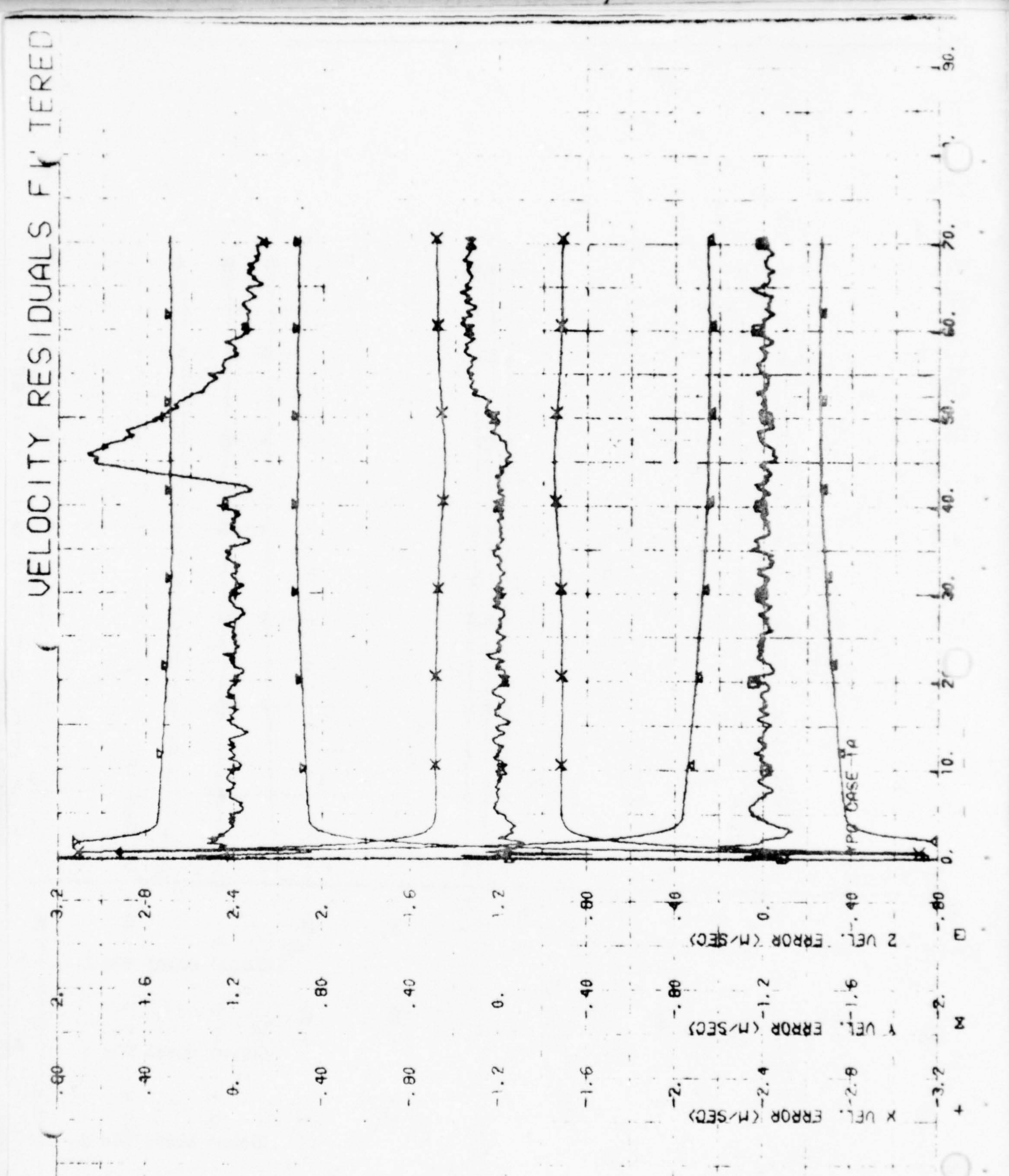
Appendix D

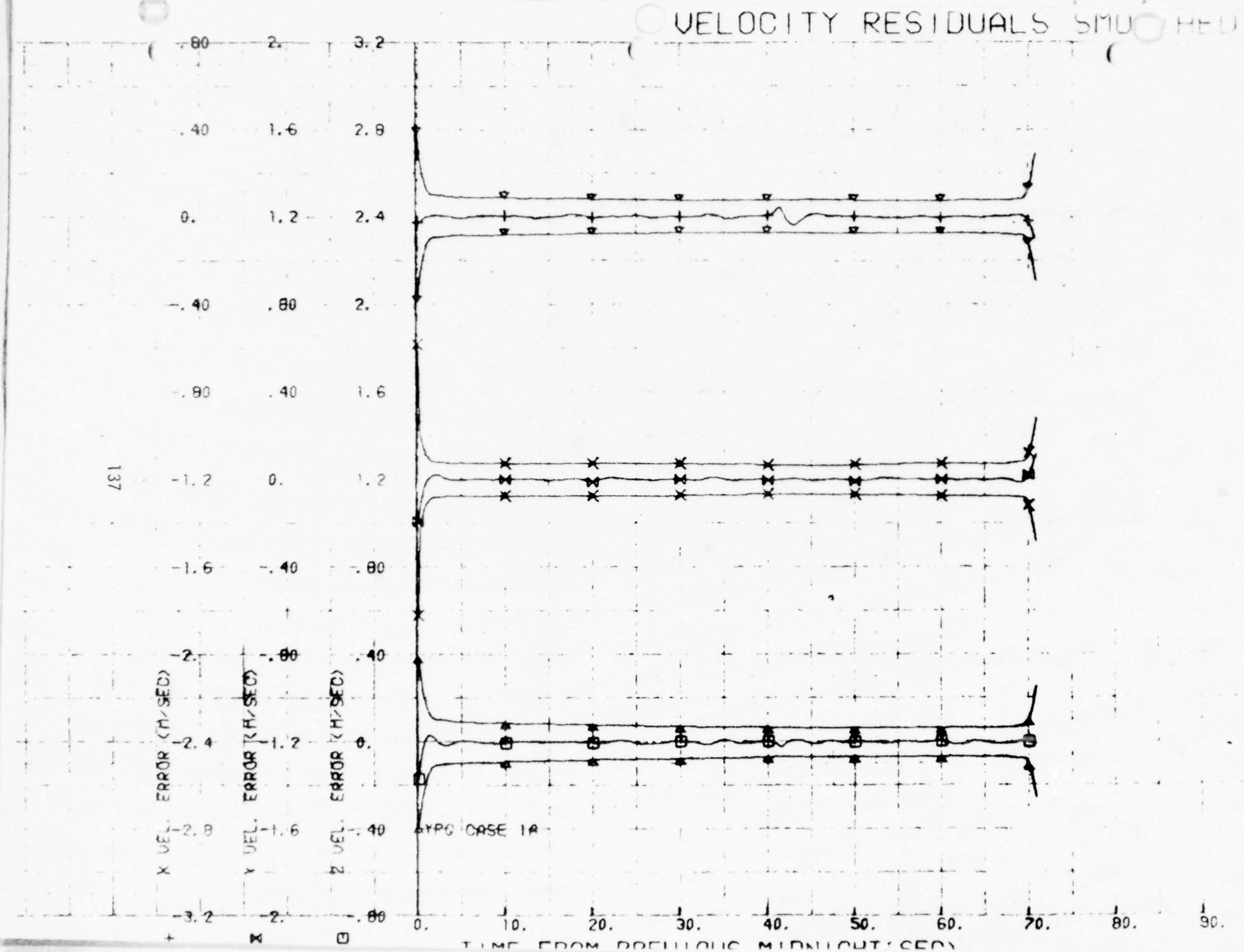
This appendix contains plots of differences between GPSBET estimates for a trajectory and the true trajectory from which simulated measurements were generated.


POSITION RESIDUALS FILTERED
YPG CASE 1A
Z POS ERROR (MTRS)
Y POS. ERROR (MTRS)
X POS. ERROR (MTRS)
0.
10.
20.
30.
40.
50.
60.
70.
80.
90.

POSITION RESIDUALS SMOOTHED

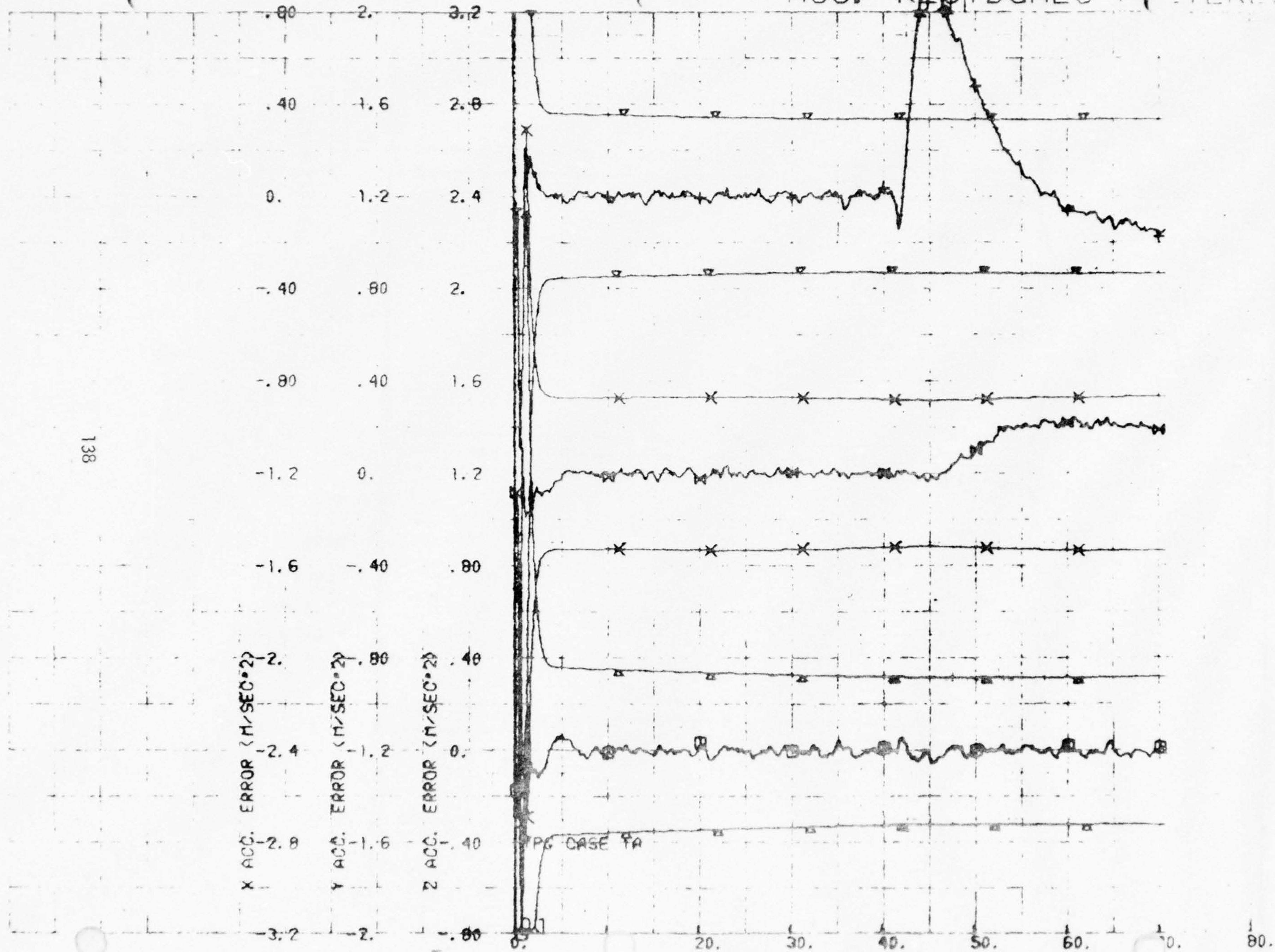


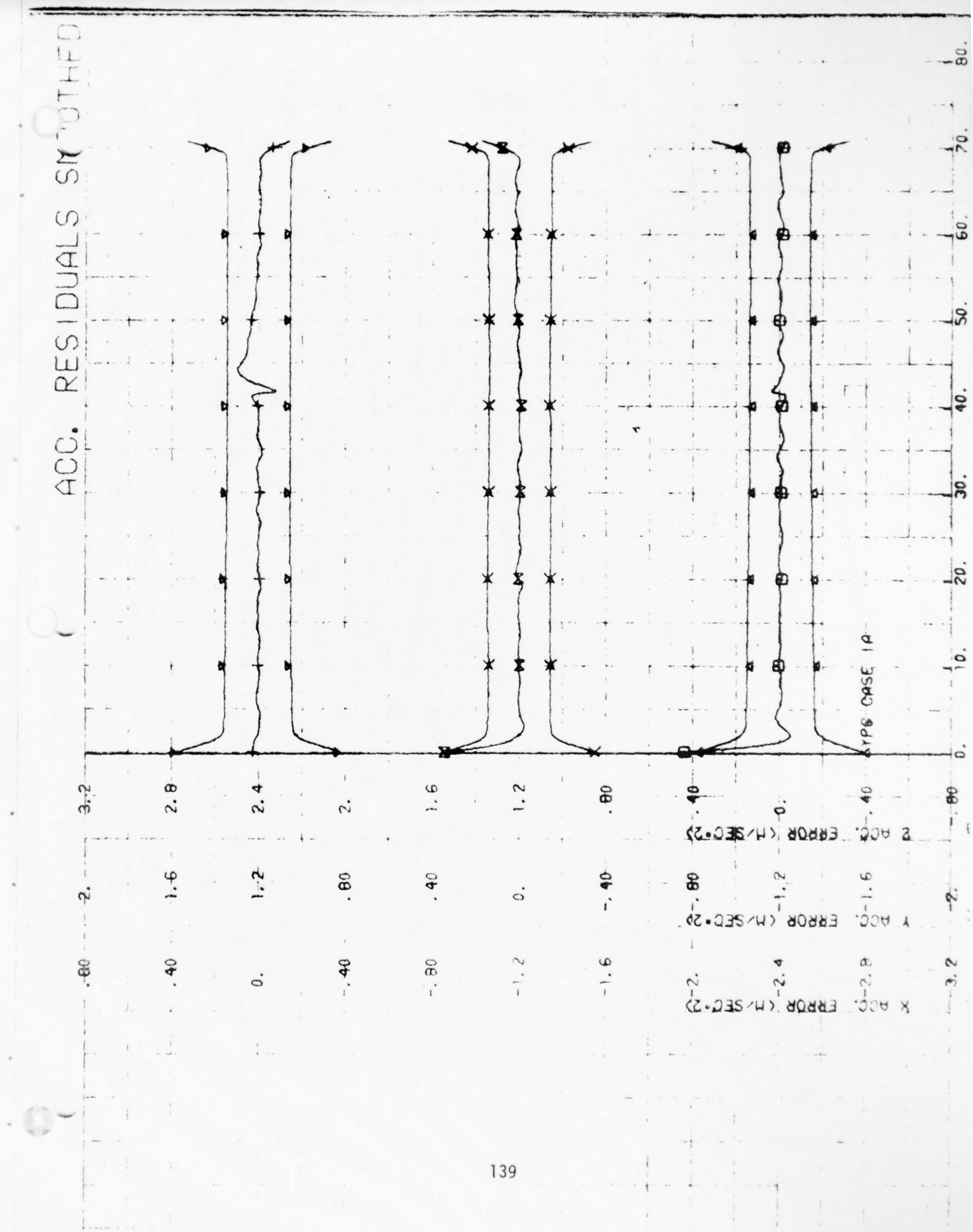

VELOCITY RESIDUALS F TERED
Z VEL. ERROR (M/SEC)
Y VEL. ERROR (M/SEC)
X VEL. ERROR (M/SEC)
PO CASE 1A

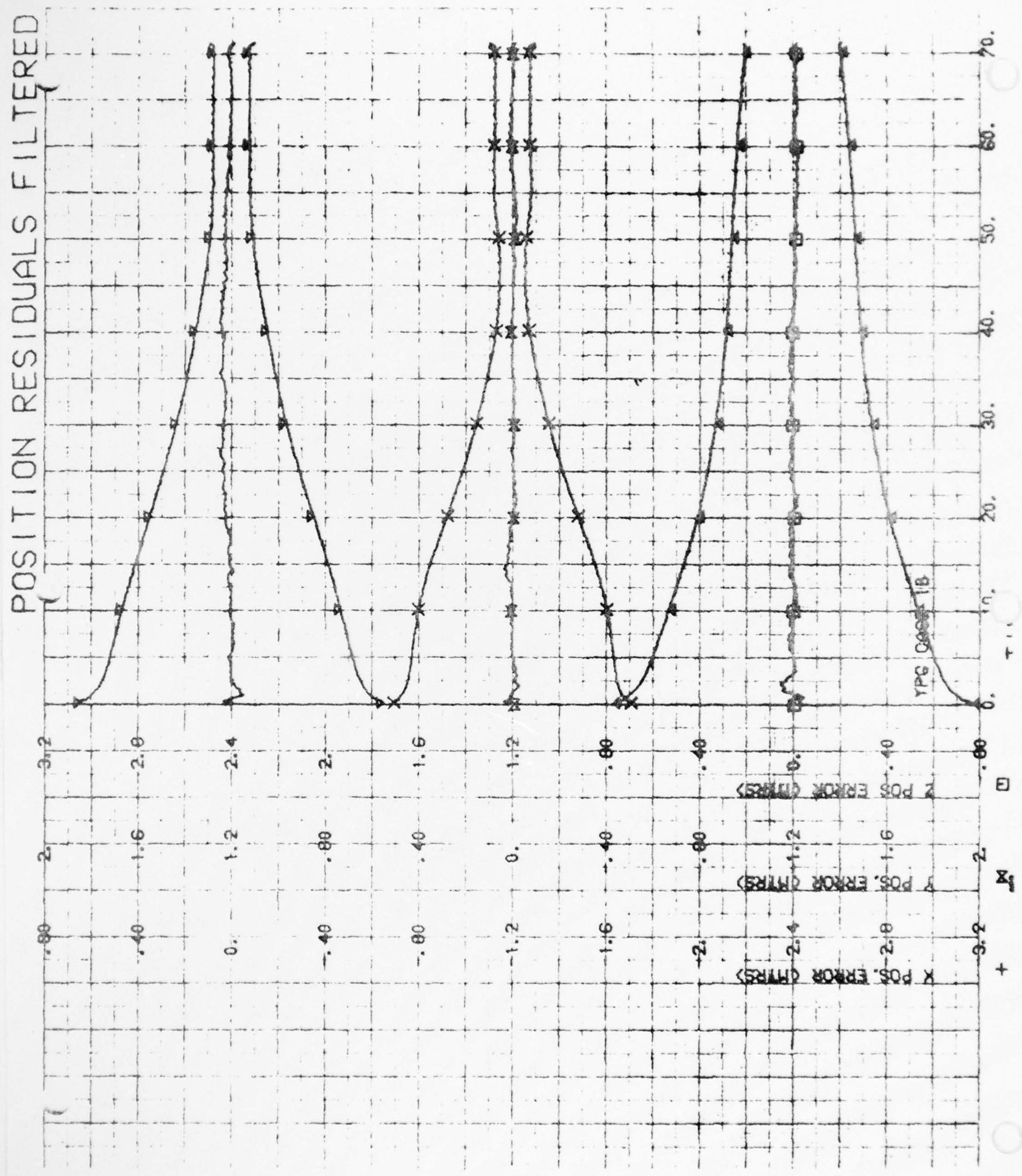


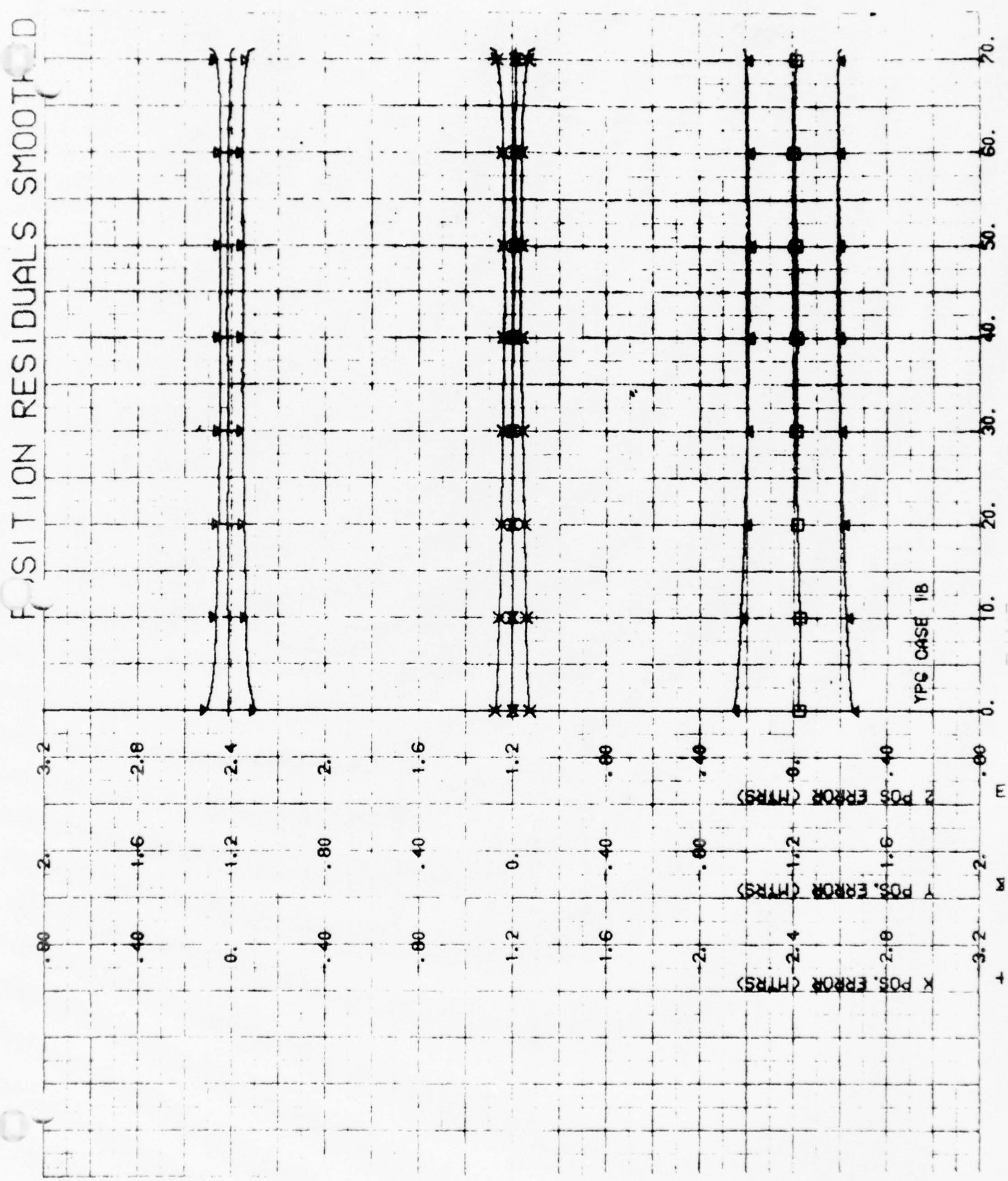
VELOCITY RESIDUALS
X VEL. ERROR (M/SEC)
Y VEL. ERROR (M/SEC)
Z VEL. ERROR (M/SEC)
TYPC CASE 1A
0.
10.
20.
30.
40.
50.
60.
70.
80.
90.

ACC. RESIDUALS FILTERED

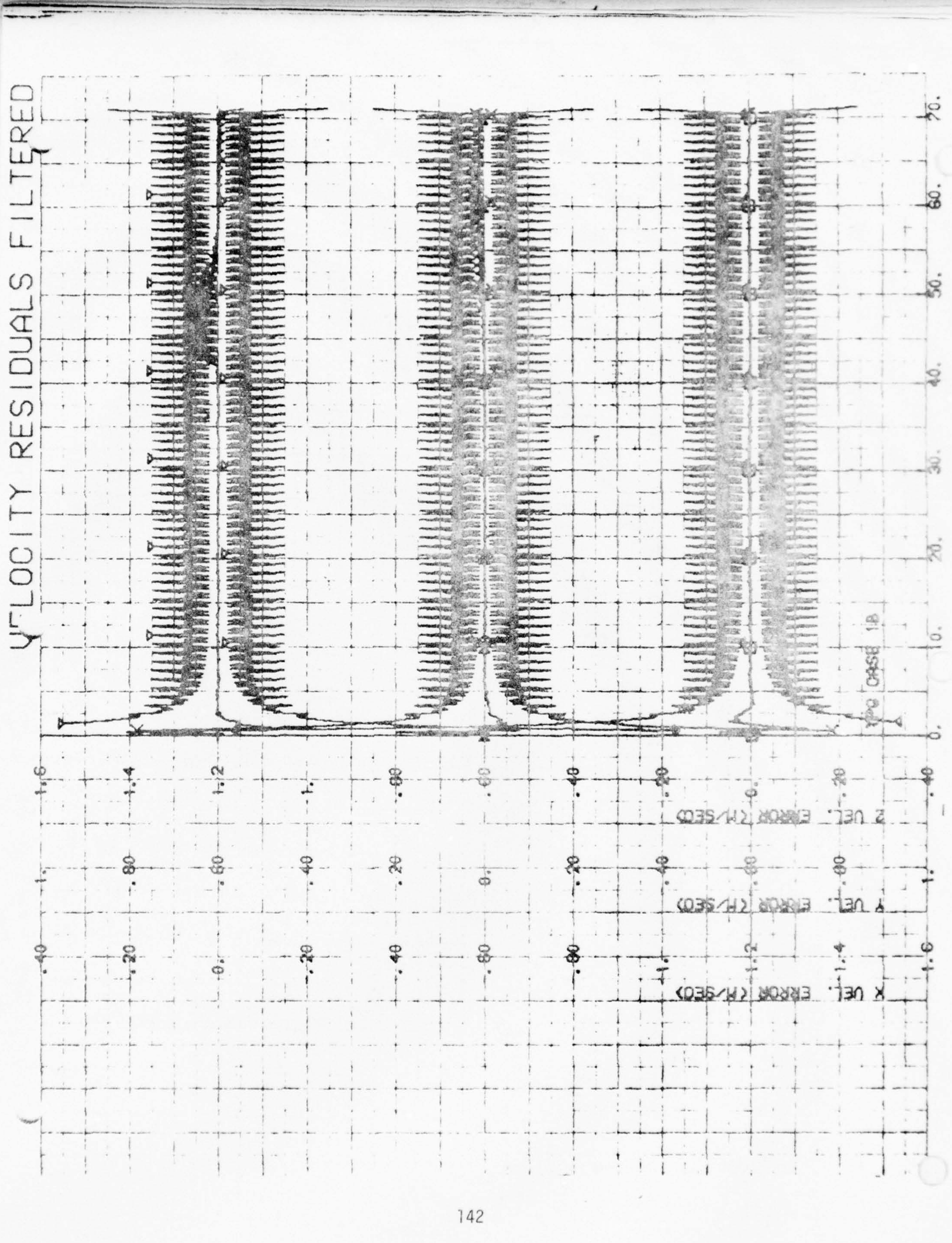


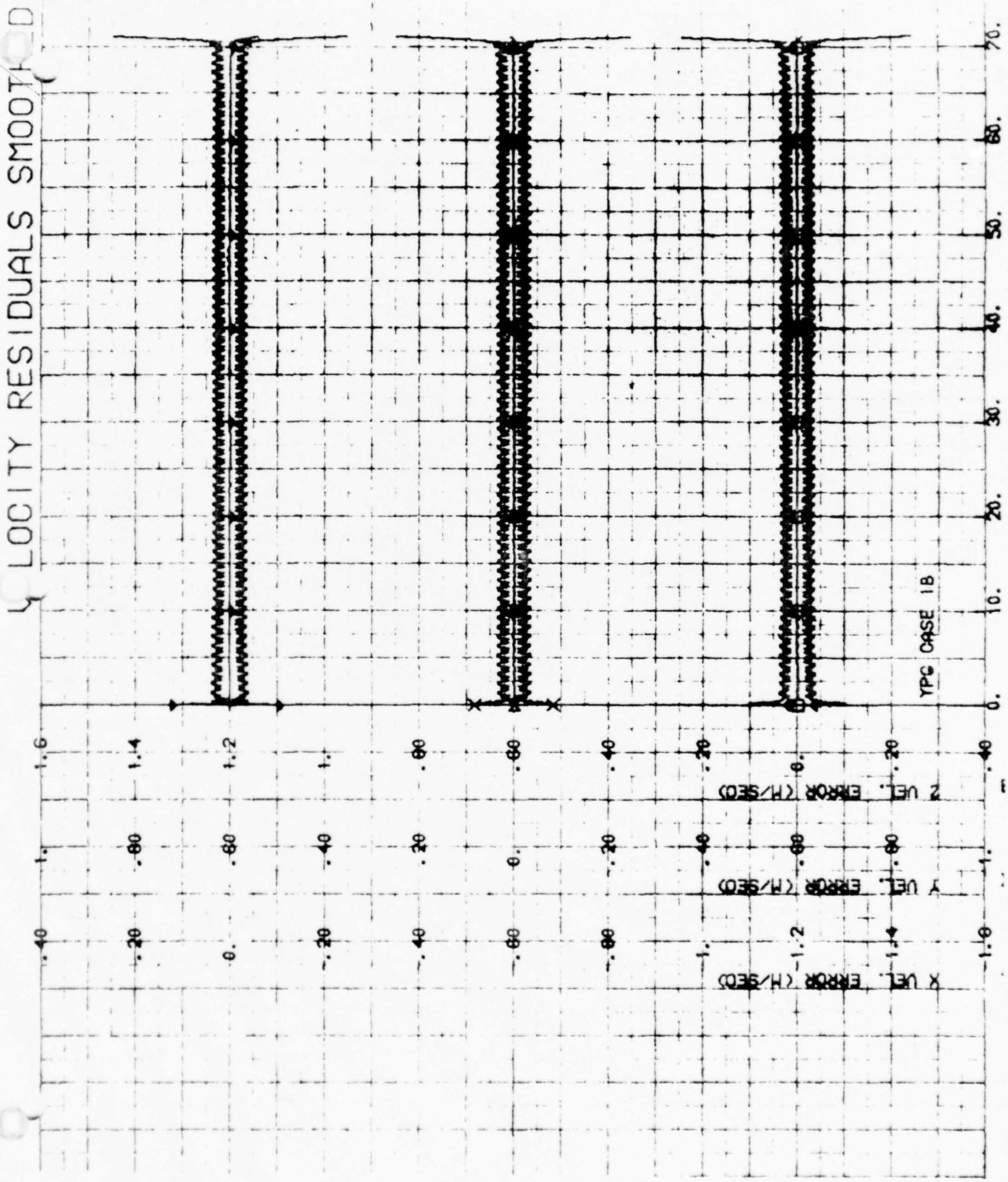

ACC. RESIDUALS SM OTHED
Z ACC. ERROR (M/SEC•2)
3.2 2.8 2.4 2. 1.6 1.2 .80 .40 0. -.40 -.80
Y ACC. ERROR (M/SEC•2)
2. 1.6 1.2 .80 .40 0. -.40 -.80 -1.2 -1.6 -2.
X ACC. ERROR (M/SEC•2)
.80 .40 0. -.40 -.80 -1.2 -1.6 -2. -2.4 -2.8 -3.2
0. 10. 20. 30. 40. 50. 60. 70. 80.
YPG CASE 1A

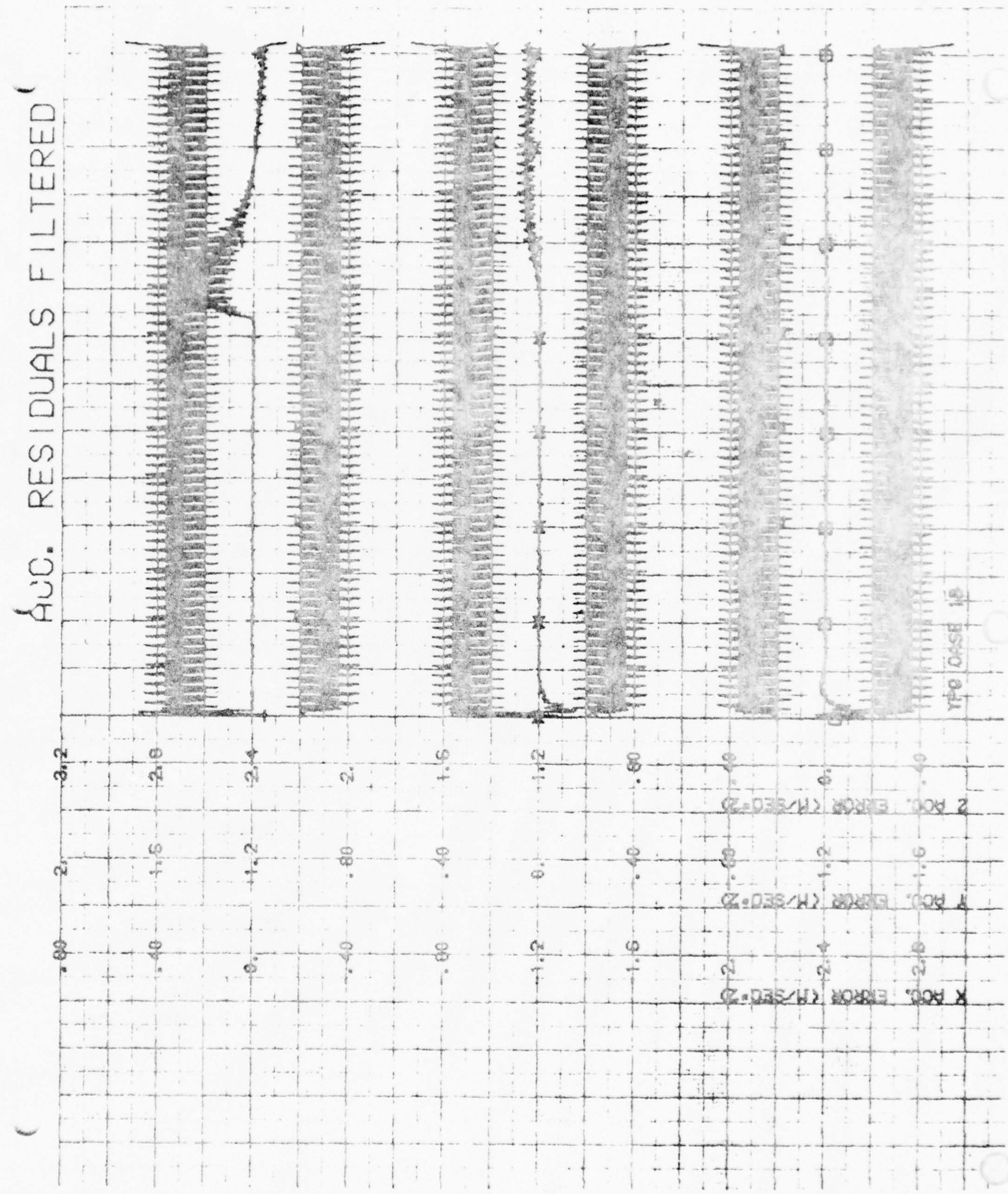

POSITION RESIDUALS FILTERED
YPC CASE 1B
X POS. ERROR (MTRS)
Y POS. ERROR (MTRS)
Z POS. ERROR (MTRS)

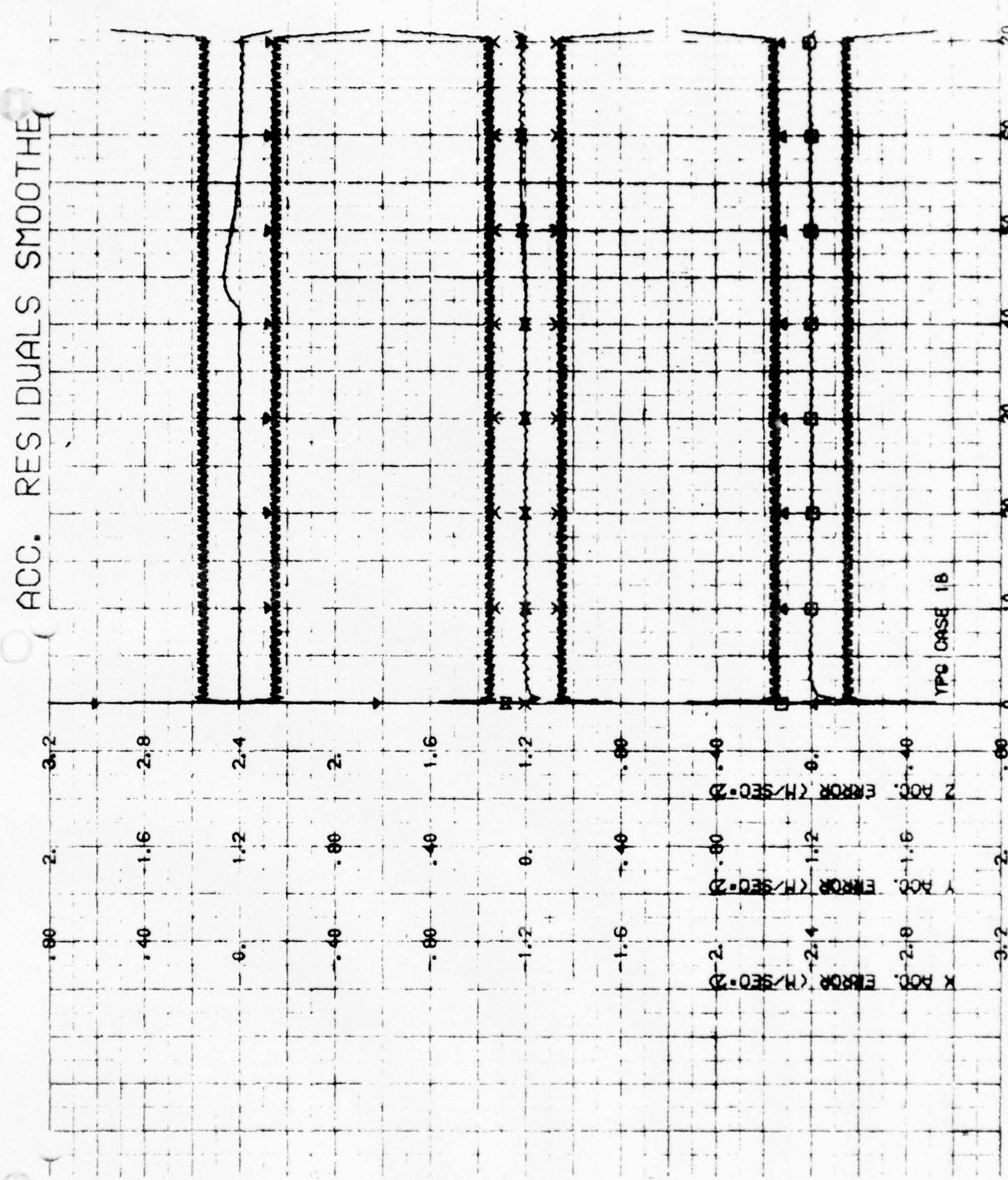


POSITION RESIDUALS SMOOTHED
YPG CASE 1B
X POS. ERROR (MTRS)
Y POS. ERROR (MTRS)
Z POS. ERROR (MTRS)
0.
10.
20.
30.
40.
50.
60.
70.


VELOCITY RESIDUALS FILTERED
Z VEL. ERROR (M/SEC)
Y VEL. ERROR (M/SEC)
X VEL. ERROR (M/SEC)
TPC CASE 1B
0.
10.
20.
30.
40.
50.
60.
70.


LOCITY RESIDUALS SMOOTH D
YPG CASE 1B
Z VEL. ERROR (M/SEC)
Y VEL. ERROR (M/SEC)
X VEL. ERROR (M/SEC)


ACC. RESIDUALS FILTERED


ACC. RESIDUALS SMOOTHE
YPC CASE 1B
X ACC. ERROR (M/SEC²)
Y ACC. ERROR (M/SEC²)
Z ACC. ERROR (M/SEC²)

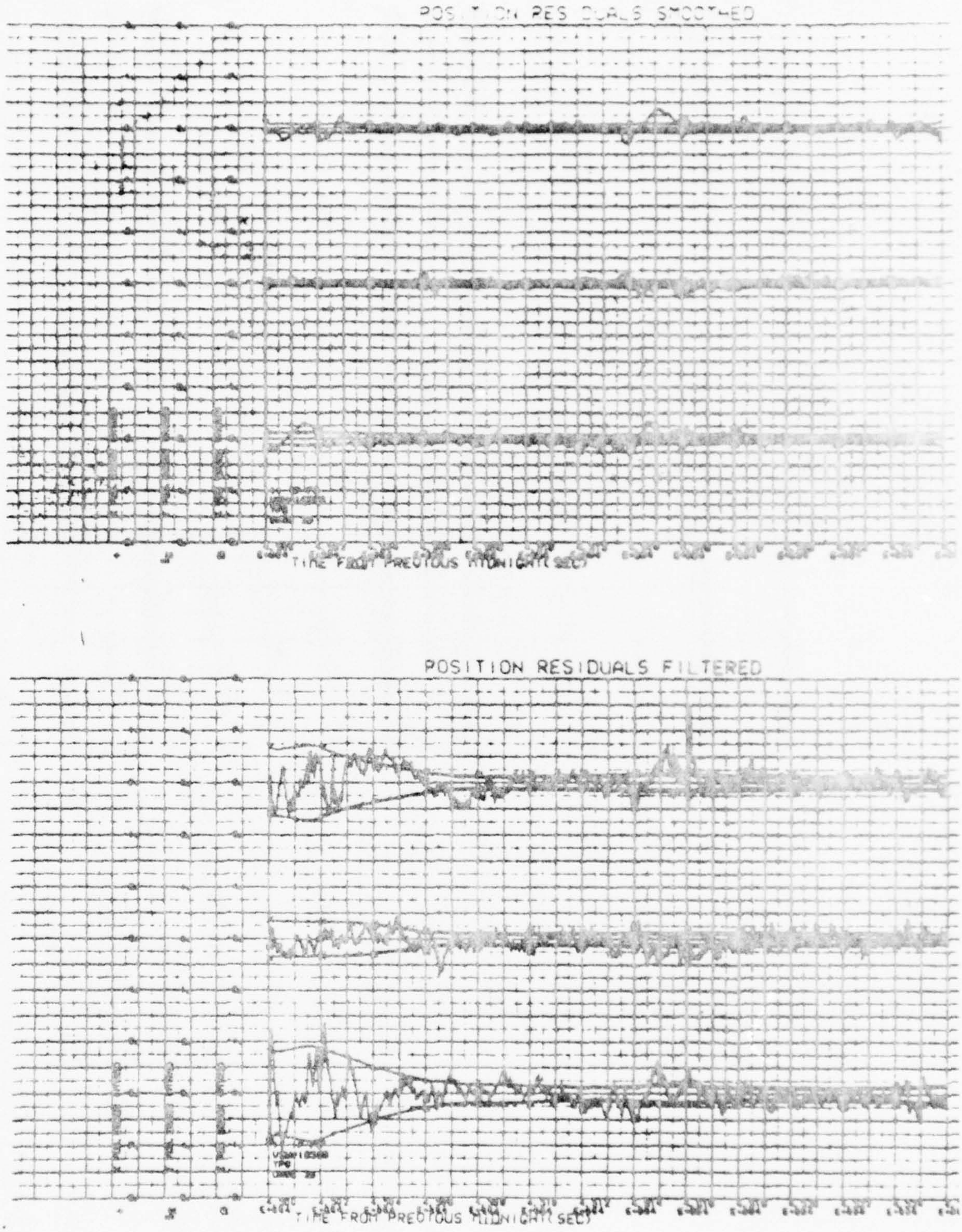

POSITION RESIDUALS SMOOTHED
TIME FROM PREVIOUS MIDNIGHT(SEC)
POSITION RESIDUALS FILTERED
TIME FROM PREVIOUS MIDNIGHT(SEC)

Figure 1

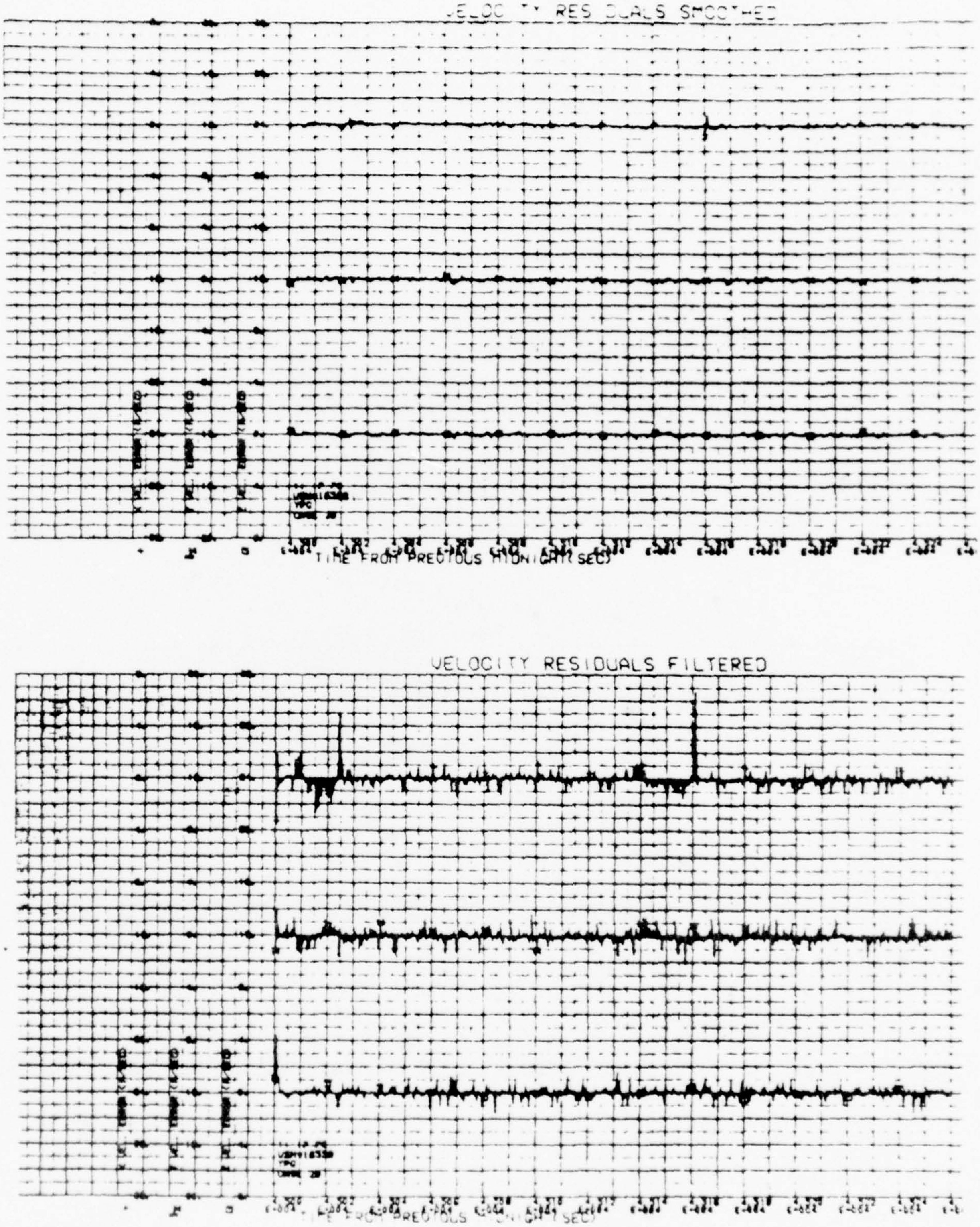
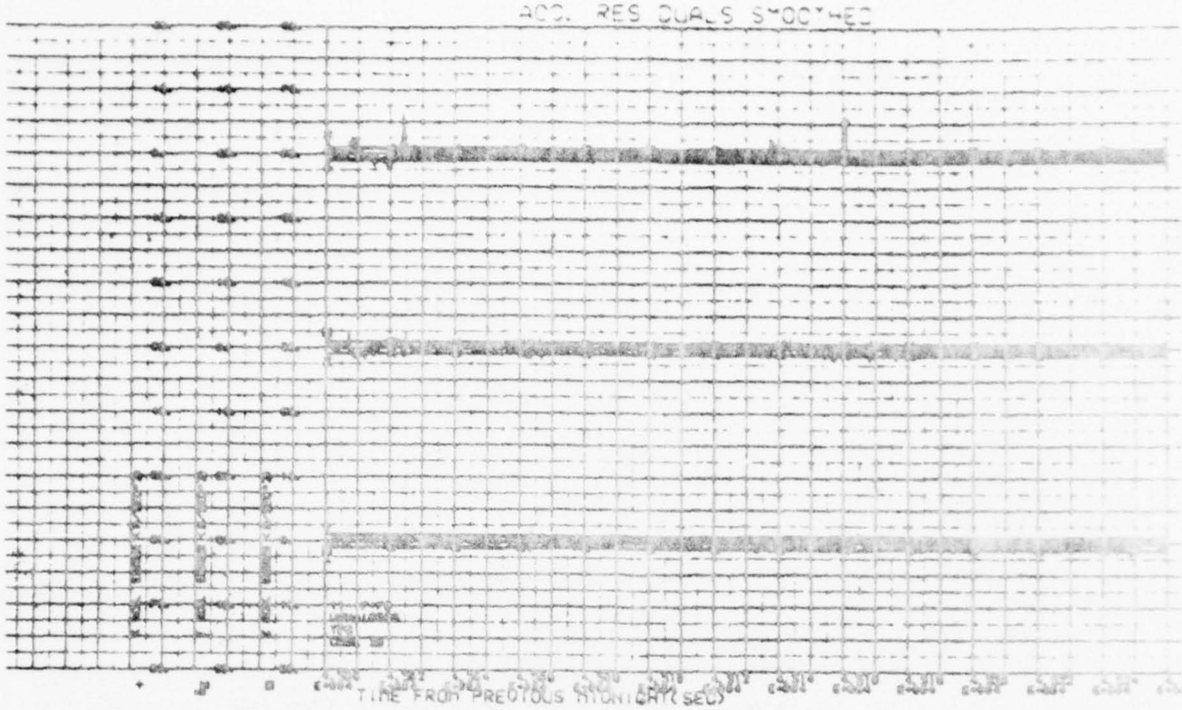

VELOCITY RESIDUALS SMOOTHED
TIME FROM PREVIOUS MIDNIGHT (SEC)
VELOCITY RESIDUALS FILTERED

Figure 2


ACC. RESIDUALS SMOOTHED
TIME FROM PREVIOUS MIDNIGHT(SEC)

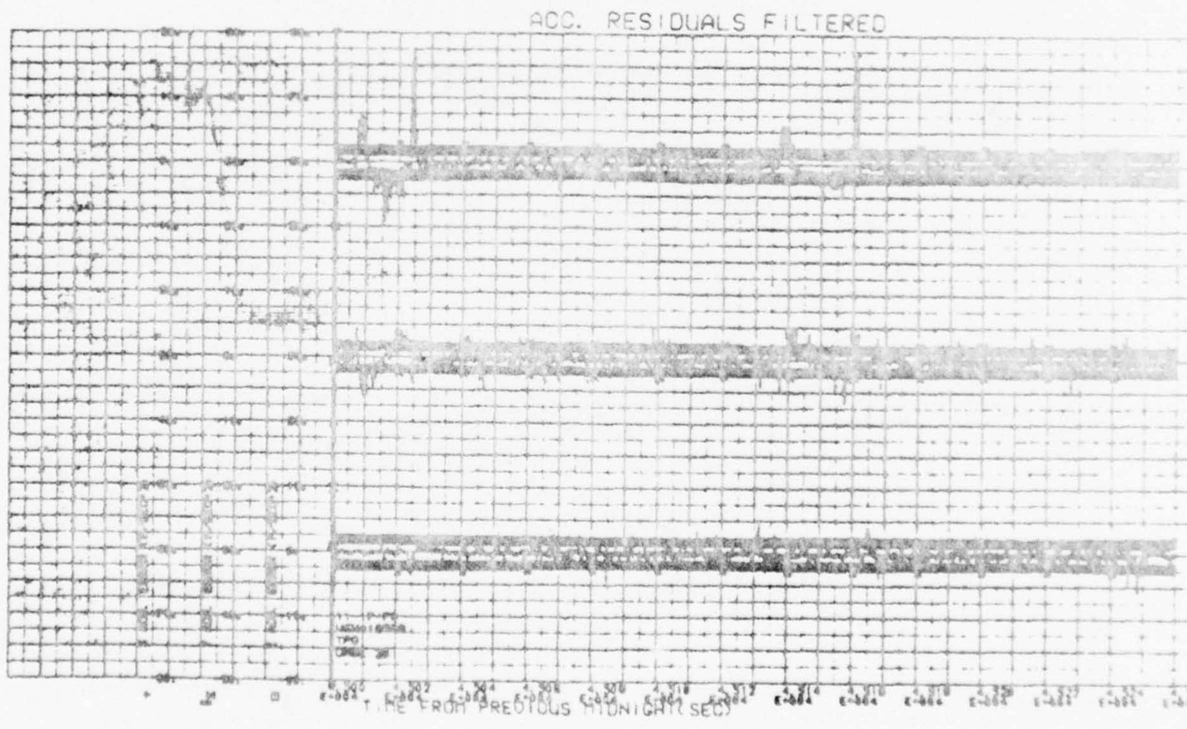
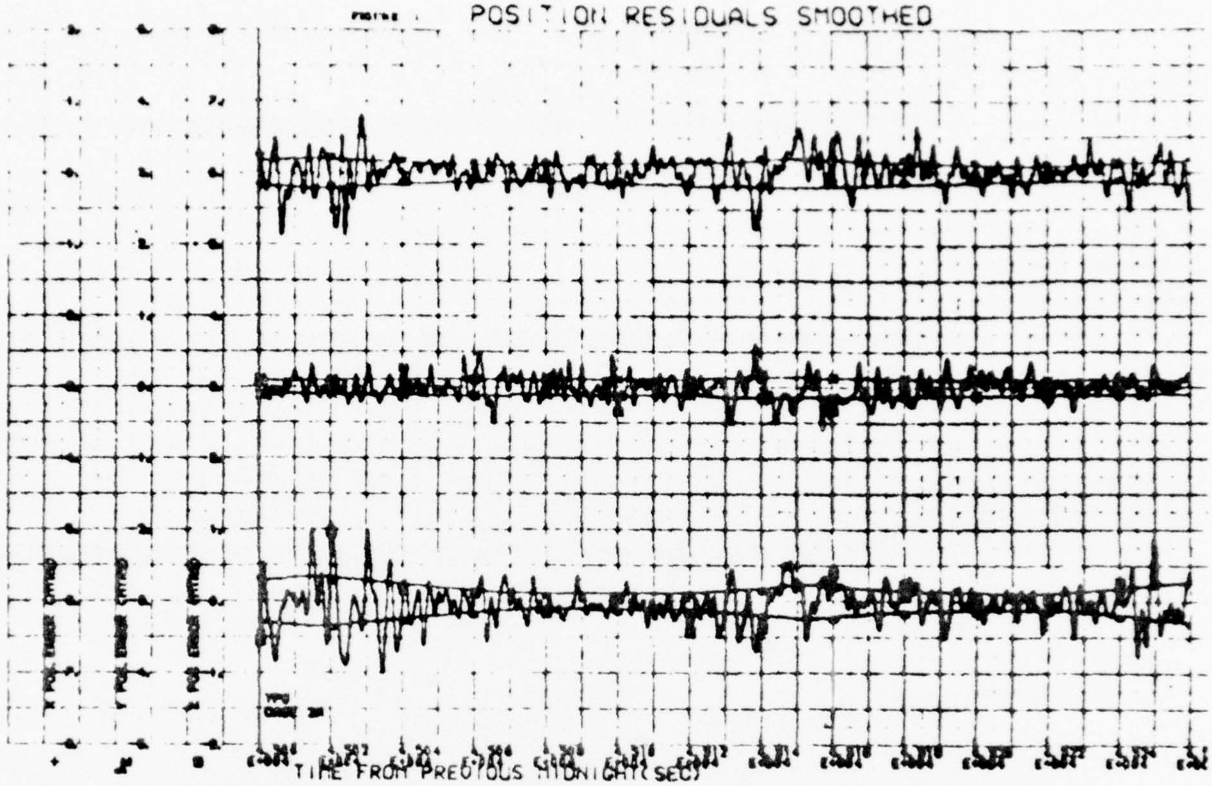
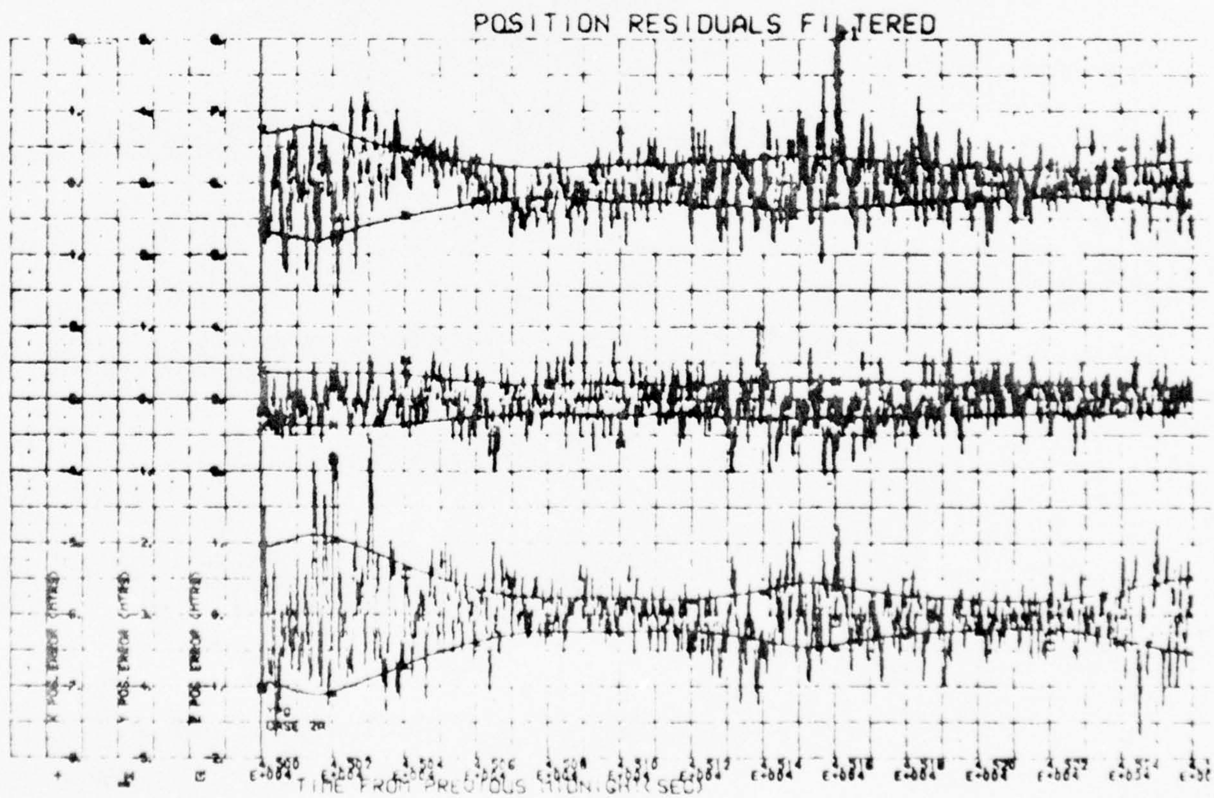

ACC. RESIDUALS FILTERED
TIME FROM PREVIOUS MIDNIGHT(SEC)

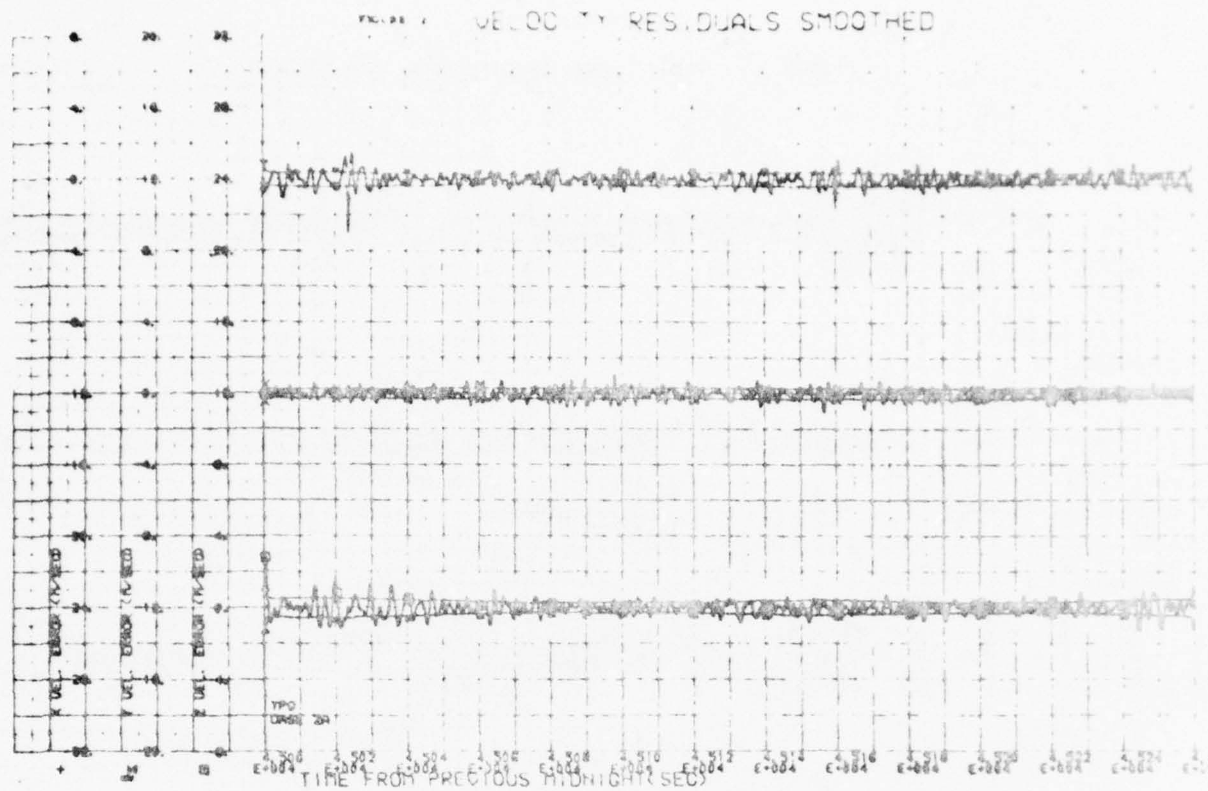
Figure 3

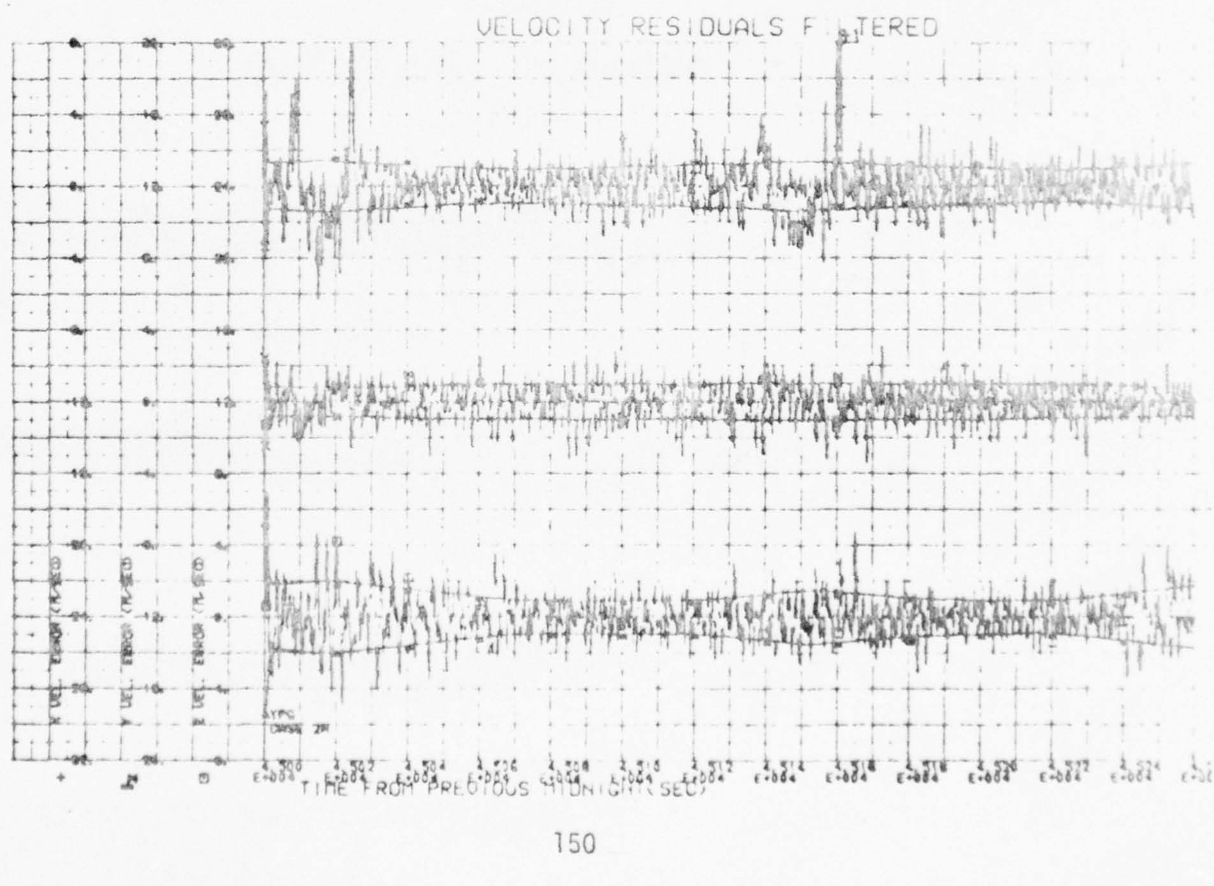


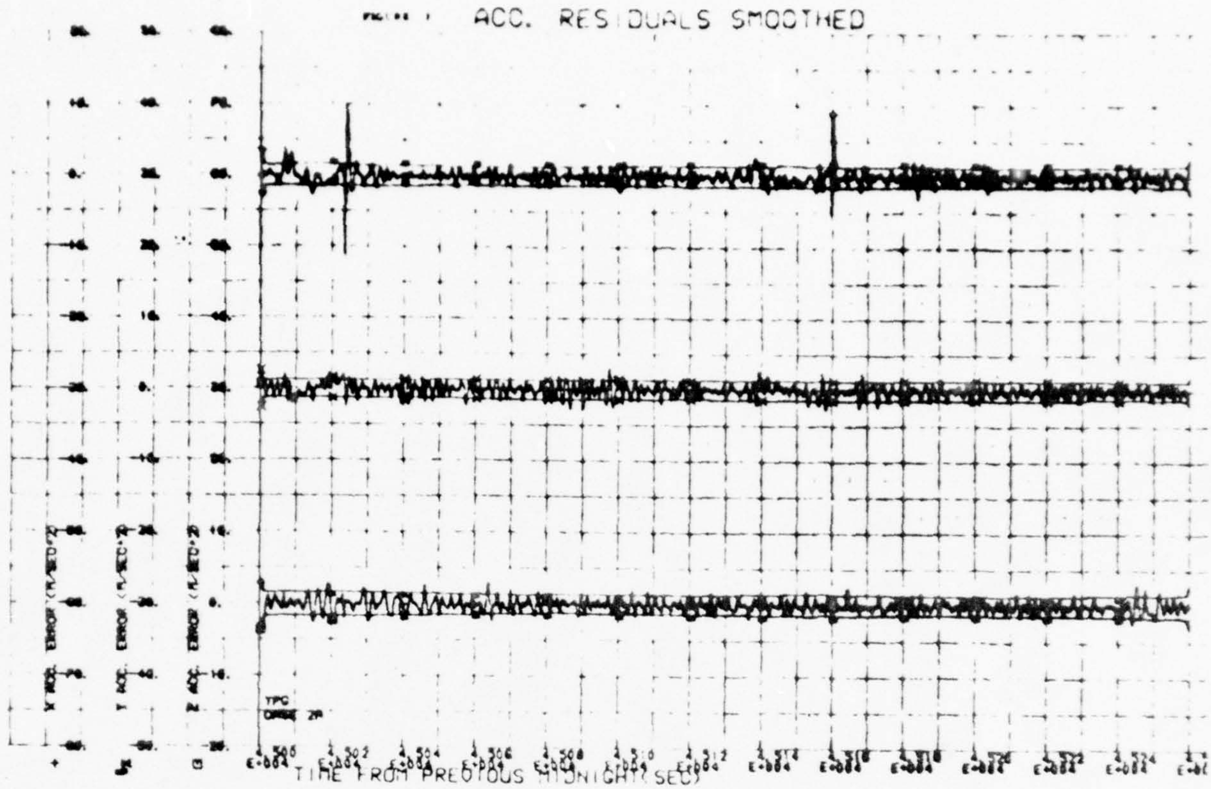
POSITION RESIDUALS SMOOTHED
TIME FROM PREVIOUS MIDNIGHT(SEC)

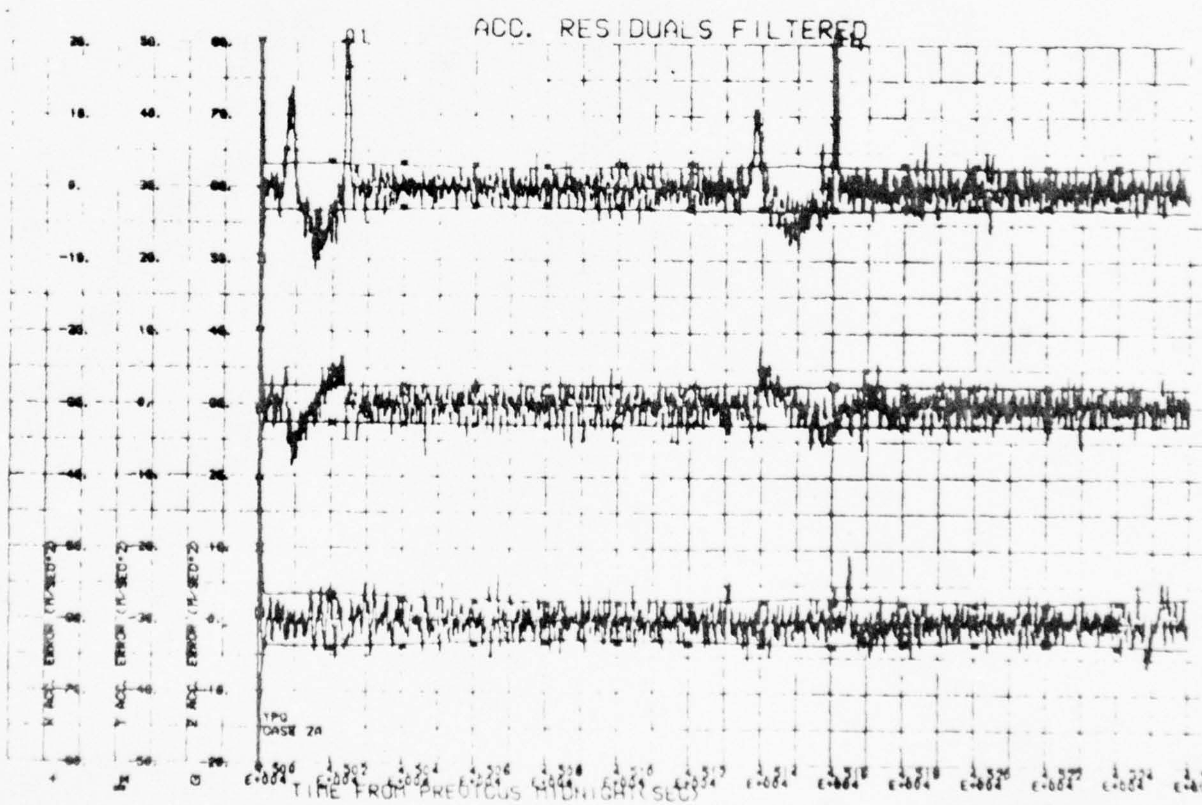


POSITION RESIDUALS FILTERED
TIME FROM PREVIOUS MIDNIGHT(SEC)


VELOCITY RESIDUALS SMOOTHED
TIME FROM PREVIOUS MIDNIGHT(SEC)


VELOCITY RESIDUALS FILTERED
TIME FROM PREVIOUS MIDNIGHT(SEC)


ACC. RESIDUALS SMOOTHED
TIME FROM PREVIOUS MIDNIGHT(SEC)
YPG
CASE 2A


ACC. RESIDUALS FILTERED
TIME FROM PREVIOUS MIDNIGHT(SEC)
YPG
CASE 2A

TECHNICAL NOTE: PA300-55

TAPP

TRAJECTORY ANALYSIS and PREDICTION PROGRAM OVERVIEW

15 AUGUST 1978

PREPARED FOR

SPACE AND MISSILE TEST CENTER (AFSC)

VANDENBERG AFB, CALIFORNIA 93437
CONTRACT NUMBER F 04703-77-C-0111

PREPARED BY

PERFORMANCE ANALYSIS DEPARTMENT

FEDERAL ELECTRIC CORPORATION
WTR DIVISION, VANDENBERG AFB, CALIFORNIA 93437

Prepared by: G. D. TRIMBLE

Reviewed by: Ray S. Fuller Date 23 Aug. 1978
RAY S. FULLER, Supervisor
Performance Analysis Department

TABLE OF CONTENTS

1.0 INTRODUCTION

The TAPP* (Trajectory Analysis and Prediction Program) is designed as a computational tool to simulate trajectory motion and tracking operations, to reconstruct model characteristics responsible for observed behavior and to perform error analyses by propagating uncertainties in initial conditions, measurements and sensor models into trajectory position, velocity and acceleration at subsequent times.

1.1 Background

In mid-1966 the TRACE-D (IBM 7094) orbit determination program was obtained at AFETR for evaluation. Through the remainder of 1966 and most of 1967 close contact was established with the Aerospace Corporation for improving and using this version of TRACE.

Late in 1967 TRACE 66** was made available for use on the CDC 6600 and 3600 computers. This program was considerably improved over TRACE-D and included years of experience gained from its predecessor. It was quoted by Aerospace to represent a $6M effort to that point.

During the summer of 1968 the TRACE 66 program was converted to run on the IBM 360/65 by RCA with several extensions aimed at improving its endoatmosphere trajectory solution capabilities. This version was renamed FIAT*** (Final Impact and Analysis of Trajectory) and used in 1970-72 on projects including: 1) The Navy Poseidon Reentry Reduction, 2) NASA Mighty Mouse Determination, 3) Foreign Object Evaluation, and 4) System Analysis Simulation.

Early in 1973 it was brought to SAMTEC by ROWS and implemented on the 360/65 to provide analysis support for real-time comparisons. Over the last three years it has been extensively extended to provide a reliable yet versatile reentry program.

---

* see reference 1 and 2
** see reference 3
*** see reference 4 and 5

Several peripheral programs were prepared at WTR to preprocess information used by the TAPP program as illustrated in Figure 1. The POT POURRI* program was written at WTR to merge asynchronous observation data from different sources onto a common tape. The RSAT** program was an AFETR program for prefitting atmospheric data. The LRGM*** program was acquired from Defense Mapping Agency to generate local a priori supplementary gravity force history. The TAL/D program was an extension of the AFETR DMDM**** program to generate high resolution drag and lift coefficient history data from observations. The POW/G program is similar to TAL/D except it generates thrust and guidance history for the boost phase of the flight. Finally, the DSTACK and CCPLOT are multiple pass printing and plotting programs written at WTR using AFETR 'destack' philosophy.

1.2 Characteristics

The TAPP program accepts information relating the vehicle character, ambient environmental conditions and the instrumentation (Figure 2). It provides as output best estimates of trajectories, trajectory comparisons, sensor error models, trajectory and sensor parameter uncertainties and trajectory and look angle predictions (Figure 3).

It is a 35,000 statement FORTRAN program consisting of 300 subroutines which operates in 59 overlays on 288K bytes of storage on an IBM 360/65 computer under IRSYS. It is retained on tape (and disk) and manipulated with a set of management routines for modifications as well as applications. Multiple copies of source libraries, object libraries and load libraries are retained on tape for backup and experimentation. The version is indicated by the release date printed on the title page of each run.

The program has four modes of operation and can be sequenced (Figure 4) by an input itinerary of numerical values 2 through 5 whose functions are defined as follows:

---

* reference 6
** reference 7
*** reference 8
**** reference 9

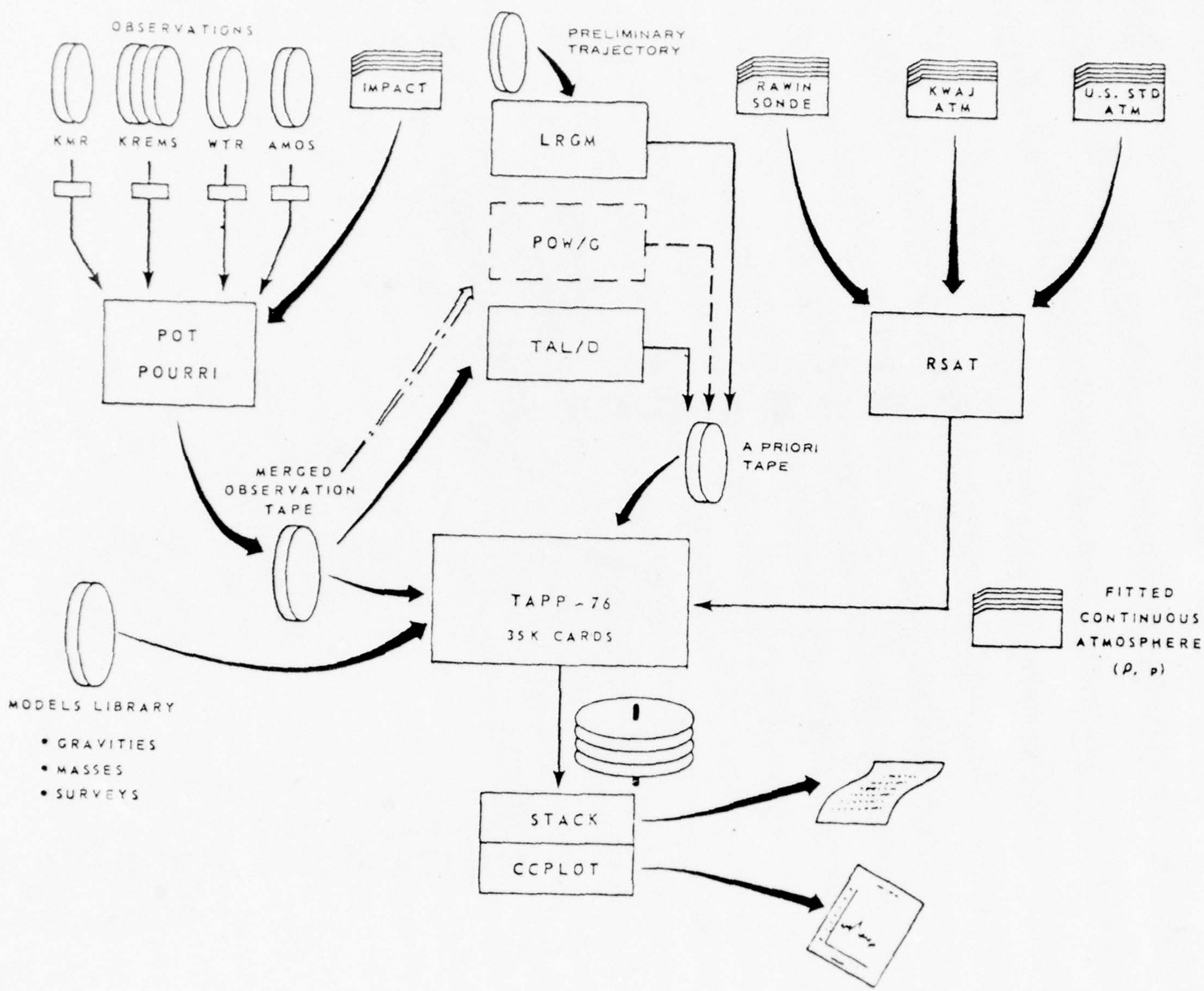


FIGURE 1 TRAJECTORY RECONSTRUCTION ASSOCIATED PROGRAMS AND FLOW

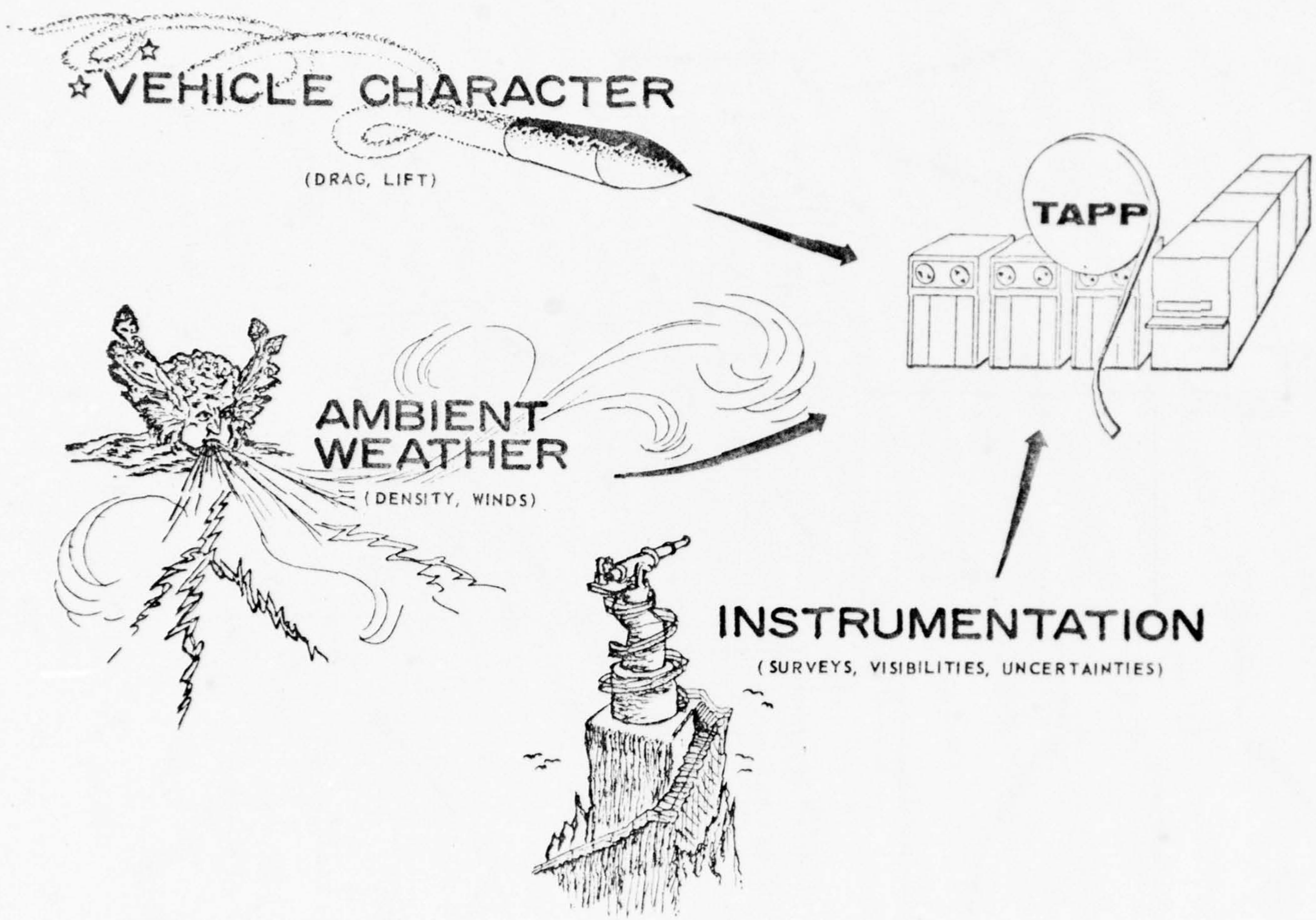


FIGURE 2 TAPP INPUT

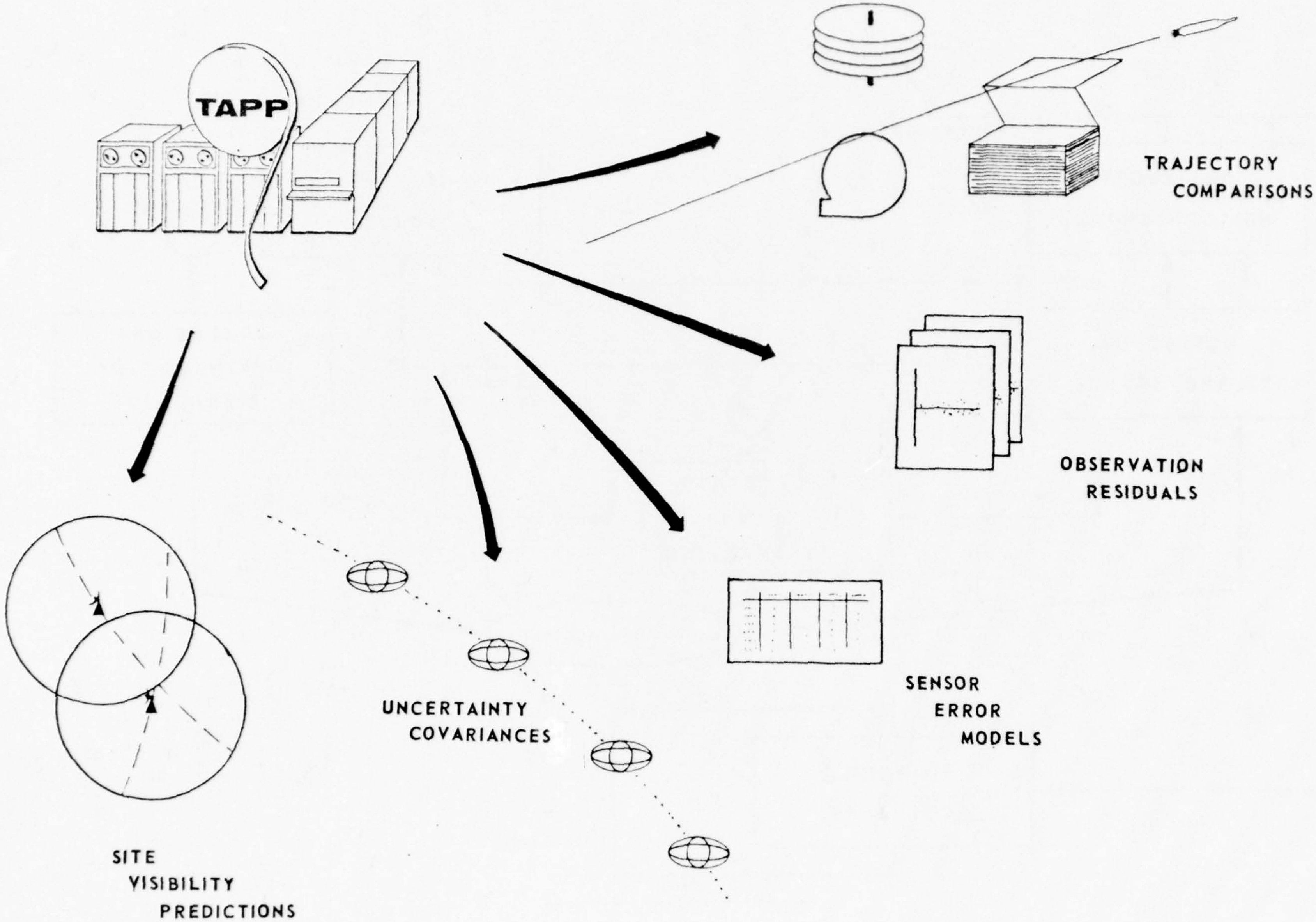


FIGURE 3 TAPP OUTPUT

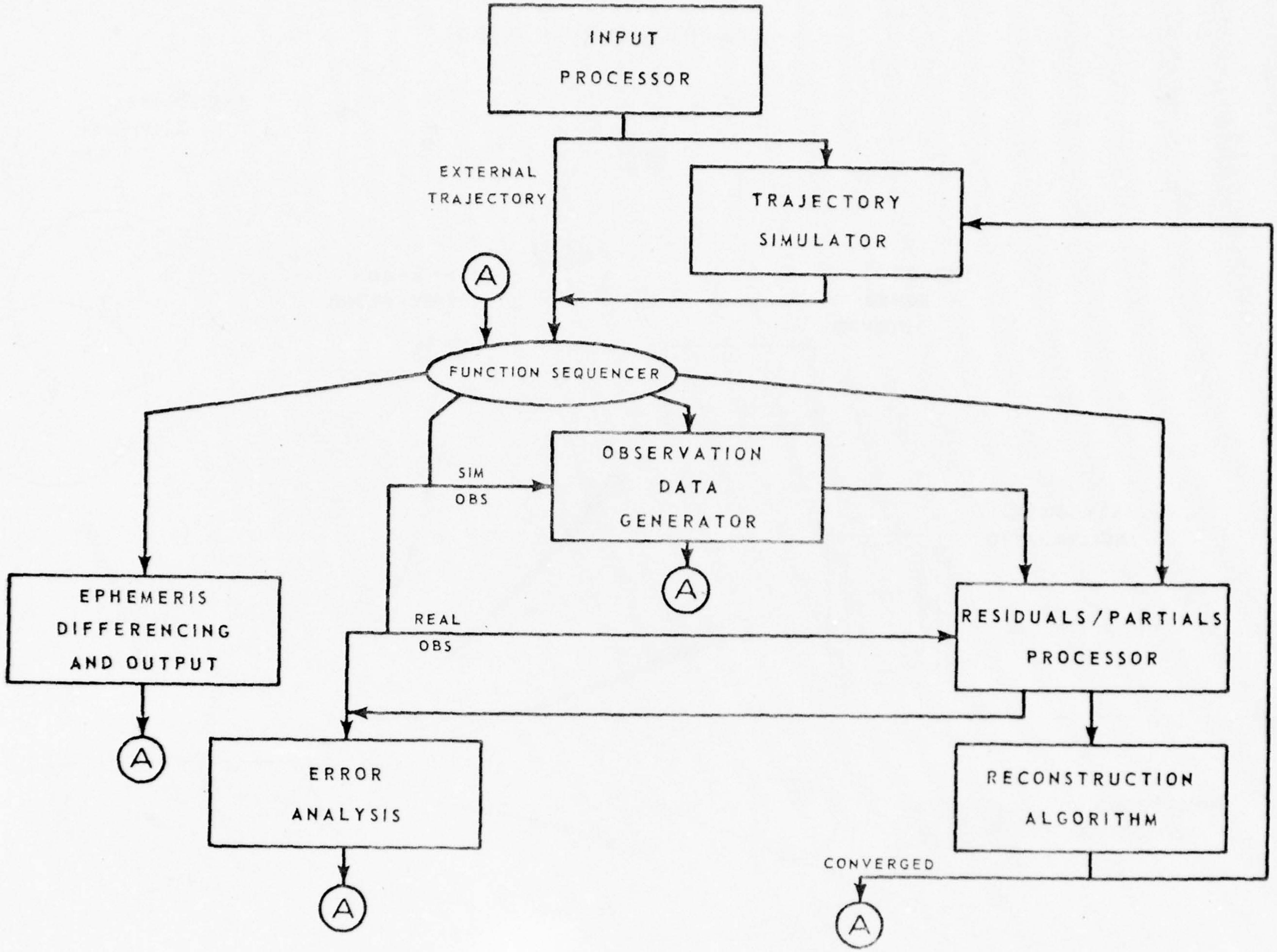


FIGURE 4 TAPP FLOW

2 Parameter Reconstruction
3 Trajectory Simulation/Comparison
4 Observation Generation
5 Error Analysis

Running times vary over a large range depending on the problem. A typical preflight error analysis with nine stations and 10,000 observations requires approximately 10 minutes of CPU time. A reconstruction run with 10,000 observations may require from 30 to 180 CPU minutes depending on whether 10 or 100 parameters are adjusted and upon the quality of the initial conditions.

1.3 Definitions

The following is a partial list of terms and their definitions to provide the reader with a better understanding of the discussion which follows:

BC	-	ballistic camera
BET	-	best estimate of trajectory
boost	-	trajectory portion from lift-off to final thrust termination
bus	-	main body of launch vehicle
epoch	-	trajectory start point (initial conditions)
free fall	-	portion of trajectory where no significant atmosphere is required
GLS	-	generalized (weighted) least squares
MM	-	Minuteman
modelled parameter	-	parameter which is subjected to adjustment in reconstruction
reconstruction	-	estimation, determination, differential correction of adjustable parameters
reentry	-	trajectory portion below 300K feet coming down
RV	-	reentry vehicle
separation	-	separation of RV from main bus

uncertainty	-	potential parameter error or measure of dispersion. Generally the rms about the mean.
unmodelled parameter	-	parameter not subjected to adjustment during reconstruction but whose uncertainty is propagated into the final parameter covariance
weight	-	certainty (squared) attached to an observation or parameter equal to the inverse of the uncertainty squared.

2.0 APPLICATIONS

The reconstruction logic of TAPP constitutes the most distinctive capability of the program and has several range applications.

The other functions namely those of simulation and error analysis are primarily subsets of this reconstruction process with some extensions added for the sake of convenience and versatility.

2.1 Vehicle Motion Determination

The requirement most often encountered is the evaluation of a BET, i.e., best estimate of trajectory, which is the motion history of the vehicle best representing available observations. Such a trajectory is forced to be continuous in position and velocity by the using of equations of motion modelling.

For such a requirement precise knowledge of the atmospheric profile and gravitation field is not required since these physical model variables may be treated as adjustable parameters in the overall solution (possibly with some a priori confidence on the assumed models to ensure a degree of reasonableness).

For instance, the atmospheric density versus height can be adjusted to eliminate excessive residuals for any altitude segments of the trajectory. An alternative to this which would give the same overall result would be the estimation of the drag or lift coefficient functions versus time since they are both linearly involved in the computation of the fluid forces.

2.2 Sensor Calibration

Another requirement of a range is the evaluation of sensor performance i.e., checking the consistency between observations on a sensor-to-sensor or operation-to-operation basis. Simultaneous with the solution for a

BET are the resolution of individual sensor biases (and other error model terms) to minimize observation disagreement (referred to as residuals).

Sensor error models can be constructed from a history of these sensor error estimates from several different BET solutions. Figure 5 is a typical set of sensor biases from an RV separation to impact solution.

2.3 Environmental Modelling

The most prevalent example of environmental modelling is the estimation of gravitational coefficients using multi-rev orbital fits. Modelling of solar flux and fluid drag affects for orbital purposes are also common, but the variability of the atmosphere at low altitudes severely limits its usefulness for reentry or boost phase environmental modelling.

2.4 Vehicle Character Estimation

When the problem is that of defining the physical character of the vehicle as well as its motion, accurate atmospheric observations are also required. The definition of only the BET does not require discerning between terms within equation (1) as to the specific reason for the fluid acceleration magnitude or in fact even between the terms of equation (2).

$$A_{DRAG} = \frac{C_D \, a \, \rho \, V^2}{2m} \qquad (1)^*$$

$$A_{TOTAL} = A_{DRAG} + A_{LIFT} + A_{THRUST} + A_{GRAVITY} + A_{OTHER} \qquad (2)$$

* where C_D = drag coefficient
a = cross section area
ρ = air density
V = vehicle velocity relative to air
m = vehicle mass
A = acceleration

OP 7269

SENSOR BIASES

SENSOR		RANGE (FT)	AZIMUTH (mrad)	ELEVATION (mrad)	RANGE RATE (fps)	TIME (ms)
OPTICS	R1A	—	.010	.005	—	—
	R3B	—	.013	.004	—	—
	R6A	—	-.005	.012	—	—
	R7A	—	.034	.005	—	—
	R9A	—	.014	.007	—	—
	AMO	—	.024	.003	—	—
RADARS	ALC	2.8	.106	.161	.03	-43.8
	TLX	2.6	-.086	.073	.07	—
	ALT	28.6	-.021	-.062	.12	—

FIGURE 5 TYPICAL SENSOR BIASES TABLE

When the physical character of the vehicle is to be determined, the C_D A/m of (1) is essentially the required parameter and knowledge of density and fluid velocity (wind) must be additional observations. Thus, no longer can C_D be allowed to absorb errors in ρ, but ρ must be a known quantity.

2.5 Predictions - Look Angles, Target Recovery

After a BET is formed for an observed period it is sometimes necessary to extrapolate the trajectory to a region where no observations exist for purposes of pointing other instrumentation or even for target recovery. The TAPP program facilitates such runs by retaining the necessary trajectory parameters from a reconstruction run to apply same in a subsequent extended observation generation run as depicted in Figure 6.

2.6 Range Planning

Part of the range function is planning future efforts including the prediction of BET quality and the allocation of sensors to satisfy accuracy requirements.

The basic statistics involved in a least squares trajectory reconstruction problem can be used to determine the effects that specified sources of uncertainty have on the confidence of the least squares estimated parameters and subsequently on the trajectory. These sources of uncertainty, for example, may be systematic inaccuracies in sensor biases, random variations in observations, uncertainties in vehicle related parameters and environmental model uncertainties.

This covariance analysis procedure does not require an actual estimation of the trajectory, nor does it require actual observations; however, site-trajectory geometry, the observation types and observation intervals are required for each sensor. A basic covariance analysis of trajectory uncertainties can then be obtained by simple matrix manipulation.

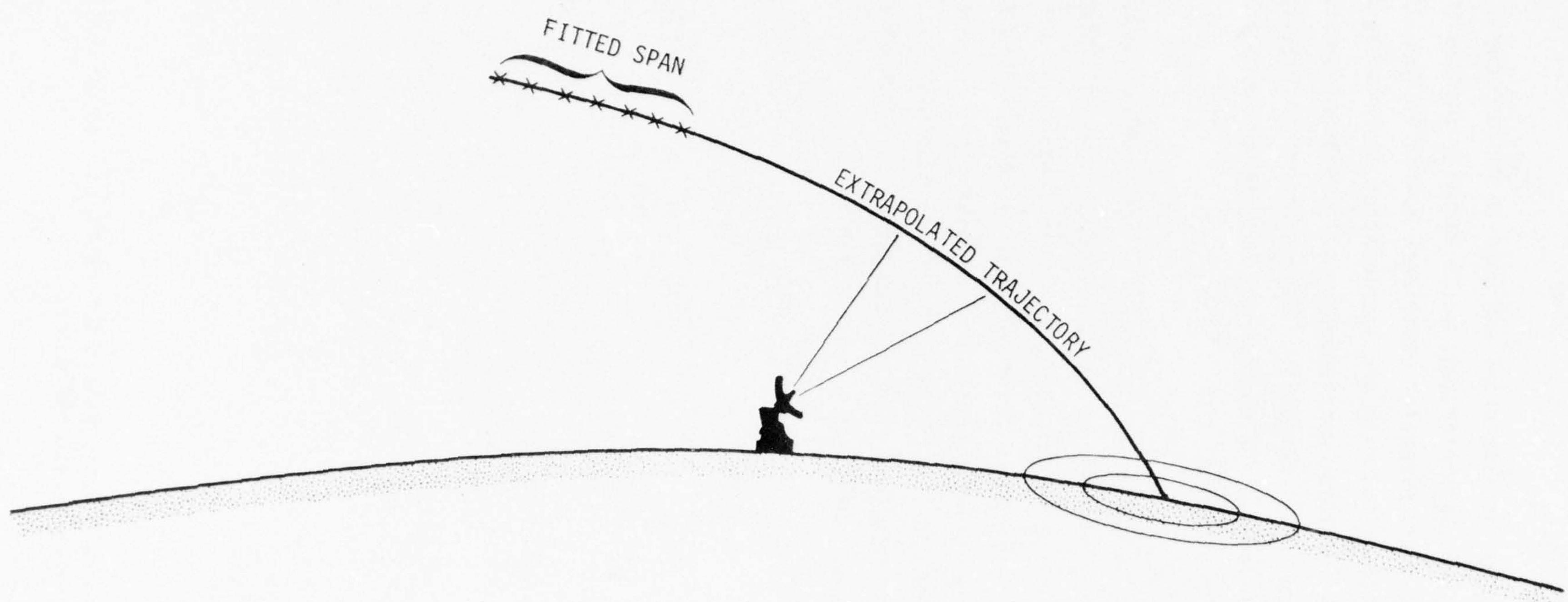
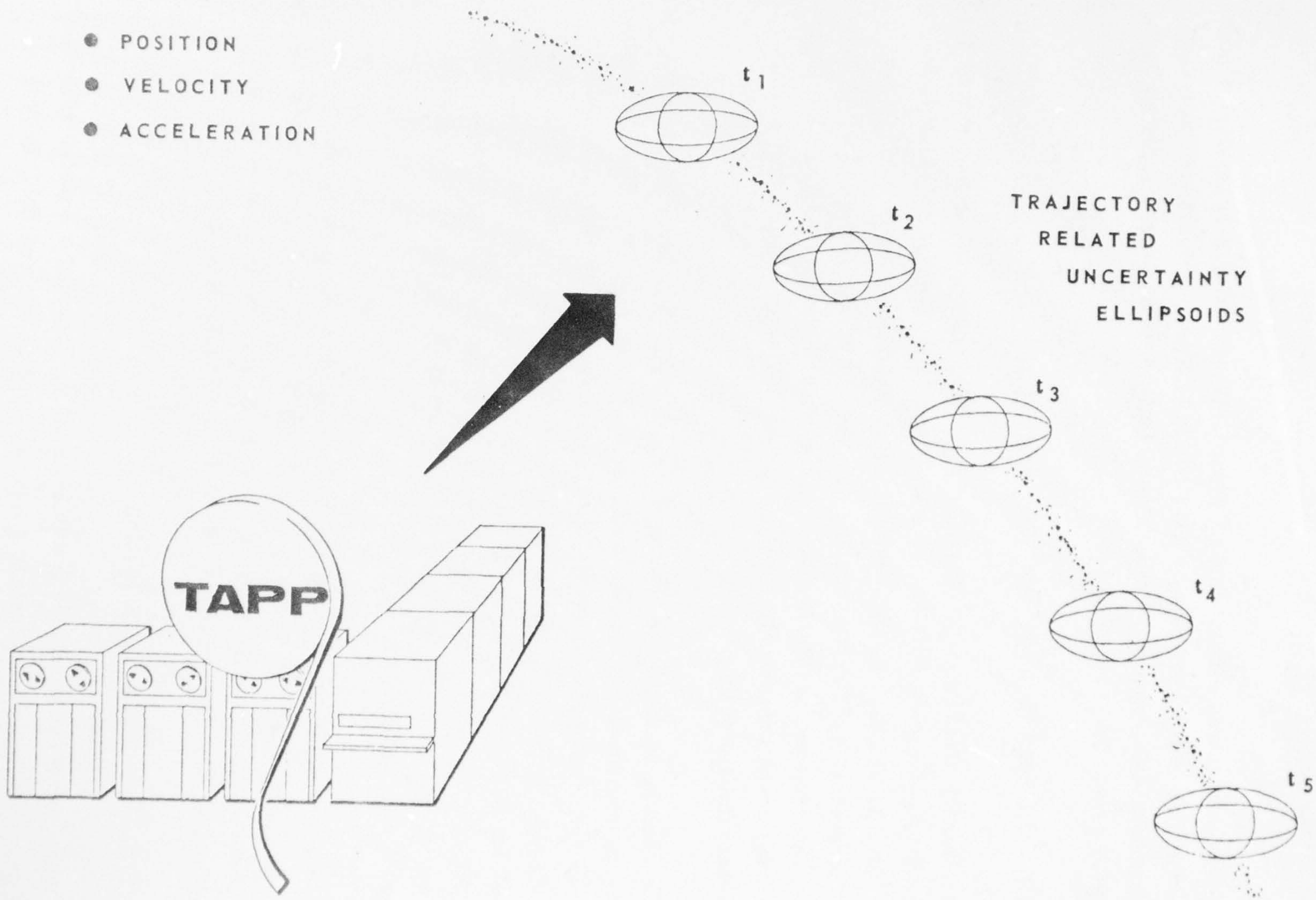


FIGURE 6 EXTRAPOLATION BEYOND THE OBSERVED TRAJECTORY SEGMENT

Essentially, observation uncertainties are used to improve the a priori uncertainties of a given parameter set, which normally include the epoch state vectors. This total parameter covariance matrix is then propagated into trajectory position, velocity and acceleration coordinates for each specific time to provide three corresponding ellipsoids of uncertainty at each time. (See Figure 7.) Generally, 1-sigma uncertainty ellipsoids are dealt with which infer a 20% confidence that the trajectory is within that ellipsoid* surface at that time.

Relative comparisons of various sensor combinations can be compared directly in terms of trajectory quality for range planning purposes although considerable care must be exercised in any interpretation of such uncertainties in an absolute sense since error correlation between observations plays a very deceptive role in covariance analyses. For instance, the use of a 20 sample-per-second (sps) data rate from a radar does not reduce the vehicle position variance (uncertainty squared) by a factor of 2 from that of using 10 sps due to serial error correlation.

* A 2.8 sigma ellipsoid would have a 95% confidence while 3.4 sigma ellipsoid would have a 99% confidence.



POSITION
VELOCITY
ACCELERATION
TAPP
t_1
t_2
t_3
t_4
t_5
TRAJECTORY
RELATED
UNCERTAINTY
ELLIPSOIDS

3.0 TECHNIQUES

The least squares reconstruction process is basically a sequential linear correction process comparable to the simple Newton-Rhapson method except in multiple dimensions. Several principal techniques are discussed below which relate to its application.

3.1 Reconstruction Quality Measures

A strictly quantitative comparison of sensor residuals is provided by the root-mean-square magnitudes of each observation channel as illustrated in Figure 8, although sensor residual plots provide the best qualitative interpretation of reconstruction success. When trended residual behavior appears for one sensor it can be compared with that of another for trend correlations which would likely indicate BET error rather than sensor observation problems.

An example on Operation 7269 near impact (1890-1892 seconds) showed a radar range component and 3 radar angle sets (Figure 9) with highly correlated non-zero residuals. Further, the range component of the ALCOR radar (Figure 10) was also positively correlated with this trend, but its angles were negatively correlated, i.e., in contradiction. As it turned out multipath had distorted these ALCOR angles at the very low elevation angles and with its deweighting, the 5 sensors' residuals fell neatly to zero mean values.

In yet another case during reentry a radar had a sudden 5 foot shift in range and disagreed strongly with 3 RADOTS. In this case a sudden vehicle event caused a sudden burst of ionization and shifted the electronic range measure leaving the optical observations unaffected.

3.2 Observation Weighting

Each observation used in a solution can be assigned a quality value (an uncertainty, i.e., actually a quality inverse) to control its relative influence in the overall solution. Also, the initial estimate of each

TABLE VI-A

SENSOR RMS OF RESIDUALS

(Separation-to-Impact)

SENSOR		RANGE (FT)	AZIMUTH (MR)	ELEVATION (MR)	RANGE RATE (fps)
OPTICS	R3B	—	.016	.017	—
	R6A	—	.016	.016	—
	R7A	—	.005	.012	—
	B1A	—	.014	.013	—
	B1B	—	.010	.009	—
	B3A	—	.016	.017	—
	B5B	—	.010	.023	—
RADARS	VAB	3.6	.113	.093	—
	PPT	2.1	.072	.073	—
	SNI	1.6	.079	.126	—
	MKM	1.5	.120	.091	—
	MKF	9.7	.715	.785	—
	ALC	2.1	.108	.120	1.0
	MPS	9.7	.314	.802	—
	TDX	6.4	.042	.065	—
	ALT	23.0	.244	.262	—

FIGURE 8 TYPICAL SENSOR RMS OF RESIDUALS

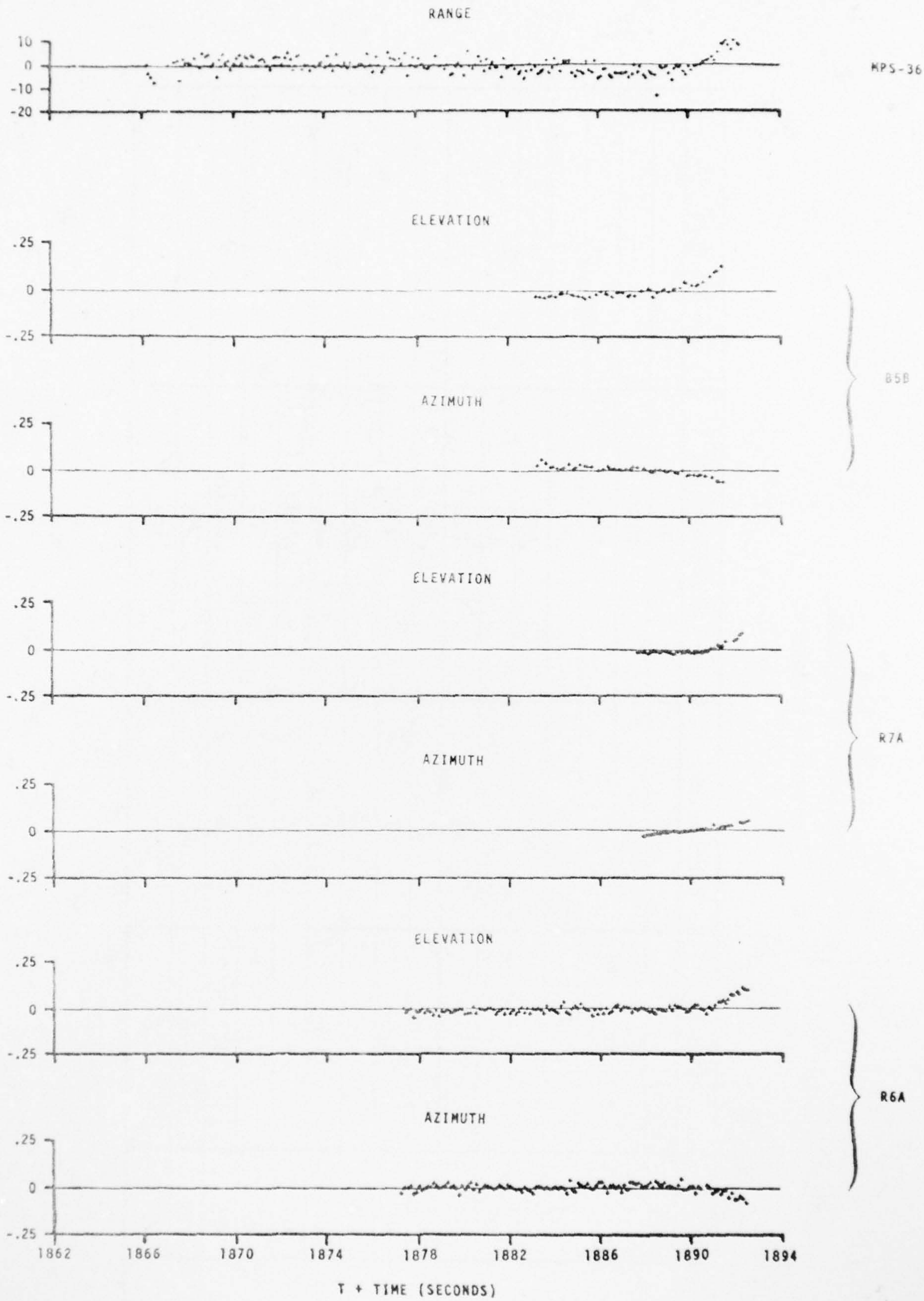


FIGURE 9 CORRELATED SENSOR RESIDUALS

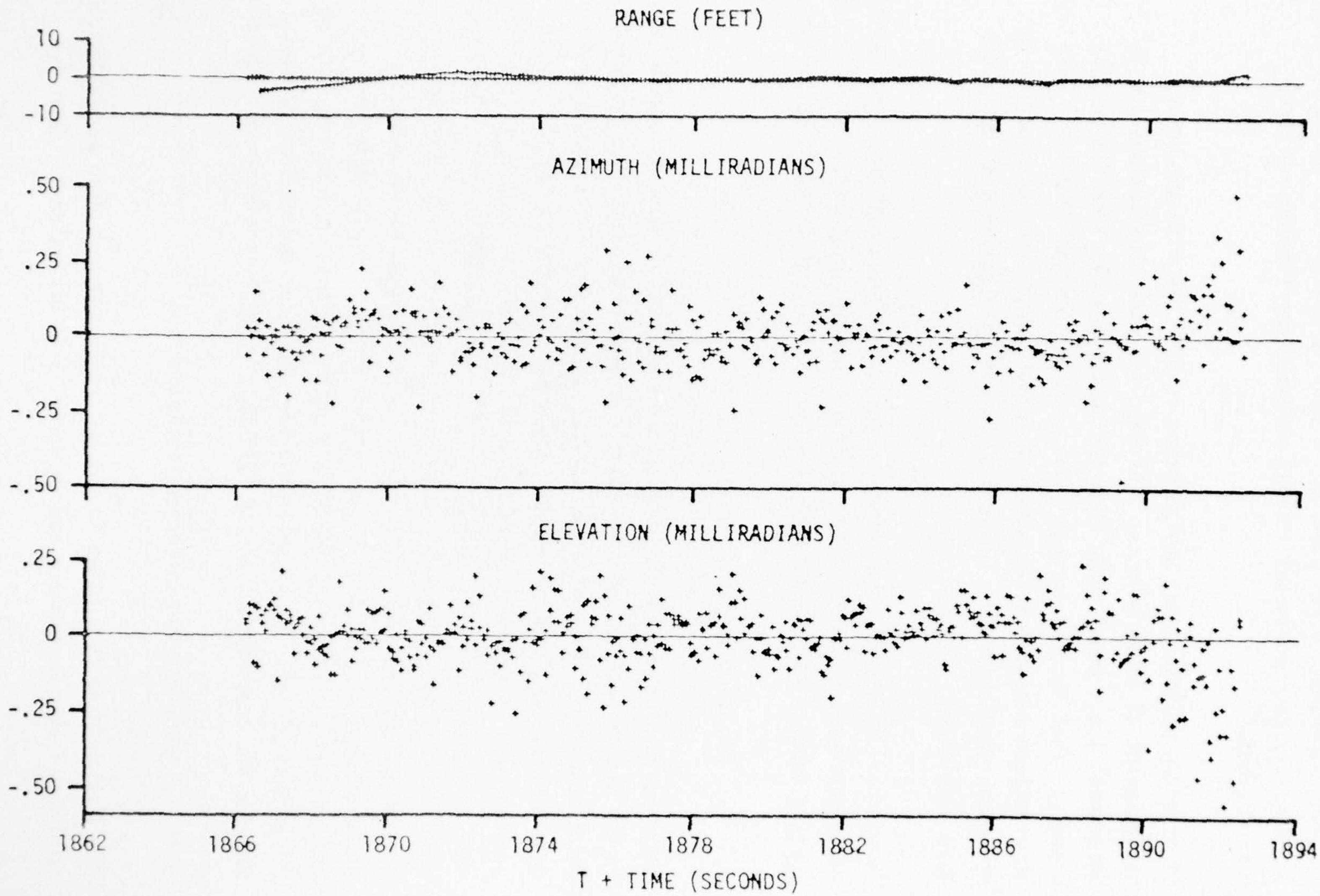


FIGURE 10 RADAR ALCOR RESIDUALS

adjusted parameter is assigned such a quality measure for the same reason (technically each parameter initial estimate constitutes one added observation).

It is often believed that any solution is possible with the proper manipulation of these weights. The fact is, when observations are in agreement, i.e., consistent, then the solution is invariant to weight changes. That is, the weighting has no effect on the solution. Only when observations disagree is weighting a factor. When they do disagree the cause can often be found, using the residual plots, and the 'erroneous' data deweighted.

In essence the use of equal weights, for disagreeing sensors, i.e., splitting the difference, is seldon a satisfactory compromise. That solution is most certain to be a wrong one since it agrees with neither set of observations.

3.3 Observation Sufficiency

Trajectory solutions constrained by the equations of motion require less in the way of observation constraints at each time point than do point by point solutions. For instance, optical azimuth and elevation observations over a sufficient time span with reasonable geometry can define a unique trajectory. If the geometry is marginal then a side condition at a single time point would normally suffice. As an example, Figure 11 illustrates a radar tracked bus (with R, A, E observations) from which an optically tracked RV (A, E observations only) separates at some time prior to the observations. Application of an initial RV position estimate from the bus trajectory constrains the RV trajectory so that it then requires the adjustment of only the separation velocity vector and RV drag parameters. Varying the separation time can further be used to minimize RV residuals.

3.4 High Resolution Trajectory Estimation

The TAPP program is limited to estimating 25 constant drag parameters, 25 horizontal lift parameters and 25 normal lift parameters*. Even for

* further limited to a total of 60 vehicle related parameters.

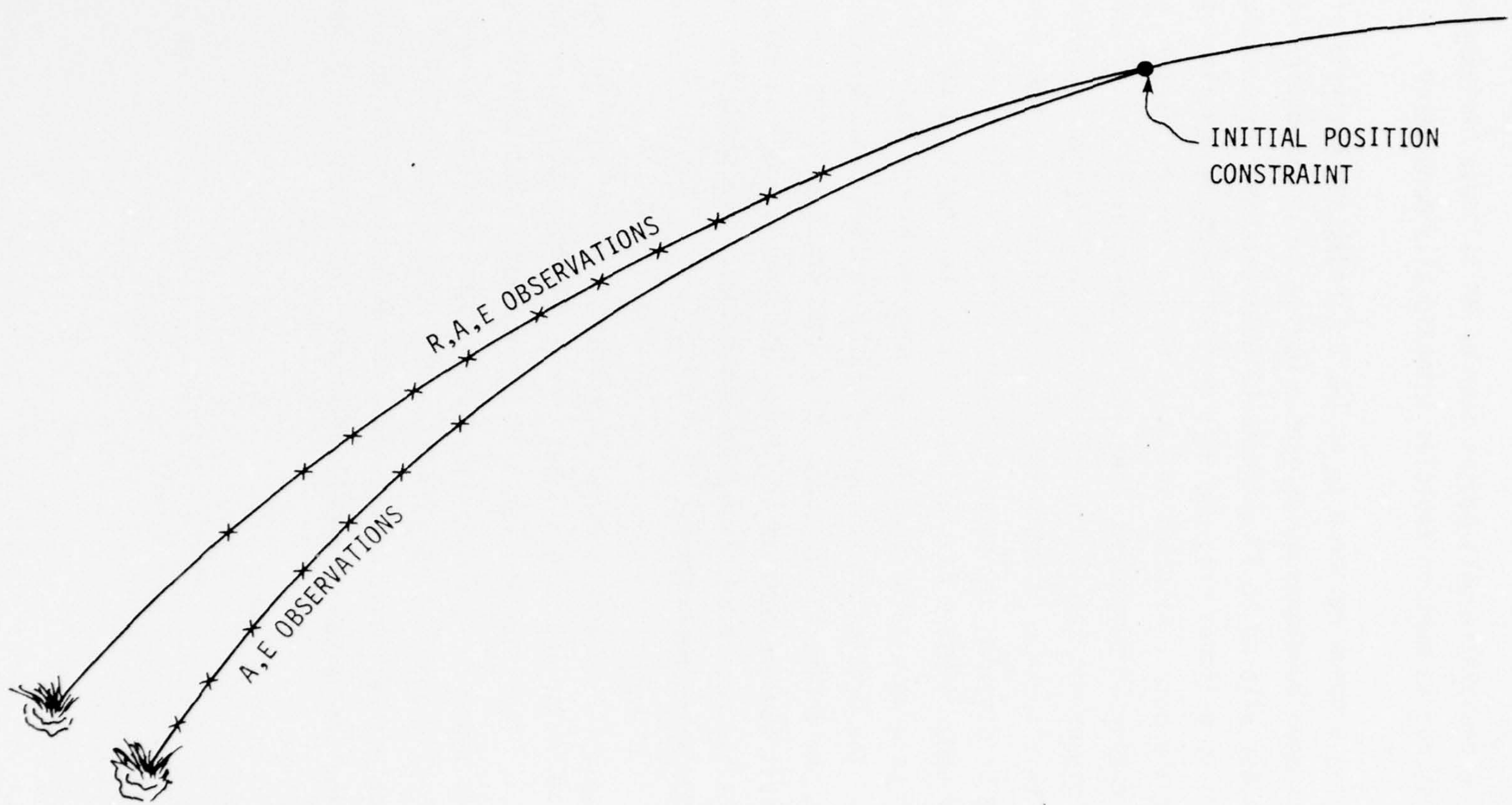
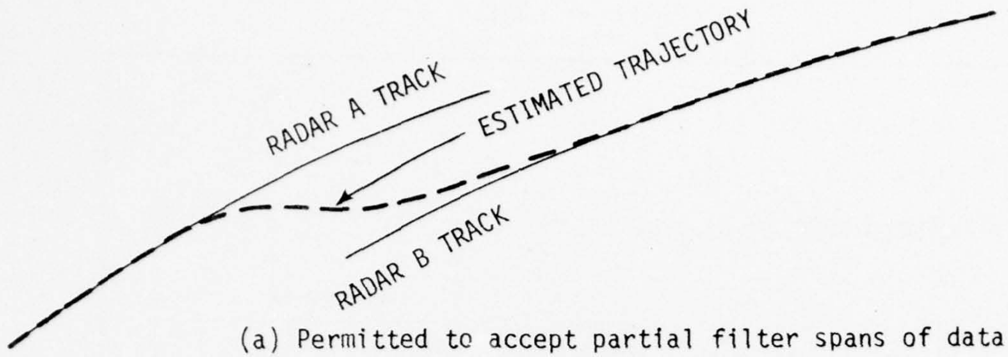


FIGURE 11 ADDITIONAL GEOMETRY CONSTRAINTS FOR REDUCED OBSERVATION SETS

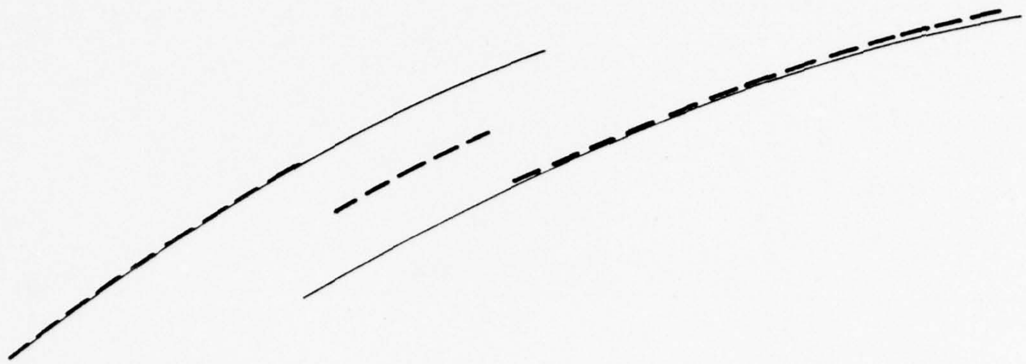
mildly lifting passive reentry bodies such as MM II these limits yield somewhat crude results, not to mention vehicles intentionally maneuvered.

A preprocessing program referred to as TAL/D (Trajectory Analysis of Lift and Drag) has been developed which uses a moving arc 9 parameters equations of motion filter with up to 57 channels of observations from a maximum of 19 sensors to estimate drag and lift accelerations at any resolution permitted by the data. Although the position and velocity vectors are part of the estimated parameters they are not necessarily useful since they may include biases not resolved in this resolution. A special precaution is taken in this process to avoid false acceleration transients caused by the starting or stopping of observation sets within a filter span. For instance, if radar A has a bias and B does not then the apparent trajectory could appear to have a false transient as depicted by the dashed line in Figure 12-a. The constraint is applied that a sensors data is used only if it covers the entire filter span. With this constraint the comparable position result would appear as in Figure 12-b and although position is discontinuous the accelerations remain continuous. The situation is comparable for rate observations.

The high resolution values of $C_D A/W$, $C_L A/W$ and θ versus time are recorded on tape by TAL/D for subsequent input into a TAPP reconstruction run (Figure 13-a). Then, in TAPP, a low frequency adjustment or biasing of these profile segments is accomplished using the 25 available parameters for both the drag and lift components as depicted in Figure 13-b. For instance, the net ballistic coefficients for each time would be obtained by multiplying the values of profile 13-b with the respective values of 13-a.



(a) Permitted to accept partial filter spans of data



(b) Constrained to accept only full filter spans of data

FIGURE 12 TAL/D MODELLING CONSTRAINT

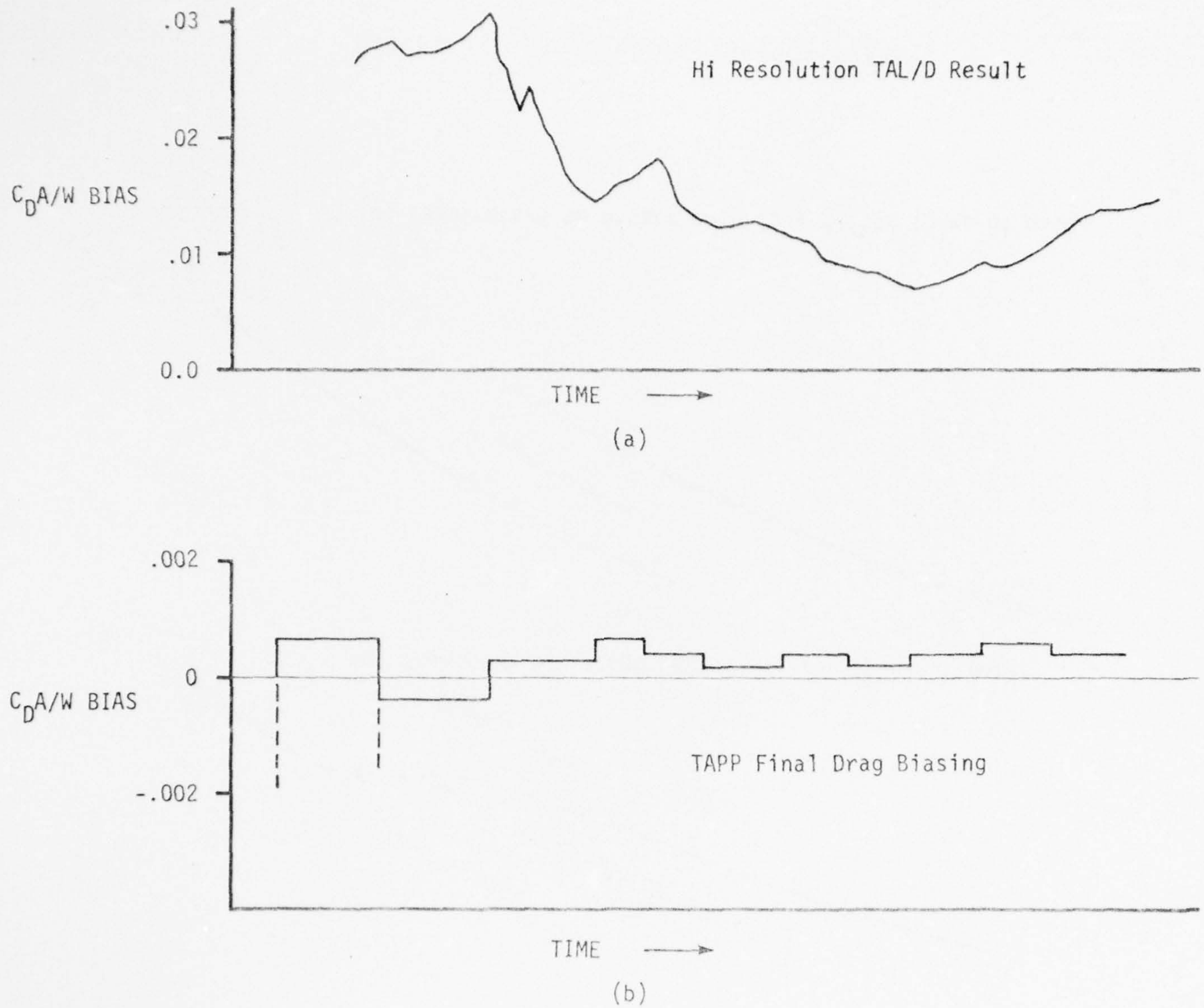


FIGURE 13 TAL/D A PRIORI BALLISTIC COEFFICIENT PROFILE and ESTIMATED TAPP BIAS HISTORY

4.0 FEATURES

Some of the salient features of the TAPP program regarding its capacity, reliability and convenience are discussed here. The more significant features are detailed first and the remaining are summarized in the final paragraph of this section.

4.1 Parameters (modelled and unmodelled)

Parameters are special variables in the TAPP program which affect the character of the trajectory and whose values are of direct interest to the analyst. Such variables include the vehicle initial state vectors and its drag and lift character, coefficients relating sensor error models and certain environmental model definitions.

For the reconstruction process only modelled (adjustable) parameters are meaningful while in the error analysis process unmodelled (unadjusted) parameters are applicable. The use of unmodelled parameters provide a means of including the uncertainties of reasonably well known parameters, such as the gravitational constant, in the final covariance matrix without subjecting it to possible alteration in value caused by unexpected correlation problems or weighting errors.

The available vehicle related parameters in TAPP are indicated in Table 1 and the available sensor and environmental parameters are shown in Tables 2 and 3 respectively.

4.2 Bounded Least Squares

The TAPP program does not rely on the generalized (weighted) least squares (GLS) process alone but utilizes what is referred to as a bounded least squares process. In the solution of a GLS problem it is not uncommon for the observed weighted residuals to not follow the corresponding predictions, or even to diverge. Conditions that could lead to such circumstances may, for example, involve inadequacies in the observational model or poor initial estimates of modelled parameters.

TABLE 1 VEHICLE PARAMETERS

• INITITAL POSITION VECTOR	ECI, SPHERICAL, KEPLERIAN
• INITIAL VELOCITY VECTOR	
• EPOCH TIME	
• BALLISTIC DRAG COEFFICIENTS	25 CONSTANT SEGMENTS OR A 5TH ORDER POLYNOMIAL COEFFICIENTS
• LIFT COEFFICIENTS	
• VELOCITY KICKS	(10 maximum)
• THRUST MAGNITUDES	(20 sets)
• GUIDANCE TERMS	
• ACCELEROMETER BIASES/ SCALE FACTORS:	(10 maximum)

TABLE 2 MODEL PARAMETERS

- LEGENDRE GRAVITY MODEL COEFFICIENTS
- POINT MASS GRAVITY COEFFICIENTS
- GRAVITATIONAL CONSTANT
- ATMOSPHERIC ROTATION RATE
- COEFFICIENTS OF THE TEMPERATURE EQUATIONS (JACCHIA 1964 ATM.)

TABLE 3 SENSOR PARAMETERS

- STATION LATITUDE
- STATION LONGITUDE
- STATION ALTITUDE
- TIME BIAS
- RANGE BIAS
- AZIMUTH BIAS
- ELEVATION BIAS
- RANGE RATE BIAS
- AZIMUTH RATE BIAS
- ELEVATION RATE BIAS
- RANGE SCALE FACTOR
- RANGE RATE SCALE FACTOR
- NORTHERLY MISLEVEL COEFFICIENT
- WESTERLY MISLEVEL COEFFICIENT
- NONORTHOGONALITY (TRUNNION) COEFFICIENT
- MISALIGNMENT (COLLIMATION) COEFFICIENT
- DROOP COEFFICIENT
- TOPOCENTRIC RIGHT ASCENSION BIAS
- TOPOCENTRIC DECLINATION BIAS
- TOPOCENTRIC HOUR ANGLE BIAS
- GEOCENTRIC RIGHT ASCENSION BIAS
- GEOCENTRIC DECLINATION BIAS
- ARGUMENT OF LATITUDE BIAS
- CROSS PLANE BIAS
- HEIGHT BIAS
- X BIAS
- Y BIAS
- Z BIAS
- P BIAS
- Q BIAS
- P SCALE FACTOR
- Q SCALE FACTOR
- P-DOT BIAS
- Q-DOT BIAS
- P-DOT SCALE FACTOR
- Q-DOT SCALE FACTOR
- DOPPLER BIAS
- DOPPLER SCALE FACTOR
- TWO WAY DOPPLER BIAS
- TWO WAY DOPPLER SCALE FACTOR
- SGLS RANGE RATE FREQUENCY
- SGLS RANGE RATE BIAS
- SGLS RANGE RATE SCALE FACTOR
- X ANTENNA ANGLE
- Y ANTENNA ANGLE

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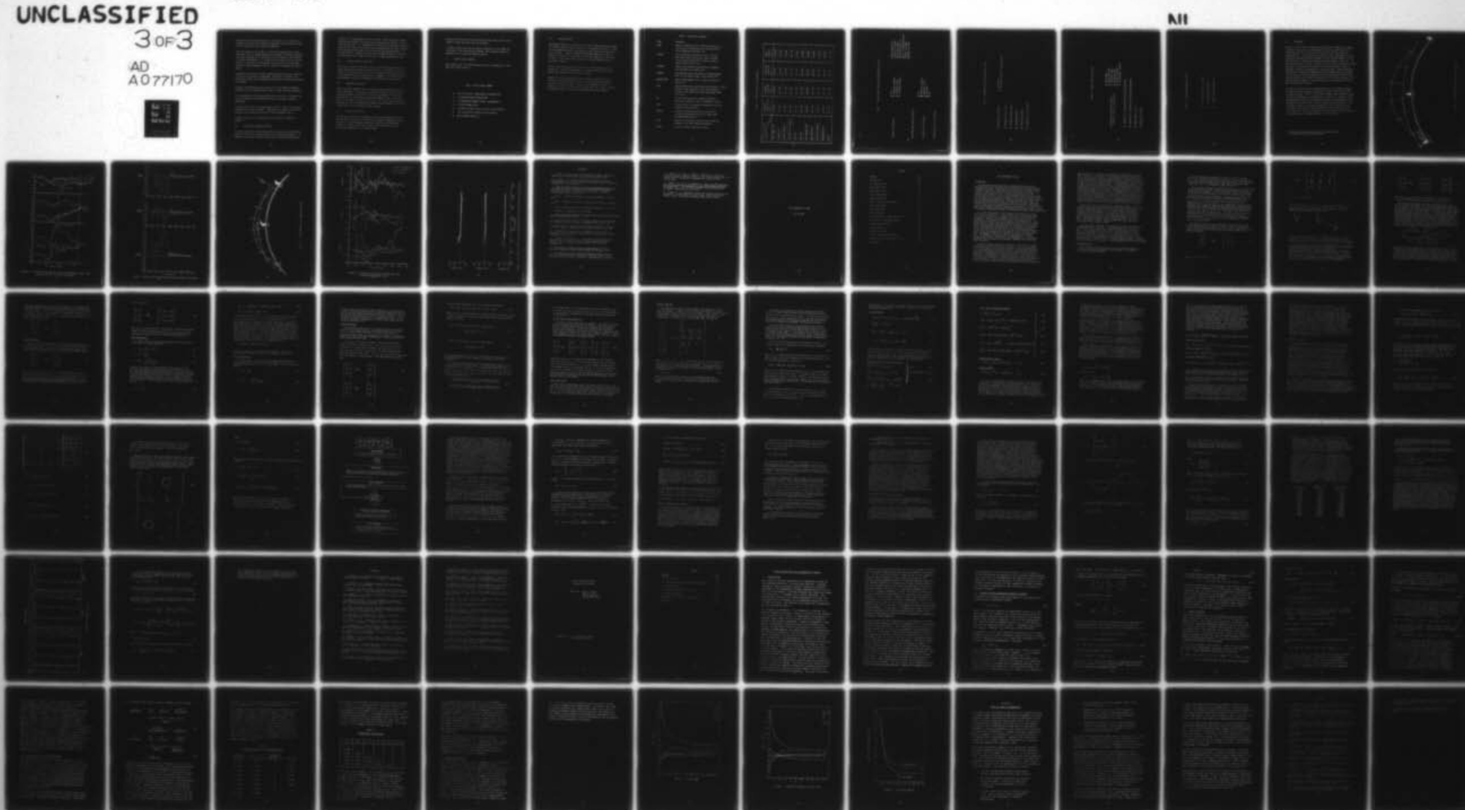

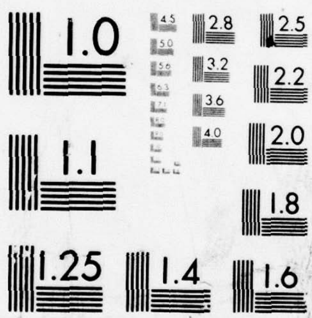

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

Where there are such manifestations of nonlinearity, it is necessary to introduce side conditions bounding the magnitude of the correction vector at each iteration to force eventual convergence.

Since the purpose of the bounds is to ensure the good behavior of each interation, they are changed dynamically according to previous performance. The overall weighted sum of the squares of all residuals is this measure of iterative performance. If it grows from one iteration to the next the solution is diverging, indicating that the corrections are too extreme. Therefore, before proceeding, the last iteration is repeated with smaller bounds.

Conversely, if the overall residual magnitude decreases from one iteration to the next the solution is converging. Therefore, to allow for the possibility of more rapid convergence through larger corrections the bounds are expanded.

To bypass the bounding process on any or all of the modelled parameters requires the use of negative bound values for those respective parameters.

If all parameters have negative bounds and an interation is divergent the run will terminate since no remedial action can be taken to promote convergence.

In general, the values of the bounds should be set to that of the expected maximum error in exact parameter. These bounds are generally halved when divergence occurs and multiplied by 1.5 when convergence occurs.

Further, discussion of the bounds logic can be found in reference 3, Volume V.

4.3 Differential Equations Solution

The TAPP program solves variational equations along with the differential equations of motion to form the partials for the vehicle parameters. Two methods are available in TAPP for numerically solving these differential equations. For exoatmospheric orbital problems a 10th order Gauss-Jackson technique with automatic step selection provides quality solutions in very short running times. For endoatmospheric solutions the differential equations of motion are solved using a drag linearized Runge-Kutta Gill (RKG) technique while the variational equations are solved with only the RKG method, i.e., no linearization. Automatic step selection procedures provide a consistent solution quality with respect to time even for high drag parachute drops.

4.4 Initial Conditions Self Start

The self-start mode for deriving initial position and velocity vectors uses range, azimuth and elevation type observations. It transforms up to 30 triads of such observations to Cartesian coordinates and least squares fits a 0th through 7th order polynomial to each component. This method enables endoatmospheric as well as exoatmospheric reconstruction self starting.

4.5 Parameter Constraints

Certain adjusted parameters may be related to other parameters rather than being independent. For instance, one stations survey may be well defined relative to another site which is itself to be estimated. In this case both stations are stipulated as adjustable parameters, but a constraint matrix is included to relate the two surveys. As a result the two surveys are moved as a pair retaining their original relative locations with respect to each other as the estimation process proceeds.

4.6 Weighting and Editing

All observations and initial parameter estimates are weighted to control their relative influence in a reconstruction problem. Observations can be weighted as a group according to type and station or are individually weighted on a point by point basis from the same input source as the observations themselves, i.e., cards or tape.

Observations editing can be controlled based either upon a priori input sigmas or sigmas derived in the last iteration.

It should be noted that only the relative magnitudes of the sigmas are significant in the reconstruction problem. Their absolute values are meaningful only in the error analysis problem.

4.7 Other Features Summary

Other capabilities of the TAPP program which can be summarized in a few words are listed in Table 4.

TABLE 4 OTHER FEATURE SUMMARY

- MULTIPLE VEHICLES (SIMULTANEOUS RECONSTRUCTION)
- 40 SENSORS/UNLIMITED OBSERVATIONS
- 100 PARAMETERS (SENSOR, VEHICLE, ENVIRONMENTAL)
- 35 OBSERVATIONAL TYPES
- 5 ATMOSPHERE MODELS (DENSITY, WIND, PRESSURE MODELS)
- 1000 GEOPOTENTIAL TERMS/20 POINT MASS MODELS
- MOVING SENSOR CAPABILITY

5.0 INPUT AND OUTPUT

Considerable effort was invested in the original TRACE program and extended in the TAPP program to make it easy to use. Large data sets such as those for the geopotential models are retained on tape libraries for access by a single mnemonic. Default constants are available when direct input is omitted. Input data is broken down into various categories as indicated in Table 5. The category input requirement for each program function is specified in Table 6.

Explicit input error messages are provided to clarify mistakes in deck preparation. Finally, documentation is available to fully describe the methods and techniques for each type of problem.

Output can be time sequenced or sorted by station. Special altitude, latitude or longitude dependent print-outs can be specified independent of time for trajectory output. Rise and set information can also be output based on first and last visibility for each site. Tables 7 through 10 provide a summary of the output available from the TAPP program.

TABLE 5 TAPP INPUT CATEGORIES

Name	Constants
MODEL	Models of ambient physical conditions which act as constraints on trajectory motion such as gravity field, atmosphere character, etc.
VEHICLE	Vehicle physical descriptions such as lift and drag coefficients versus mach, time, or height, initial position and velocity vectors, weights, areas, thrusts, kicks, etc.
STATIONS	Geodetic and astronomic observation or reference site positions, ID, indices, etc.
SENSORS	Observational sensor adjustable or fixed parameters such as site survey, biases, scale factors, etc.
OBSERVATIONS	Observed measurements such as range, azimuth, elevation, etc.
DATA	Simulated observation generation specifications - Type II consists of the observation types to be produced and Type I data generation cards denote when output is to occur.
ATA	A priori (ATA) matrix for modelled parameters expressing a measure of their confidence or quality.
COVQ	A priori matrix for unmodelled parameters expressing a measure of their quality.
REJECTS	Information specifying observation data points or spans to be deleted from use as inputs under the OBSERVATION identifier.
BLIST	A matrix of constraints relating certain modelled parameters with other modelled parameters.
VISCON	Visibility contour input data section.

TABLE 6 FUNCTION INPUT REQUIREMENTS

FUNCTION / CATEGORY	PARAMETER RECONSTRUCTION ITIN = 2	EPHEMERIS SIMULATION ITIN = 3	OBSERVATION GENERATION ITIN = 4	ERROR ANALYSIS ITIN = 5
BASIC MODEL DATA	REQUIRED	REQUIRED	REQUIRED	REQUIRED
STATION DATA	REQUIRED	NOT USED	REQUIRED	OPTIONAL
SENSOR PARAMETERS	OPTIONAL	NOT USED	OPTIONAL	OPTIONAL
CONSTRAINT DATA (BLIST)	OPTIONAL	NOT USED	NOT USED	NOT USED
ATA DATA	NOT USED	NOT USED	NOT USED	OPTIONAL
COVQ DATA	NOT USED	NOT USED	NOT USED	OPTIONAL
BASIC VEHICLE DATA	REQUIRED	REQUIRED	REQUIRED	REQUIRED
DATA GENERATION	NOT USED	NOT USED	REQUIRED	OPTIONAL
OBSERVATION REJECTS	OPTIONAL	NOT USED	NOT USED	OPTIONAL
OBSERVATION DATA	REQUIRED	NOT USED	NOT USED	OPTIONAL
VISCON	NOT USED	NOT USED	OPTIONAL	NOT USED

TABLE 7 TRAJECTORY SIMULATION/COMPARISON OUTPUT

- VEHICLE TRAJECTORY
  - ECI
  - SPHERICAL INERTIAL
  - SPHERICAL FIXED
  - KEPLERIAN ELEMENTS

  TIME DEPENDENT OR PRINTED AT EQUATOR CROSSINGS APOGEE - PERIGEE MIN-MAX ALT. GEOC. LATITUDE/LONGITUDES SPECIAL ALTITUDES OBSERVATION TIMES

- VARIATIONAL EQUATION PARTIALS
- TRAJECTORY DIFFERENCES
  - ECI
  - SPHERICAL INERTIAL
  - SPHERICAL FIXED
  - KEPLERIAN
  - HCL
- PRINTER PLOTS OF DIFFERENCES

TABLE 8 PARAMETER RECONSTRUCTION OUTPUT

- OBSERVATION RESIDUALS
  - INDIVIDUAL
  - ROOT MEAN SQUARE BY STATION AND TYPE
- PARAMETER CORRECTIONS
- PARAMETER UNCERTAINTIES
- COVARIANCE MATRIX
- CORRELATION MATRIX
- PRINTER PLOTS OF RESIDUALS

TABLE 9 ERROR ANALYSIS OUTPUT

- TRAJECTORY COVARIANCE MATRICES (Position, Velocity, Acceleration)
  - ORBIT PLANE
  - TRAJECTORY PLANE
  - SPHERICAL PLANE
  - CARTESIAN PLANE
  - KEPLERIAN ELEMENT
  - PERIOD/APOGEE, PERIGEE
- EIGENVALUES & EIGENVECTORS OF POSITION COVARIANCE
- SQUARE ROOT OF DIAGONALS OF TRAJECTORY COVARIANCES
- IMPACT ERROR ELLIPSE
- OBSERVATION PARTIALS

TABLE 10 OBSERVATION GENERATION OUTPUT

- VISIBILITY RISE AND SET TIMES
- TIME SORTED OBSERVATIONS
- STATION SORTED OBSERVATIONS

6.0 EXPERIENCE

On April 6, 1977 a special JRIAIG* working group met at SAMSO in Los Angeles to compare BET results for the Minuteman III Operation 6290 separation to impact trajectory for the purpose of estimating first order trajectory uncertainties**. The solutions from five participants, Kentron/KMR, Lincoln Laboratory, TRW and Xonics were compared with that from FEC/SAMTEC using the TAPP program for a trajectory shape approximately defined in Figure 14. The position differences in terms of intrack (X) (along earth fixed velocity vector), crosstrack (Y) and normal (Z) (normal to X and Y) are shown in Figure 15. Reference 10 details the parameters of each participant in these comparisons and reference 11 details the 6290 solution work accomplished at WTR leading to this solution including comparisons of the effects using three different gravity models on the solution. One additional illustration included at the meeting was that comparing a point-by-point BET with an equation of motion BET, as provided in Figure 16.

In light of the success of this JRIAIG meeting, the 1st Strategic Aerospace Division sponsored a meeting on November 26, 1977 at VAFB to compare BET's for Operation 7269, an MM II flight, where somewhat fewer observations were available from midrange and uprange sensors (Figure 17). Autonetics, AVCO, Kentron/KMR and Lincoln Lab trajectories were again compared against that generated by the TAPP program. Position comparisons are provided in Figure 18. Again a point by point versus equations of motion comparisons were illustrated using only the data where RADOT and BC coverage was present as in Figure 19. Reference 12 details the presentations from these uncertainties and reference 13 documents details of the FEC solution.

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* Joint Range Instrumentation Accuracy Improvement Group
** differences from all possible sources.

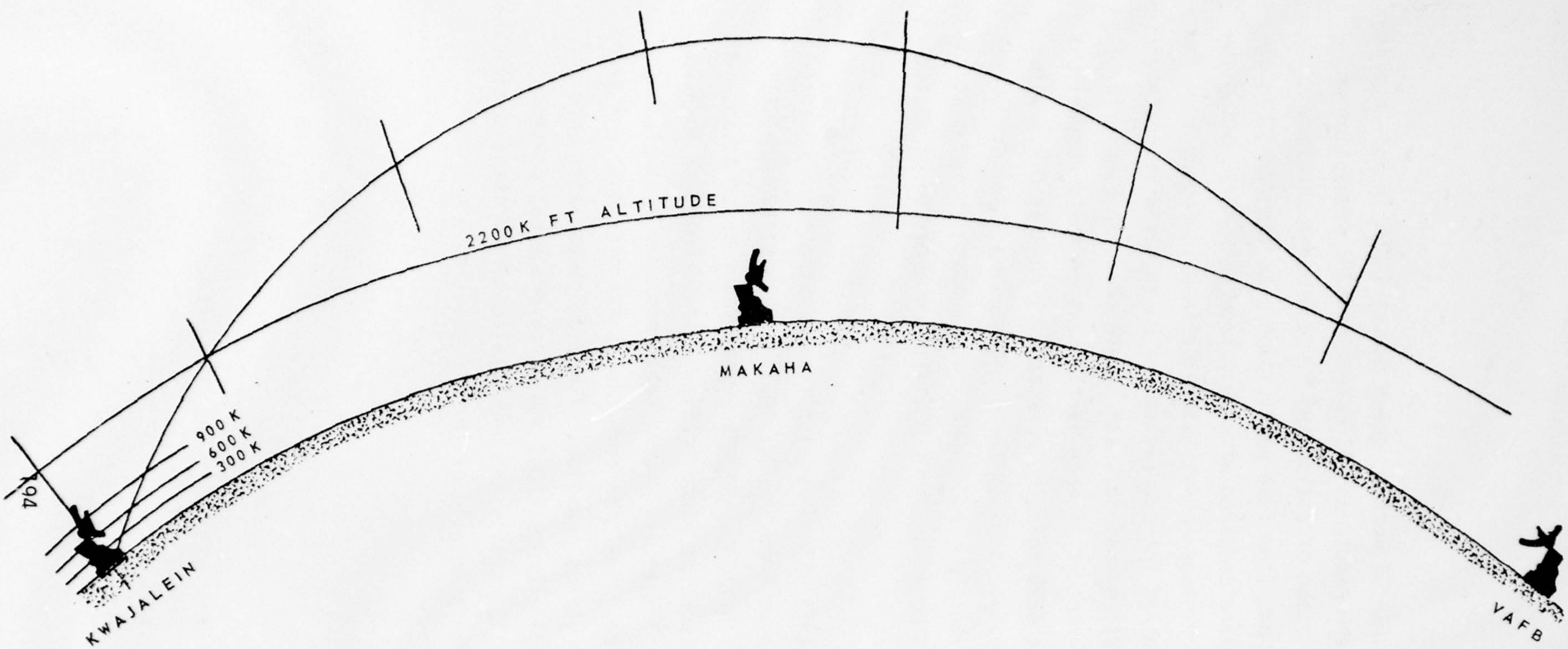

FIGURE 14 APPROXIMATE 6290 TRAJECTORY SHAPE

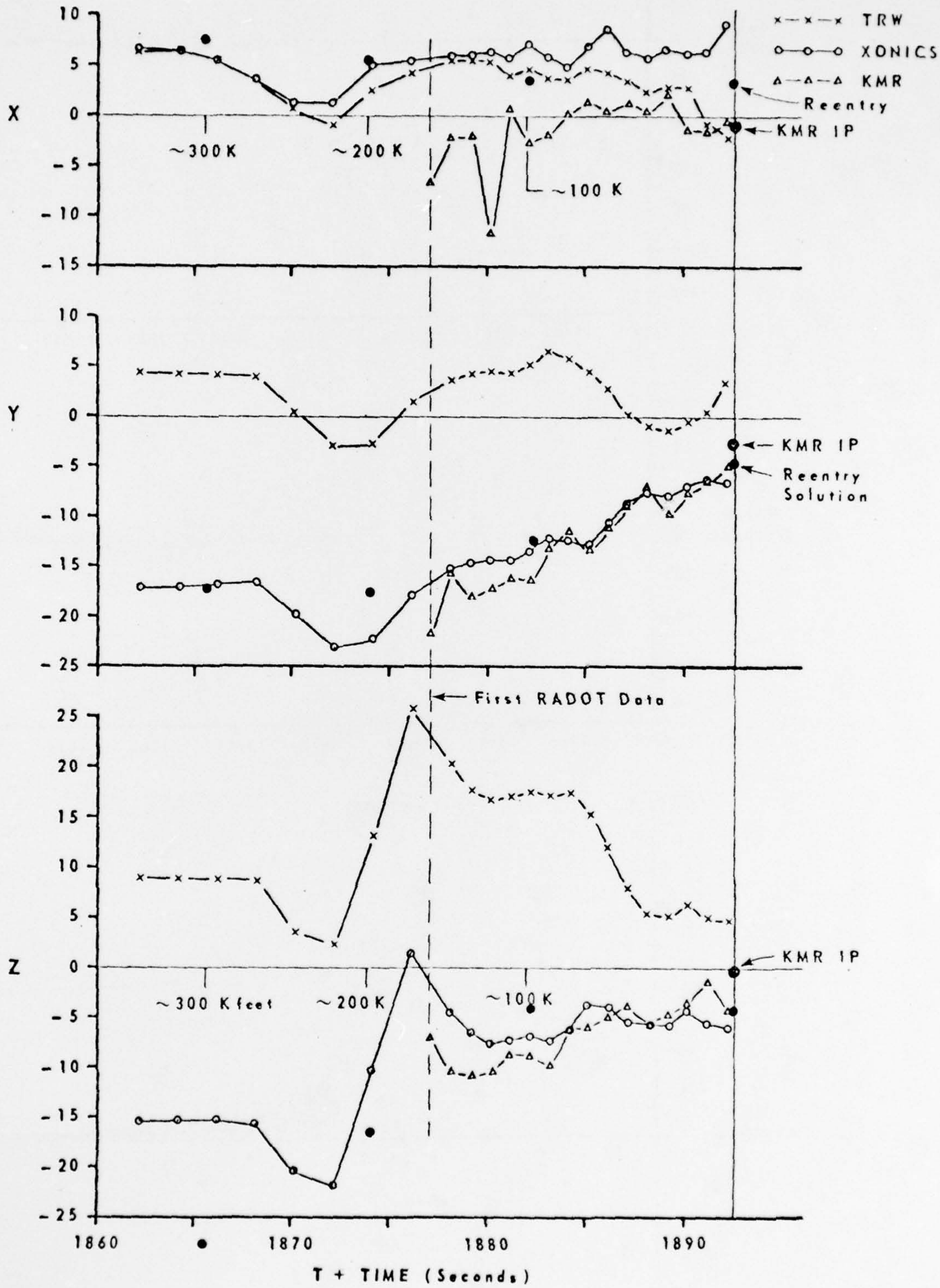

FIGURE 15 TRAJECTORY PLANE POSITION DIFFERENCES VERSUS TIME
(SAMTEC SEPARATION - TO - IMPACT REFERENCED)

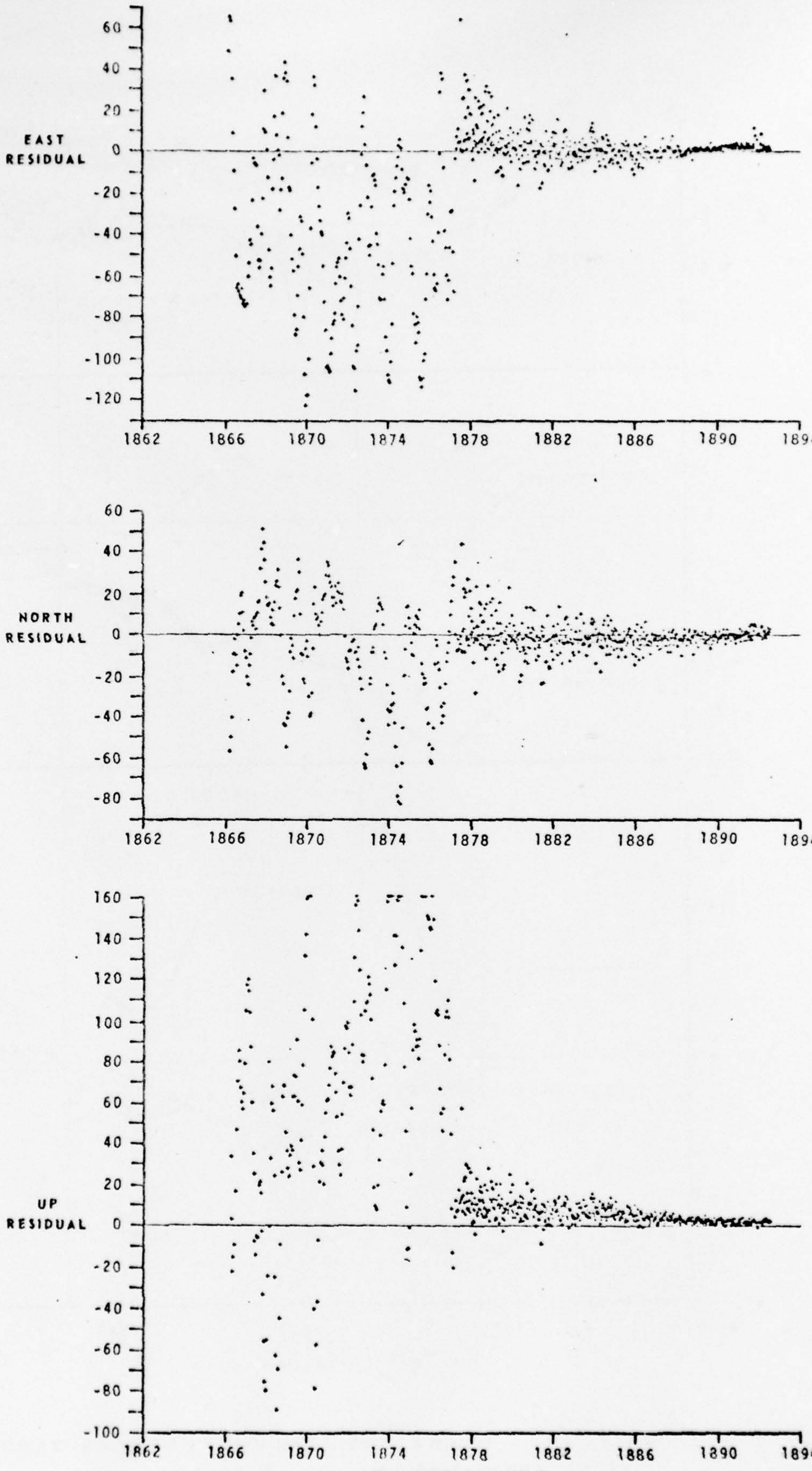

FIGURE 16 KMR POINT BY POINT TRAJECTORY MINUS WTR REENTRY TRAJECTORY

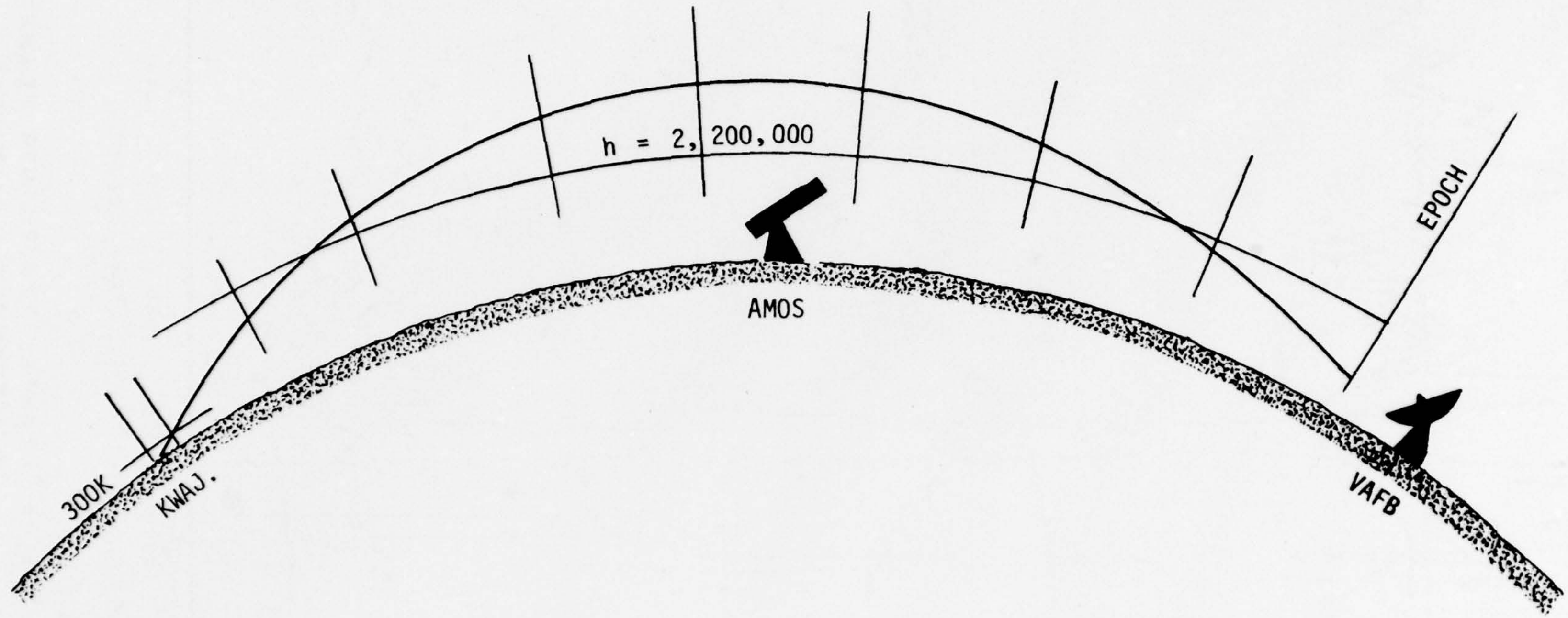

FIGURE 17 APPROXIMATE 7269 TRAJECTORY SHAPE

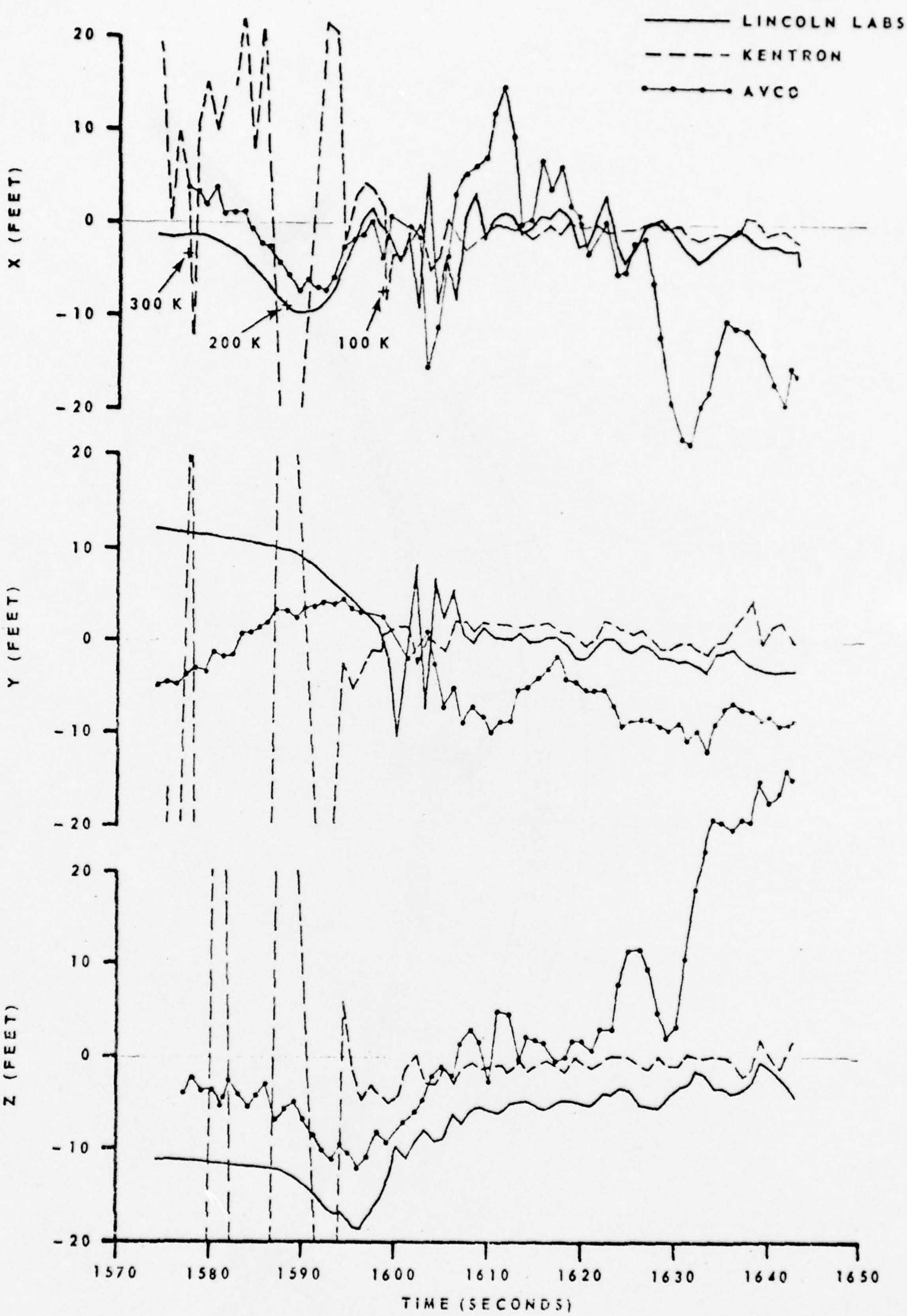

FIGURE 18 REENTRY TRAJECTORY PLANE POSITION DIFFERENCES VERSUS TIME

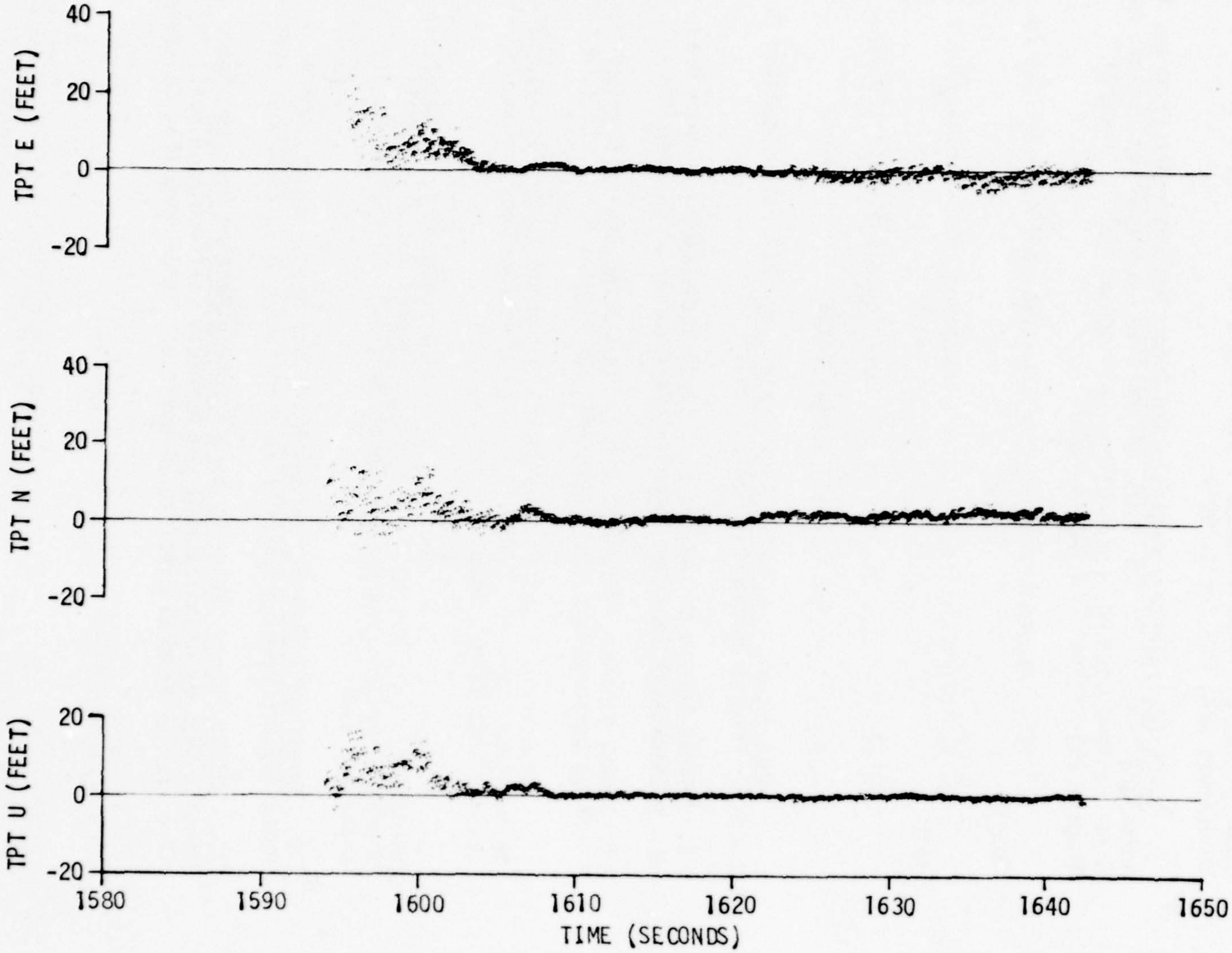

FIGURE 19 EAST-NORTH-UP COMPARISON WITH KMR/KENTRON MULTISENSOR REPORT POINT BY POINT SOLUTION

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BET DEVELOPMENT AT WSMR

William Agee

CONTENTS

BET DEVELOPMENT AT WSMR

Introduction

The development of Best Estimate of Trajectory (BET) programs at WSMR began more than ten years ago and these programs have been evolving in several directions. Extensive use has been made of these BET programs at WSMR in support of various missile and aircraft test programs. The impetus for development of a BET program was provided by the SAM-D (now Patriot) project and also by the navigational satellite test program. The BET program is still used in support of the Patriot and other missile test programs and has been used in support of several navigational satellite test programs including 621B, INI and INHI test programs. Other test programs at WSMR for which the BET has been used to provide support are Nike Hercules, the Navy SM2 and ASMD missile test programs, the Tomahawk and Air Launched and Cruise Missile (ALCM) test programs. The BET has also been used in support of the B1 avionics test program, Lance missile testing, and Completely Integrated Reference Instrumentation System (CIRCUS) system tests.

In initial development of a BET one must first decide as to what the BET must be able to do. A very optimistic but naive person might want his BET to estimate any parameter even remotely associated with a trajectory. Obviously, to accomplish such a task would require a prohibitively large amount of instrumentation and computer time. More realistically one might group the parameters to be estimated into three classifications, those associated with the trajectory or vehicle being tested, the parameters associated with the instrumentation, and parameters associated with the environment. The trajectory parameters include not only the usual position, velocity, and acceleration parameters of the trajectory, but also parameters such as attitude, angular rates, aerodynamic coefficients, guidance and control, parameters, etc. The parameters associated with the instrumentation include biases, scale factors, survey errors, misalignments, etc. Atmospheric parameters are an example of environmental parameters which one might want to estimate.

From each of these groupings one chooses a relatively small number of parameters which he wants a BET to estimate. The actual parameters which are estimated on a given test might be a program option depending on the instrumentation available on the test. The parameters which can be reliably estimated by a BET are entirely dependent on the type and geometry of instrumentation available on the test. Unfortunately, the high cost of instrumenting a trajectory and the relatively few types of instrumentation available at a range greatly restricts the parameters that can be estimated. If one is able to incorporate measurements made on board the test vehicle obtained either by telemetry or on board recording, the possibilities for parameter estimation are significantly increased. Thus, if there is a variation of instrumentation available from test to test and also a wide variation in the geometry of the trajectory relative to the instrumentation, one would probably want to include sufficient program flexibility in terms of options on the parameters to be estimated to match the variable instrumentation. In the early development of a BET at WSMR there was considerable interest in developing a highly flexible program that could utilize Patriot telemetry in conjunction with range instrumentation data to do a relatively complete estimation of aerodynamic and control parameters of this system. To develop a BET in this way would require an extreme amount of program flexibility if the program were to be used for other than Patriot trajectories. Thus, we decided to pursue a more general development of a BET that required much less flexibility and would apply to the wide variety of trajectories flown at WSMR and be able to utilize any available range instrumentation and have a limited amount of ability to utilize measurements made on board the test vehicle.

We currently have two operational BET programs at WSMR. The first of these two programs to be developed was our Kalman filtering BET. This program has been operational since 1971 and has been used extensively in support of WSMR test programs. It is seldomly used at the present time because we currently favor the use of our other BET program for reasons which we will discuss. Our second BET program is a nonlinear least square batch processing type of algorithm which we shall call MISTE. We began development on the MISTE program about 1973 and it became operational in 1975. Since mid-1976 it has been used almost exclusively for BET support at WSMR.

The intention of this paper is to provide an overview description of the WSMR BET programs including coordinate systems used, trajectory models, basic flow diagrams, basic estimation equations, input requirements, output options, data editing, etc. Our BET methods are under continuous development. One of the most important aspects of our BET efforts is continuous improvement. Thus, this paper will also describe our current directions of efforts at improving our BET programs. Another important aspect of WSMR BET efforts has been programs which were developed as derivatives of the BET programs. These developments will also be described.

Coordinate Systems

Many different coordinate systems are used in BET data reduction. The two most prominent coordinate reference systems in the WSMR BET's are the WSCS and the BET systems.

The WSCS coordinate system has its origin at 33° 05' latitude, 106° 20' W longitude and altitude at sea level. The x-axis is positive east, y-axis positive north, and the z-axis positive up. The WSCS system is the basic cartesian system used at WSMR and all instrument sites, launch sites, etc., are referenced to WSCS coordinates.

The trajectory coordinates estimated by the BET program are referenced to the BET cartesian system. The BET system is the basic coordinate system of the BET program. The origin of the BET system is at latitude λ_B, longitude μ_B, and altitude H_B. The x-axis is positive east, y-axis positive north, and z-axis positive up. Equivalently the origin of the BET system may be defined by specifying its WSCS coordinates ($x_{B/W}$, $y_{B/W}$, $z_{B/W}$). For a ground launched missile trajectory the most commonly chosen origin for the BET is the launch point. For an aircraft trajectory it is common to choose the BET system to be the WSCS system. The matrix which rotates from the BET system to the WSCS system will be denoted by M_{WB}. M_{WB} is computed from a knowledge of the latitudes and longitudes of the BET and WSCS systems. The specific form of this 3x3 rotation matrix will not be given here but may be found in [21]. The position translation vector from the BET system to the WSCS system has components ($x_{B/W}$, $y_{B/W}$, $z_{B/W}$) and are the components of the BET origin in the WSCS system.

Let ($\dot{x}$, $\dot{y}$, $\dot{z}$) be the velocity components in the BET system of the target reference point with respect to the BET origin. Let the origin of a trajectory Cartesian system be located at some reference point on the target. Define the rotation matrix M_{TB} from the BET system to the trajectory system as

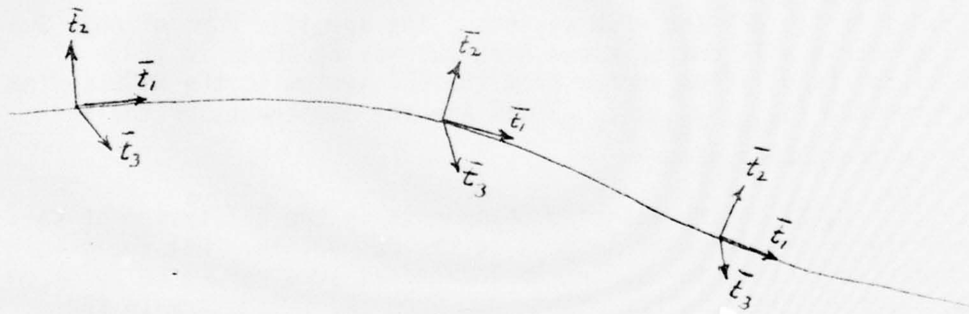
$$\begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} = M_{TB} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{1}$$

where $v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}}$

$$M_{TB} = \begin{bmatrix} \frac{\dot{x}}{V} & \frac{\dot{y}}{V} & \frac{\dot{z}}{V} \\ \frac{-\dot{x}\dot{z}}{V_G V} & \frac{-\dot{y}\dot{z}}{V_G V} & \frac{V_G}{V} \\ \dot{y}/V_G & \frac{-\dot{x}}{V_G} & 0 \end{bmatrix} \tag{2}$$

$$V_G = (\dot{x}^2 + \dot{y}^2)^{\frac{1}{2}}$$

The unit base vector $\overline{t}_1$ of this trajectory system is tangent to the target trajectory. The orientation of the trajectory system base vectors along a trajectory is shown below.



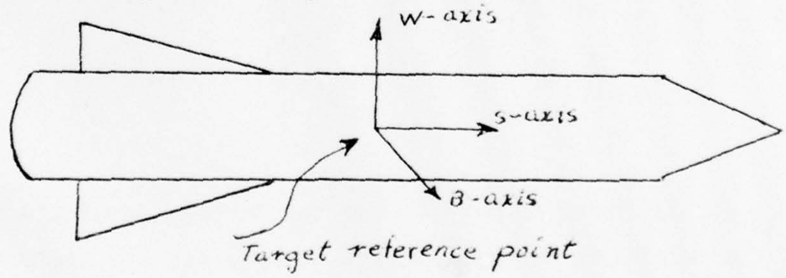
Note that the transformation M_{TB} is singular when $V_G=0$, i.e., when the trajectory is vertical. In order to avoid this singularity we choose a different trajectory coordinate system for nearly vertical trajectories.

At each range instrumentation site we define a cartesian coordinate system with origin having the geodetic coordinates (λ_i, μ_i, H_i) and with the x-axis positive east, the y-axis positive north, and the z-axis positive up. The rotation matrix from an instrument coordinate system to the BET system is denoted by M_{Bi} and is computed from the latitude and longitude of the two origins. The coordinates of the origin of an instrument coordinate system with respect to the BET system origin, denoted by $(x_{i/B}, y_{i/B}, z_{i/B})$, are computed from

$$\begin{bmatrix} x_{i/B} \\ y_{i/B} \\ z_{i/B} \end{bmatrix} = M_{BW} \left\{ \begin{bmatrix} x_{i/W} \\ y_{i/W} \\ z_{i/W} \end{bmatrix} - \begin{bmatrix} x_{B/W} \\ y_{B/W} \\ z_{B/W} \end{bmatrix} \right\}, \qquad (3)$$

where $(x_{i/W}, y_{i/W}, z_{i/W})$ are the coordinates of the origin of the instrument coordinate system with respect to the WSCS origin.

The trajectory estimated by the BET program must refer to some specific point on the target. In a single instrumentation system type of data reduction this target reference point would usually be the point on the target to which the measurement is referenced, e.g., DOVAP antenna, radar antenna, painted cross for optics. For a BET with several types of measurement systems tracking the target it is absolutely necessary to translate each measurement to a common point on the target. This point may be specified by the range user. This point is called the target reference point and is the origin of the target coordinate system. For an elongated target such as a missile or an aircraft the target 1-axis or s-axis is parallel to the long axis and positive toward the nose. The target 2-axis or w-axis is normal to the s-axis and positive upward. The 3-axis or b-axis completes a right handed system.



Since the target coordinate system is rigidly tied to the target, the orientation of this coordinate system is unknown unless there are attitude or gimbal angle measurements relating the target orientation to a coordinate system of known orientation. In the absence of such measurements, it is necessary to assume that the target coordinate system axes are parallel to the trajectory coordinate axes previously defined. If gimbal angle measurements are available, the transformation M_{BA} from the target coordinate system to the BET system can be computed. The computation of this transformation is peculiar to the type of inertial measurement unit (IMU) being used. If no angle measurements are available, it is necessary to set $M_{BA} = M_{TB}^T$. If the coordinates of a measurement reference point in the target coordinate system are (d_s, d_w, d_b), then the coordinates of this point in the BET system are

$$\begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} = M_{BA} \begin{bmatrix} d_s \\ d_w \\ d_b \end{bmatrix} \qquad (4)$$

Measurement Models

Let (x, y, z) be the coordinates of the target reference point in the BET coordinate system. Also, let (d_{xi}, d_{yi}, d_{zi}) be the BET coordinates of the measurement reference point on the target (radar antenna, optical cross, etc.) with respect to the origin of the target coordinate system.

$$\begin{bmatrix} d_{xi} \\ d_{yi} \\ d_{zi} \end{bmatrix} = M_{BA} \begin{bmatrix} d_{si} \\ d_{wi} \\ d_{bi} \end{bmatrix} , \qquad (5)$$

where M_{BA} is the rotation matrix from the target coordinate system to the BET coordinate system and (d_{si}, d_{wi}, d_{bi}) are the coordinates of the measurement reference point in the target coordinate system. Let the coordinates of the measurement reference point on the target with respect to the instrument coordinate system be denoted by ($x_{T/i}$, $y_{T/i}$, $z_{T/i}$).

They are given by

$$\begin{bmatrix} x_{T/i} \\ y_{T/i} \\ z_{T/i} \end{bmatrix} = M_{Bi}^{T} \begin{bmatrix} x + d_{xi} - x_{i/B} \\ y + d_{yi} - y_{i/B} \\ z + d_{zi} - z_{i/B} \end{bmatrix} \tag{6}$$

where M_{Bi} is the rotation matrix from the local instrument coordinate system to the BET coordinate system and $(x_{i/B}, y_{i/B}, z_{i/B})$ are the coordinates in the BET system of instrument origin given by (3).

Radar Measurements

The range, azimuth, and elevation measurements of the FPS-16 and MPS-36 radars are ideally modelled as

$$R = (x_{T/i}^2 + y_{T/i}^2 + z_{T/i}^2)^{\frac{1}{2}} \tag{7}$$

$$A = \tan^{-1} \frac{x_{T/i}}{y_{T/i}} \tag{8}$$

$$E = \tan^{-1} \frac{z_{T/i}}{(x_{T/i}^2 + y_{T/i}^2)^{\frac{1}{2}}} \tag{9}$$

These are ideal models of the measurements which assume that the systematic errors such as refraction, zero set, collimation, tilt, etc. have been removed from the measurements by calibration. Although the radar measurements have been calibrated and the estimated systematic errors removed from the data, there is usually a significant amount of systematic error remaining in the data. Thus, one must use an error model of the radar measurement and estimate the unknown parameters of this error model. A fairly complete error model for radar measurements is given by

$$\Delta R = r_0 + r_1\dot{R} \tag{10}$$

$$\Delta A = a_0 - a_1 \tan E \cos A - a_2 \tan E \sin A + a_3 \sec E + a_4 \dot{A} \tag{11}$$

$$\Delta E = e_0 + a_1 \sin A - a_2 \cos A + e_1 \dot{E} \tag{12}$$

In the above error model (r_0, a_0, e_0) are zero set error coefficients, a_1 and a_2 are tilt errors, a_3 is a collimation error, and r_1, a_4, e_1 are a combination of timing error and servo lag errors. Other error terms could also be included in this model. The relative geometry of the trajectory and instrumentation plan will determine which coefficients in the error model can be reliably estimated. For most trajectories at WSMR, it is not possible to estimate more than just the zero set error coefficients (r_0, a_0, e_0). This situation is acceptable if the other error coefficients have been reliably estimated by the premission calibration procedure. For many WSMR trajectories the relative geometry will not even allow the estimation of the zero set coefficients. Some of the radars available at WSMR have the capability to provide a range rate measurement. The ideal model for this measurement is

$$\dot{R} = \frac{x_{T/i}\dot{x}_{T/i} + y_{T/i}\dot{y}_{T/i} + z_{T/i}\dot{z}_{T/i}}{R} \tag{13}$$

No errorsare estimated for range rate measurements. However, an approximation for range rate refraction is removed from the data.

Optical Measurements

The fixed cameras and cinetheodolites provide measurements of the azimuth and elevation angles of the line of sight from the camera coordinate system origin to a point on the target. The ideal measurement models are

$$A = \tan^{-1} \frac{x_{T/i}}{y_{T/i}} \tag{14}$$

$$E = \tan^{-1} \frac{z_{T/i}}{(x_{T/i}^2 + y_{T/i}^2)^{\frac{1}{2}}} \tag{15}$$

The above model assumes that systematic components of error have been reliably estimated by premission calibrations and removed from the data and that the measurements have been corrected for refraction. The premission optical calibrations are usually reliable so that it is not necessary to estimate coefficients in a complete error model. At most it may be necessary to estimate zero set error coefficients from the trajectory measurement data.

Dovap Measurements

The Dovap measurement system is a two way CW doppler system which measures a loop range change or alternatively an average loop range rate of a target between consecutive sampling times t_i and t_{i+1}. Although the Dovap system has been phased out at WSMR, it has been used extensively in BET reductions at WSMR and well illustrates the use of doppler measurements in a BET.

Let (x_T, y_T, z_T) be the coordinates of the ground transmitter antenna in the BET coordinate system. Also, let (x_R, y_R, z_R) be the coordinates of a ground receiver antenna. Let (d_{xT}, d_{yT}, d_{zT}) and (d_{xR}, d_{yR}, d_{zR}) be the BET coordinates of the Dovap transmitting and receiving antennas on the target. Let (d_{sT}, d_{wT}, d_{bT}) and (d_{sR}, d_{wR}, d_{bR}) be the coordinates of these antennas in the target coordinate system. Then

$$\begin{bmatrix} d_{xT} \\ d_{yT} \\ d_{zT} \end{bmatrix} = M_{BA} \begin{bmatrix} d_{sT} \\ d_{wT} \\ d_{bT} \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} d_{xR} \\ d_{yR} \\ d_{zR} \end{bmatrix} = M_{BA} \begin{bmatrix} d_{sR} \\ d_{wR} \\ d_{bR} \end{bmatrix} \tag{17}$$

The loop range change model for a Dovap receiver measurement is

$$s(t_i, t_{i+1}) = R_T(t_{i+1}) + R_R(t_{i+1}) - R_T(t_i) - R_R(t_i) \tag{18}$$

where $R_T(t_i)$ is the range between the ground and target transmitter antennas at time t_i and $R_R(t_i)$ is the range between the ground and target receiver antennas at time t_i.

$$R_R(t_i) = [(x(t_i)+d_{xR}(t_i)-x_R)^2+(y(t_i)+d_{yR}(t_i)-y_R)^2 +(z(t_i)+d_{zR}(t_i)-z_R)^2]^{\frac{1}{2}} \tag{19}$$

$$R_T(t_i) = [(x(t_i)+d_{xT}(t_i)-x_T)^2+(y(t_i)+d_{yT}(t_i)-y_T)^2 +(z(t_i)+d_{zT}(t_i)-z_T)^2]^{\frac{1}{2}} \tag{20}$$

The Dovap measurements are corrected for refraction as described in [21]. No other systematic errors are assumed to be present in the Dovap measurements.

An alternative to the above position type measurement model for Dovap is a velocity type measurement model. If the dovap measurements are divided by $(t_{i+1} - t_i)$, the result is an average loop range rate over the interval (t_i, t_{i+1}). This average loop range rate can be used to approximate the loop range rate at $t_{AVE} = (t_i + t_{i+1})/2$. The loop range rate model of the Dovap measurements is

$$\dot{L} = \frac{(\dot{x}+\dot{d}_{xT})(x+d_{xT}-x_T)+(\dot{y}+\dot{d}_{yT})(y+d_{yT}-y_T)+(\dot{z}+\dot{d}_{zT})(z+d_{zT}-z_T)}{R_T} + \frac{(\dot{x}+\dot{d}_{xR})(x+d_{xR}-x_R)+(\dot{y}+\dot{d}_{yR})(y+d_{yR}-y_R)+(\dot{z}+\dot{d}_{zR})(z+d_{zR}-z_R)}{R_R} \tag{21}$$

In the above model all time varying quantities are evaluated at t_{AVE}. The quantities $(\dot{d}_{xR}, \dot{d}_{yR}, \dot{d}_{zR})$ and $(\dot{d}_{xT}, \dot{d}_{yT}, \dot{d}_{zT})$ can be approximated by differencing or differencing and smoothing the quantities defined in (16) and (17).

LTN-51 Inertial Measurement Unit

The LTN-51 inertial measurement unit (IMU) is one example of an IMU whose measurements have been used in a WSMR BET. The LTN-51 provides inertial acceleration measurements from accelerometers mounted on a 4-gimbal stable platform. In addition, the angle readings, which measure the orientation of the target with respect to a coordinate system established by the platform, are also used in the BET program. In terms of the tangential, normal and lateral accelerations along the trajectory the measured accelerations are (approximately)

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = M_{P_0T} \begin{bmatrix} A_T \\ A_N \\ A_L \end{bmatrix} + D_u \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} - S_u \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (22)$$

where (A_T, A_N, A_L) are the acceleration components in the trajectory coordinate system, (d_1, d_2, d_3) are constant unknown scale factor errors which are to be estimated, $(\delta_1, \delta_2, \delta_3)$ are constant, unknown platform misalignment errors to be estimated, and (b_1, b_2, b_3) are constant bias errors to be estimated. The 3x3 matrices M_{P_0T}, D_u, and S_u are known quantities depending mainly on the latitude and longitude of the platform, the latitude and longitude of the origin of the BET coordinate system, and the velocity of the target.

Other Measurements

Other types of measurements such as telemetered acceleration, angle measuring equipment (AME) measurements which are direction cosines, velocimeter, and laser tracker (R, A, E) measurements have been used at WSMR in the BET. The laser tracking measurements will probably be more often used in the future. It is a relatively simple matter to add capability to either of the BET programs so that they can process additional types of measurements.

Kalman Filter BET

The Kalman filter assumes a dynamic model of the trajectory in terms of a state vector x. A model of the systematic measurement error components is specified as a function of a bias vector b. Also, a measurement model is specified as function of the state vector x and the bias vector b. The state vector x is takento be a 9-vector of positions, velocities, and accelerations. The state vector and trajectory model are given by

$$\begin{aligned} x_1 &= x \\ x_2 &= y \\ x_3 &= z \\ x_4 &= \dot{x} \\ x_5 &= \dot{y} \\ x_6 &= \dot{z} \\ x_7 &= A_T \\ x_8 &= A_N \\ x_9 &= A_L \end{aligned} \qquad \dot{x} = f(x) = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ \frac{x_4 x_7}{v} - \frac{x_4 x_6 x_8}{v_G v} + \frac{x_5 x_9}{v} \\ \frac{x_5 x_7}{v} - \frac{x_5 x_6 x_8}{v_G v} + \frac{x_4 x_9}{v} \\ \frac{x_6 x_7}{v} + \frac{x_8 v}{v} + g \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

where $v_G = (x_4^2 + x_5^2)^{\frac{1}{2}}$, $v = (v_G^2 + x_6^2)^{\frac{1}{2}}$ and A_T, A_N, A_L are the components of acceleration in the trajectory coordinate system. Thus, we are modelling the trajectory as a constant acceleration system. We compensate for this mismodelling by adding an uncertainty term to the state equation so that we actually assume

$$\dot{x} = f(x) + w \tag{24}$$

where w is a random vector with zero mean and covariance matrix Q. Q is a 9x9 matrix with zero entries except for the 7th, 8th, and 9th diagonal entries which we use to compensate for mismodelling the acceleration.

The Kalman filter provides an optimal estimate of the state $x(t_k)$ of a linear system corrupted by additive Gaussian noise conditioned on all observations up to time t_k. In trajectory estimation we do not have a linear system but rather a nonlinear system due to the nonlinear nature of the trajectory equations and the nonlinear relationship between the measurements and the state variables.

The Kalman filter equations can be extended to this nonlinear situation and while the resulting equations and estimates are no longer optimal, they still provide good estimates which are easily implemented on the computer. The resulting filter, which is called an extended Kalman filter, is a most popular way of implementing a recursive estimation procedure for nonlinear systems and has been the subject for considerable development and evaluation in engineering literature, [1] - [4].

Let $x(t_k)$ be the state vector of the trajectory at time t_k. The trajectory dynamics are governed by the nonlinear dynamic equation

$$\dot{x}(t_k) = f\big(x(t_k)\big) + w_k \tag{25}$$

where $f(\cdot)$ is a known nonlinear function specified in (23) and w_k is a vector of Gaussian noise with mean zero and covariance matrix Q_k. The ith trajectory observation at time t_k is

$$z_i(t_k) = h_i\big(x(t_k)\big) + g_i^T(t_k)b(t_k) + v_i(t_k) \tag{26}$$

where $h_i(\cdot)$ is a known nonlinear function, $v_i(t_k)$ is zero mean Gaussian noise with covariance $R_i(t_k)$, $b(t_k)$ is a constant m-vector of biases to be estimated, and $g_i^T(t_k)$ is a known 1xp matrix. We assume and process only scalar observations with no loss of generality. We use the decomposition procedure of B. Friedland [5] - [7] to separate state estimation from bias estimation.

In the following we denote by $\hat{x}(k)$ the estimate at time t_k given all measurements up to time t_k and by $\hat{x}(k/k-1)$ the estimate at time t_k given all measurements up to time t_{k-1}. $\hat{b}(k)$ is the estimate of the bias vector given measurements up to time t_k.

Thus, $\hat{x}(k|k-1)$ is an estimate predicted from $\hat{x}(k-1)$. $\bar{x}(k)$ is an estimate obtained assuming no biases are present. The estimation equations for our Kalman filter BET are

At a Time Update

$$\bar{x}(k+1|k) = \bar{x}(k) + f(\bar{x}(k))\Delta t_{k+1} + J(\bar{x}(k))f(\bar{x}(k))\frac{\Delta t_{k+1}^2}{2} \tag{27}$$

$$\underset{9x9}{J(\bar{x}(k))} = \left[\frac{\partial f}{\partial x}\right]_{\bar{x}(k)}$$

$$\bar{P}_{k+1/k} = \Phi_k(\bar{P}_k + .5Q_k\Delta t_{k+1})\Phi_k^T + .5Q_k\Delta t_{k+1} \tag{28}$$

$$\Phi_k = I + J(\bar{x}(k))\Delta t_{k+1} + J^2(\bar{x}(k))\frac{\Delta t_{k+1}^2}{2} \tag{29}$$

In the above $\bar{P}_x$ is the 9x9 covariance matrix of $\bar{x}(k)$ and $\bar{P}_{k/k-1}$ is the covariance matrix of $\bar{x}(k/k-1)$. In obtaining the prediction equation (27) for $\bar{x}$ a second order Taylor series integration of (23) was used. The matrix Ricatti equation which governs the evolution of the covariance matrix is integrated using a trapezoidal rule to obtain (28).

Zero Bias Filter at Measurement Update

$$\left.\begin{array}{ll} \bar{P}_k^{(i)} = (\bar{P}_k^{(i-1)^{-1}} + H_i^T H_i/R_i(t_k))^{-1} & i=1,m \qquad (30) \\ \bar{w}^{(i)}(k) = \bar{P}_k^{(i)} H_i^T/(R_i(t_k) + H_i P_k^{(i-1)} H_i^T) & m=\#\text{ measurements at } t_k \qquad (31) \\ \bar{x}^{(i)}(k) = \bar{x}^{(i-1)}(k) + \bar{w}^{(i)}(k)\,(z_i(t_k) - h_i(\bar{x}^{(i-1)}(t_k))) & \qquad (32) \end{array}\right\}$$

$$H_i = \left[\frac{\partial h_i}{\partial x}\right]_{x=\bar{x}^{(i-1)}(t_k)} \qquad \bar{x}^{(0)} = \bar{x}(k/k-1)$$

Bias Filter at Measurement Update

$$
\left.
\begin{array}{ll}
s_i^T = g_i^T(t_k) + H_i T_{i-1}(k) & \\[2ex]
a_i(k) = R_i(t_k) + H_i P_k^{(i-1)} H_i^T \quad = \text{variance of residual} & \\[2ex]
P_b^{(i)}(k) = \left(P_b^{(i-1)^{-1}}(k) + s_i^T s_i / a_i(k)\right)^{-1} & \\[2ex]
w_b^{(i)}(k) = P_b^{(i-1)}(k) s_i(k) / \left(a_i(k) + s_i^T P_b^{(i-1)}(k) s_i\right) & \\[2ex]
\bar{r}_i(k) = z_i(k) - h\left(\bar{x}^{(i-1)}(k)\right) \quad = \text{residual from zero bias filter} & \\[2ex]
\hat{b}^{(i)}(k) = \hat{b}^{(i-1)}(k) + w_b^{(i)}(k) \left(\bar{r}_i(k) - s_i^T(k) \hat{b}^{(i-1)}(k)\right) &
\end{array}
\right\} i=1,m
\qquad
\begin{array}{r}
(33) \\[2ex] (34) \\[2ex] (35) \\[2ex] (36) \\[2ex] (37) \\[2ex] (38)
\end{array}
$$

COMBINING MATRIX UPDATE

$$T_i(k) = T_{i-1}(k) - \bar{w}^{(i)}(k)\, s_i^T(k) \qquad i=1,m \tag{39}$$

OPTIMAL ESTIMATE

$$\hat{x}^{(i)}(k) = \bar{x}^{(i)}(k) + T_i(k) \hat{b}^{(i)}(k) \qquad i=1,m \tag{40}$$

The direct implementation of the above Kalman filter equations will often lead to serious numerical stability problems. This instability, called filter divergence in the literature [8] - [10], is primarily caused by mismodelling of the target dynamics, mismodelling of the measurements, or by errors in the noise statistics used by the filter. The divergence due to these sources have been successfully treated by several methods, most commonly by the addition of ficticious process noise covariance Q_k. Another contributor to the filter divergence problem is numerical inaccuracies in the filter computations. These numerical inaccuracies may act in such a way that the filter covariance matrices lose their positive definite character causing divergence of the filter. One very effective method of preventing divergence caused by numerical errors is to implement a square root version of the Kalman filter equations [5], [6], [11] - [14]. A square root filter computes a lower triangular square root matrix L such that $P=LL^T$ or such that $P=LDL^T$ where D is a diagonal matrix and P is a covariance matrix. In the square root implementation the square root matrix is computed and propagated in time rather than the covariance matrix. All filter computations are expressed in terms of the square root matrix. Our Kalman filter BET is implemented by using a numerically efficient square root formulation of the Kalman filter equations. The result is a stable filter that is almost as numerically efficient as the basic Kalman filter computations but has much more accuracy.

The dimension of the bias vector b can be quite large sometimes as large as 40. Then the computation of and computations involving the bias covariance matrix, for each scalar measurement and each time point along the trajectory sometimes presents a great computational burden.

The implementation of the Kalman filter for a BET is certainly not difficult. There are, however, two important features which must be implemented in any BET program and whose successful implementation is a must for good BET performance. These two features are measurement data editing and adaptive filtering. In our Kalman filter BET and in other Kalman filtering programs a rather simple method of measurement editing has been used. In this method an observation whose absolute predicted residual,

$$\left| z_i(t_k) - H_i \bar{x}(k)^{(i-1)} - g_i^T(k)\hat{b}(k)^{(i-1)} \right|$$

is greater than,

$$\alpha a_i(k) = \alpha(R_i(t_k) = H_i P_k^{(i-1)} H_i^T),$$

where α is a suitable scalar usually in the range $3 \leq \alpha \leq 6$, is not processed. This is an oversimplification of the measurement editing process which at least in a multi-instrument BET Kalman filter must be considerably more complex. In a Kalman filtering BET the editing must attempt to answer the questions: Did the residual fail the criterion because of a jump in bias, because of a sudden change in noise level, or because of a localized wild data point? How many consecutive times may a measurement fail the criterion before we decide the failures are not localized wild data but are caused by a bias jump or noise level jump? What do we do if we decide a bias or noise jump? Should measurement noise covariance be modified when criterion failure occurs? Our measurement editing technique answers all these questions and while quite successful, it consists of several ad hoc techniques and special conditions peculiar to measurement type. A more unified method of measurement editing would be desirable. We are now working on some filtering techniques which will be discussed later where measurement editing is inherent in the filter derivation. Except for some special conditions steps in the BET measurement editing are given by the following:

(1) If $|\bar{r}_i(k) - g_i^T(k)\hat{b}^{(i-1)}(k)| \leq 4\sigma_r$, process measurement and update computation of $R_i(t_k)$,

(2) If $4\sigma_r < |\bar{r}_i(k) - g_i^T(k)\hat{b}^{(i-1)}(k)| \leq 12\sigma_r$, do not process measurement, but update computation of $R_i(t_k)$,

(3) If $|\bar{r}_i(k) - g_i^T(k)\hat{b}^{(i-1)}(k)| > 12\sigma_r$, do not process measurement and do not update computation of $R_i(t_k)$,

In the above, σ_r^2 is the variance of the residual, $\bar{r}_i(k) - g_i^T(k)\hat{b}^{(i-1)}(k)$.

(4) If condition in (2) occurs eight consecutive times, decide bias has changed and reinitialize corresponding zero set bias component in bias filter with old bias estimate plus average of last eight residuals.

(5) If condition (3) occurs four consecutive times, assume instrument failure and reinitialize filter to reflect this condition. Measurement may try to reenter solution.

The second filter feature which is necessary for good performance is that the filter must be able to change its parameters in order to adapt to changes in measurement noise, process noise, and errors made in mismodelling dynamics and measurements. Compensation for errors in modelling are usually made thru proper selection of the process covariance Q_k. Since the principle source of modelling error in the WSMR BET filter is due to the assumption of constant accelerations, we add process noise only to these acceleration states by choosing the

7th, 8th, and 9th diagonal elements of the matrix Q_k. All other elements of Q_k are zero. We choose the level of these elements depending upon the level of acceleration (assuming high levels must mean a high future rate of change of acceleration) along various parts of the trajectory. This is not done automatically in our filter, but is set either by a knowing what acceleration levels are expected for the mission or by running a preliminary filter over the trajectory data to determine approximate acceleration levels. The measurement noise covariance used by the Kalman filter program is computed for each measurement by applying a fading memory filter to the scalar measurement residuals. There have been many efforts to construct Kalman filter algorithms which will automatically adapt to maneuvering targets. However, none of the methods we have seen seem to be satisfactory for our Kalman filter. There are some current research efforts in adaptive estimation which offer some promise and we are eagerly awaiting the results of the research.

Batch Processor BET

Let $x(t_k)$ be a state vector for a trajectory. If we have only position measurements available, we let $x(t_k)$ be the 3-vector with components (x, y, z), i.e., the BET coordinates of the trajectory. If in addition to position measurements, we also have range rate measurements available, we let $x(t_k)$ be a 6-vector with components $(x, y, z, \dot{x}, \dot{y}, \dot{z})$. The batch processor, which we call MISTE, combines measurements from radars, fixed cameras, cinetheodolites, and laser trackers to estimate the state vector $x(t_k)$ of the target. Attitude measurements may also be used in the batch processor to relate each measurement to the common target reference point. Batch processor in this case means that measurements for the entire trajectory are processed simultaneously rather than sequentially in time. We obtain an estimate $\bar{x}(t_k)$ simultaneously for each t_k along the trajectory and also obtain estimates of the measurement bias terms. Acceleration states or velocity and acceleration states of the trajectory are obtained by running an adaptive variable lag smoother over the raw trajectory estimates obtained from the batch processor.

There are some advantages to the batch processor formulation of the BET. Since no trajectory model is assumed by the batch processor, there are no errors in the position estimates or in the measurement bias estimates induced by trajectory mismodelling. The influence of mismodelling errors on position and bias estimates are very significant in the Kalman filter BET. We also have found that the batch processor BET is considerably more efficient in terms of computer time than the Kalman filter BET.

Let the scalar measurement model be as before, i.e.,

$$z_i(t_k) = h_i(x(t_k)) + g_i^T(k)b + v(t_k) \tag{41}$$

Assume we have measurements along the trajectory at times t_k, k=1, 2,---,n and for i=1,2,---,m. Again let $R_i(t_k)$ be the variance of $v(t_k)$. Then the batch processor estimates the p-vector b and the state vector $x(t_k)$ so that

$$\sum_{i=1}^{m} \sum_{k=1}^{n} \frac{1}{R_i(t_k)} \left(z_i(t_k) - h_i(x(t_k) - g_i^T(k)b\right)^2 \tag{42}$$

is minimized. Since we have a nonlinear estimation problem due to the nonlinear relation (41) between the measurements and trajectory state, we will be required to linearize the estimation at some stage of the derivation. It is most convenient to linearize in (42) now so as to obtain the common Gauss-Newton iteration sequence. Let $x^\circ(t_k)$, k=1, 2,---,n be a guess trajectory. We will discuss the sources of this guess later. Also, let $\delta x(t_k) = x(t_k) - x^\circ(t_k)$ and

$$H_i(k) = \left[\frac{\partial h_i(x(t_k))}{\partial x(t_k)}\right]_{x^\circ(t_k)}.$$

Then instead of minimizing (42) we will minimize

$$\sum_{i=1}^{m} \sum_{k=1}^{n} \frac{1}{R_i(t_k)} \left(r_i(t_k) - H_i(k)\delta x(t_k) - g_i^T(k)b\right)^2, \tag{43}$$

where $r_i(t_k) = z_i(t_k) - h(x_i^\circ(t_k))$. We proceed in the usual way by differentiating (43) with respect to $\delta x(t_k)$ and b and manipulating. This procedure leads to the set of normal equations,

$$
\begin{bmatrix}
A_1 & 0 & 0 & 0 & A_{1,n+1} \\
0 & A_2 & 0 & & A_{2,n+1} \\
 & 0 & A_3 & & A_{3,n+1} \\
 & & 0 & & \\
 & & & 0 & \\
0 & 0 & 0 & A_n & A_{n,n+1} \\
A_{1,n+1}^T & A_{2,n+1}^T & A_{3,n+1}^T & A_{n,n+1}^T & A_{n+1}
\end{bmatrix}
\begin{bmatrix}
\delta x(t_1) \\ \delta x(t_2) \\ \delta x(t_3) \\ \\ \\ \delta x(t_n) \\ b
\end{bmatrix}
=
\begin{bmatrix}
y(1) \\ y(2) \\ y(3) \\ \\ \\ y(n) \\ y(n+1)
\end{bmatrix}
\tag{44}
$$

where

$$A_k = \sum_{i=1}^{m} H_i^T(k) H_i(k) / R_i(t_k) \tag{45}$$

$$A_{k,n+1} = \sum_{i=1}^{m} H_i^T(k) g_i^T(k) / R_i(t_k) \tag{46}$$

$$A_{n+1} = \sum_{i=1}^{m} \sum_{k=1}^{n} g_i(k) g_i^T(k) / R_i(t_k) \tag{47}$$

$$y(k) = \sum_{\alpha=1}^{m} H_i^T(k) r_i(t_k) / R_i(t_k) \tag{48}$$

$$y(n+1) = \sum_{i=1}^{m} \sum_{k=1}^{n} g_i(k) r_i(t_k) / R_i(t_k) \tag{49}$$

The matrices A_k are either 3x3 are 6x6, $A_{k,n+1}$ are 3xp or 6xp depending on the dimension of the state vector $x(t_k)$. The vectors $y(k)$ are either 3-vectors or 6-vectors. $y(n+1)$ is p-vector and A_{n+1} is pxp.

Although (44) may be of very large dimension due to the large number of trajectory time points t_k, the simple structure of (44) allows a simple decomposition which leads to a simple solution. Let L be a block lower triangular matrix and let U be a block upper triangular matrix. Then we can find an L and U such that the coefficient matrix A in (44) can be written as A = LU. Equating corresponding blocks of A with those of the LU product gives

$$L = \begin{bmatrix} A_1 & & & \\ 0 & A_2 & \bigcirc & \\ & 0 & & \\ & & & \\ 0 & 0 & A_n & \\ A^T_{1,n+1} & A^T_{2,n+1} & A^T_{n,n+1} & Q \end{bmatrix} \tag{50}$$

$$U = \begin{bmatrix} I & 0 & & 0 & p_1 \\ & I & 0 & 0 & p_2 \\ & & & & \\ & \bigcirc & & & p_n \\ & & & & I \end{bmatrix} \tag{51}$$

where

$$P_k = A_k A_{k,n+1}^{-1} \tag{52}$$

$$Q = A_{n+1} - \sum_{k=1}^{n} A_{k,n+1}^T P_k \tag{53}$$

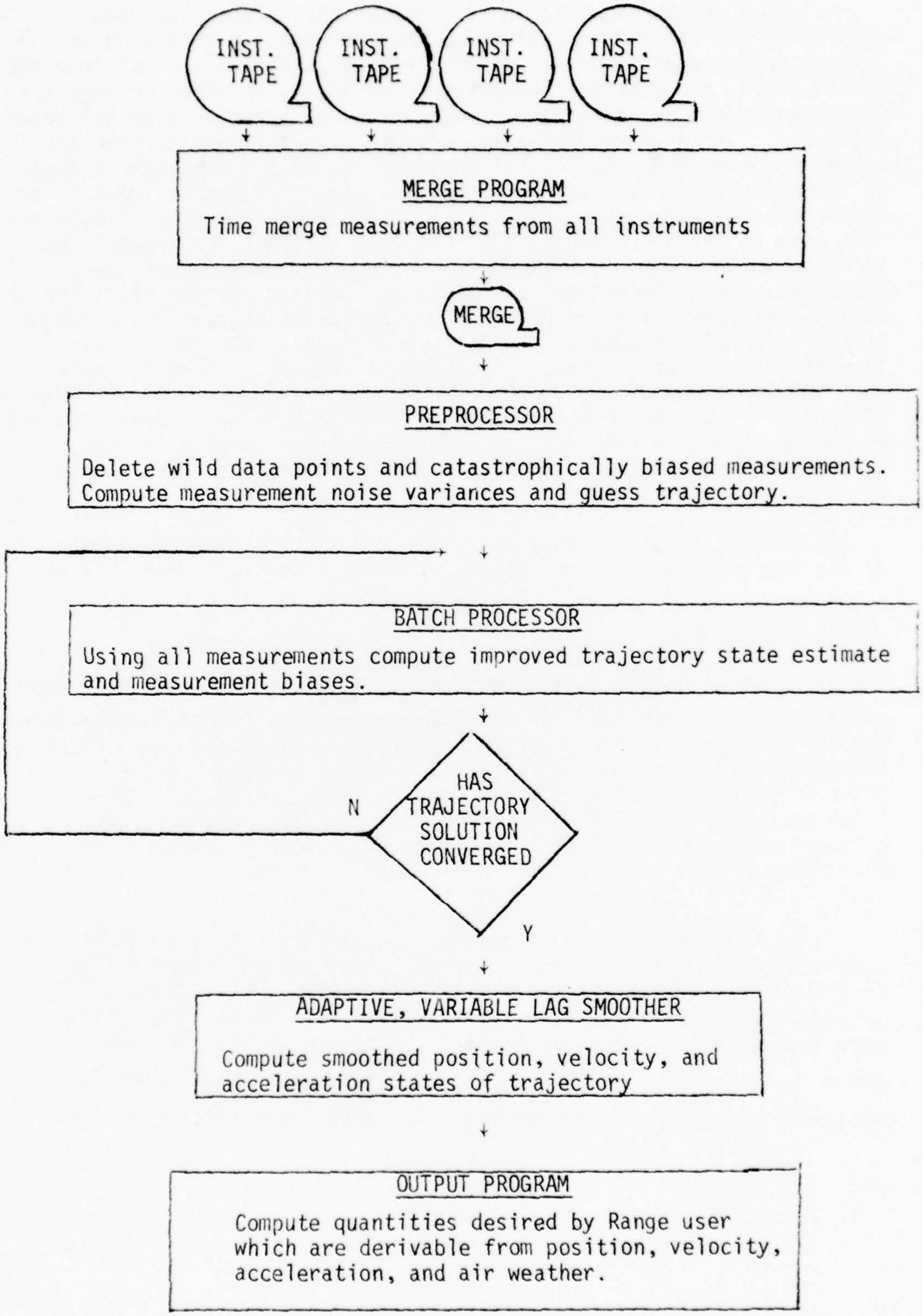
The solution of (44) leads to the solution of simple set of equations which reduces to

$$A_k \delta\bar{x}(t_k) = y(i) \quad i=1,n \tag{54}$$

$$Q\hat{b} = y(n+1) - \sum_{k=1}^{n} A_{k,n+1}^T \delta\bar{x}(t_k) \tag{55}$$

$$A_k \delta\hat{x}(t_k) = \sum_{i=1}^{m} H_i^T(k)(r_i(k) - g_i^T(k)\hat{b})/R_i(t_k) \tag{56}$$

Covariance estimates of $\delta\hat{x}(t_k)$ and $\hat{b}$ are also easily computed. The above linear least squares minimization is repeated with $x_i^\circ(t_k)$ replaced with $x_i^\circ(t_k) + \delta\hat{x}(t_k)$ until convergence. The batch processor is incorporated into a BET as shown by the following flow diagram.


INST. TAPE
INST. TAPE
INST. TAPE
INST. TAPE
MERGE PROGRAM
Time merge measurements from all instruments
MERGE
PREPROCESSOR
Delete wild data points and catastrophically biased measurements. Compute measurement noise variances and guess trajectory.
BATCH PROCESSOR
Using all measurements compute improved trajectory state estimate and measurement biases.
HAS TRAJECTORY SOLUTION CONVERGED
N
Y
ADAPTIVE, VARIABLE LAG SMOOTHER
Compute smoothed position, velocity, and acceleration states of trajectory
OUTPUT PROGRAM
Compute quantities desired by Range user which are derivable from position, velocity, acceleration, and air weather.

As shown in the flow diagram a preprocessor is used for data editing in the batch processor BET. The preprocessor fits a curve to each measurement sequence, deletes points which are too far from the fitted curve, and computes the variance of the differences between the measurements and curve fit to use as a processing variance in the batch processor. The preprocessor also provides a guess trajectory to the batch processor and interpolates measurements to a common set of times if the measurements are not synchronized. The measurements need to be synchronized in the batch processor in order that each type of measurement have an influence on the bias estimate obtained for another type of measurement. Thus, for example, if the radar measurements were processed at a different set of times than optical measurements, the optical measurements would not have any influence on the determination of the radar bias estimates and vice versa. It is the biases that furnish a tie between times in the batch processor. It is obvious from the structure of the batch processor equations given in (44) and (46) that if we have no bias terms to estimate then the (n+1)st row and column of blocks in (44) will be zero so that the batch processor reduces to an n(k)-station solution at each time point with n(k) is the number of instruments at t_k.

At each t_k a point $x^\circ(t_k)$ along the guess trajectory is computed by the preprocessor. If n_k, $n_k \geq 2$, optics stations are available at t_k, $x^\circ(t_k)$ is an n_k-station optical solution. If $n_k < 2$, $x^\circ(t_k)$ is computed from radars such that each component of position of $x^\circ(t_k)$ is the median of the corresponding components of the individual radar cartesian position. If any measurements considered for the guess are too far from the guess position solution, these measurements are temporarily removed from consideration and the guess trajectory solution repeated. If velocities are also being estimated by the batch processor, a guess solution for the velocity components is computed using the range rate measurements and the cartesian guess position.

Since the batch processor produces only a trajectory position solution or a position and velocity solution, an additional estimator is required to produce the derivative states for the trajectory. The estimator used is an adaptive, optimal variable lag smoother which uses the raw batch processor state estimates to obtain smoothed estimates $(x_s, \dot{x}_s, \ddot{x}_s)$, $(y_s, \dot{y}_s, \ddot{y}_s)$, $(z_s, \dot{z}_s, \ddot{z}_s)$. The smoothed estimates are obtained independently for each coordinate direction.

Let $s(t_k)$, a 3-vector, represent any of three coordinates of the trajectory, i.e., $s = (x, \dot{x}, \ddot{x})$ or $s = (y, \dot{y}, \ddot{y})$ or $s = (z, \dot{z}, \ddot{z})$. Assume that $s(t_k)$ obeys the discrete state equation,

$$s(t_{k+1}) = \Phi(\Delta)s(t_k) + \gamma(\Delta)u. \tag{57}$$

It is assumed that measurements of $s(t_k)$ are equally spaced in time with $t_{k+1} - t_k = \Delta$, the measurements being a position coordinate or position and velocity coordinate from the batch processor state estimate. Assuming constant acceleration the state transition matrix is

$$\Phi(\Delta) = \begin{bmatrix} 1 & \Delta & \Delta^2/2 \\ 0 & 1 & \Delta \\ 0 & 0 & 1 \end{bmatrix} \tag{58}$$

In (57) u is an unknown constant scalar forcing function. $\gamma(\Delta)$ is the vector

$$\gamma^T(\Delta) = [\Delta \quad \Delta^2/2 \quad \Delta^3/6] \tag{59}$$

In order to avoid complexity of notation, we consider the smoother equations when the batch processor is estimating only a 3-vector of position so that a position is the only measurement input to the smoother. Let $\hat{s}(k)$ be the smoothed estimate at time t_k given measurements $m(t_1), m(t_2), ---, m(t_{k+n_k})$. Thus, the smoother lags by n_k data points where n_k is variable. Let $\hat{\Delta}(k/k)$ be the optimal estimate at time t_k given measurements $t_1, t_2, ---, t_k$, i.e., the filtered estimate. Then

$$P^{-1}(k) = P^{-1}(k/k) + [\sum_{i=1}^{n_k} \Phi^T(i\Delta)H^T H\Phi(i\Delta)]/R(k) \tag{60}$$

$$\hat{s}(k) = \hat{s}(k/k) + P(k)[\sum_{i=1}^{n_k} \Phi^T(i\Delta)H^T\left(m(t_{k+i}) - H\Phi(i\Delta)\hat{s}(k/k)\right)]/R(k) \tag{61}$$

$H = [1\ 0\ 0\ ]$, $R(k)$ = measurement noise variance

$$\hat{s}(k+1/k) = \Phi(\Delta)\hat{s}(k/k) \tag{62}$$

$$P(k+1/k) = \Phi(\Delta)P(k/k)\Phi^T(\Delta) + q(k)\gamma(\Delta)\gamma^T(\Delta) \tag{63}$$

$$P^{-1}(k+1/k+1)=P^{-1}(k+1/k)+H^T H/R(k) \tag{64}$$

$$\hat{s}(k+1/k+1) = \hat{s}(k+1/k) + P(k+1/k+1)H^T\big(m(k+1)-H\hat{s}(k+1/k)\big)/R(k) \tag{65}$$

Equations (60) - (65) are not implemented directly. Instead, these equations are implemented in a matrix square root formulation in order to ensure smoother and filter stability. As in the matrix square root formulation of the Kalman filter BET all covariance matrices, $P(k)$, $P(k+1/k)$, and $P(k/k)$ are written as LL^T or LDL^T where L is lower triangular and D is diagonal. The filtering and smoothing equations (60) - (65) are then expressed in terms of L and D and a covariance is only computed in order to output error estimates.

The smoother adapts to changing conditions by computing $R(k)$ from filter residuals and estimating $q(k)$ from the forward residual stack, $r(k+1) = m(t_{k+1}) - H\Phi(1\Delta)\hat{s}(k/k)$, $1=1,n$. The lag n_k in (60) and (61) is less than the fixed integer n and is a function of $q(k)$; $0 \le n_k \le n$ with equality $n_k = 0$ for small $q(k)$ and $n_k = n$ for large $q(k)$.

Optimal Instrumentation Planning

The relative geometry of a trajectory and an instrumentation plan (IP) determines the quality of output of a BET. It is natural then for someone who is producing BET's to be concerned with the geometry of the instrumentation plan used on a mission. It is also a natural extension of a BET program to develop a program to compute an optimal instrumentation plan. Thus, as a derivative of our batch processor BET we have developed optimal instrumentation planning programs for DOVAP, cines, and radar. The algorithm for optimal instrumentation planning will be briefly described for the radar case. The basic algorithm also applies to cines and Dovap with modifications which will be mentioned.

Given a set of n time points t_i which entirely span a nominal flight path specified by a range user and the corresponding position vectors $\bar{x}_i$, i=1, n, to the flight path a set of M existing radar sites is selected which minimizes

$$C_M = \sum_{i=1}^{n} w_i t_r \operatorname{cov}(\bar{x}_i)$$

The quantity C_M may be interpreted as the weighted sum of error estimates which would result if radar measurements from the M radars were processed by the batch processor BET program. The w_i are a set of weights used to attach more importance to some trajectory points than to others. $\operatorname{Cov}(\bar{x}_i)$ are the 3x3 covariance matrices of the errors in the estimates of the position vectors $\bar{x}_i$.

The effect of changing the number of radar sites M to be used in instrumenting the flight path is easily computed using the optimal site selection program. Using a minimum number of sites M_1 and a maximum number os sites M_2, the selection program sequentially obtains the optimal IP for M_1 sites, M_1+1 sites,---, M_2 sites so that the effect of additional sites on the error estimates can be readily determined. Besides the relative geometry of the radars and flight path, the selection program also considers past radar performance statistics in the form of measurement noise variances.

Rather than pursuing a global minimum of C_M which would require the solution of a very large combinatorial programming problem, a local minimum of C_M is achieved through the use of an IP improvement algorithm. The following sequence of steps are executed in the IP improvement algorithm.

a. Starting with an initial IP having M arbitrary radar sites construct a modified IP having M+1 sites by adding the site from among all remaining radar sites which results in the smallest value of C_{M+1}.

b. Delete the radar site from the modified IP which results in the smallest value of C_M.

c. Repeat steps a and b until no further decrease in the value of C_M in step b can be achieved.

The algorithm will terminate when the radar added in step a is the same radar deleted in step b. The minimum of C_M achieved in step b is local in the sense that it is dependent on the arbitrary initial IP with which the algorithm started. The existing sites from which the radars in the IP are selected must satisfy some basic constraints. For a radar to be considered for selection the elevation angles cannot be too small or too large over a large portion of the trajectory.

Optimal instrumentation planning programs have also been written for DOVAP and cinetheodolite. The DOVAP instrumentation planning program was the first developed and was used for all DOVAP IP's for some time before DOVAP was phased out at WSMR. The use of this instrumentation planning algorithm resulted in an average of 25% saving in the number of DOVAP receiver sites required to achieve a given quality of trajectory data. The application of this algorithm to cine instrumentation planning is considerably more difficult. The difficulty does not occur in the application of the IP improvement algorithm but in implementing the several constraints which must be met before a cine site can be considered for inclusion in an IP. Constraints which must be satisfied for each cine site considered are: minimum image size readable on film, maximum tracking rates, sun angle, and flight safety evacuation area. Development of these optimal IP programs are described in [22] - [25].

Directions of BET Development at WSMR

The development of BET techniques at WSMR is continual. As mentioned previously measurement data editing and adaptive filtering are two difficult features of a BET program which must be successfully implemented to insure good BET performance.

We have been very active at WSMR in the development of robust estimation methods and their application to several measurement editing problems in data reduction, [15] - [18]. We have been highly successful in these applications of robust estimation to measurement editing [15]. We have applied these methods to the preprocessor in the MISTE program, to instrument calibration problems, the N-station Davis solution, and are currently developing some robust filtering methods.

Before describing the application of robust estimation methods we need to answer: What are robust estimation methods and how do they apply to data editing? In answer to the first part of the question we can briefly describe robust statistical methods as those which perform well under a wide variety of underlying probability density functions of the measurements or in the presence of measurements from contaminating distributions. In answer to the second part of the question, we are probably not very concerned about the performance of data reduction procedures under a wide variety of underlying distribution functions for the measurements, but are mainly concerned about the performance of our methods in the presence of observations from contaminating distributions, i.e., outliers or wild data points. In data reduction we are usually interested in estimating the parameters in some postulated linear or nonlinear model of a process. Thus, in data reduction we are specifically interested in developing methods for linear and nonlinear regression which are insensitive to a large percentage of outlying observations. The usual methods of least squares, weighted least squares, maximum likelihood, etc, used in data reduction for estimating parameters in a model become useless in the presence of outliers. To quote Huber [19], "even a single grossly outlying observation may spoil the least squares estimate and moreover outliers are much harder to spot in the regression case than in the simple location case."

The most popular and most developed of the robust methods for use in parameter estimation problems are the M-estimates of Huber. Given the linear model

$$y_i = \sum_{j=1}^{P} x_{ij}\theta_j + e_i \qquad i=1,n \tag{66}$$

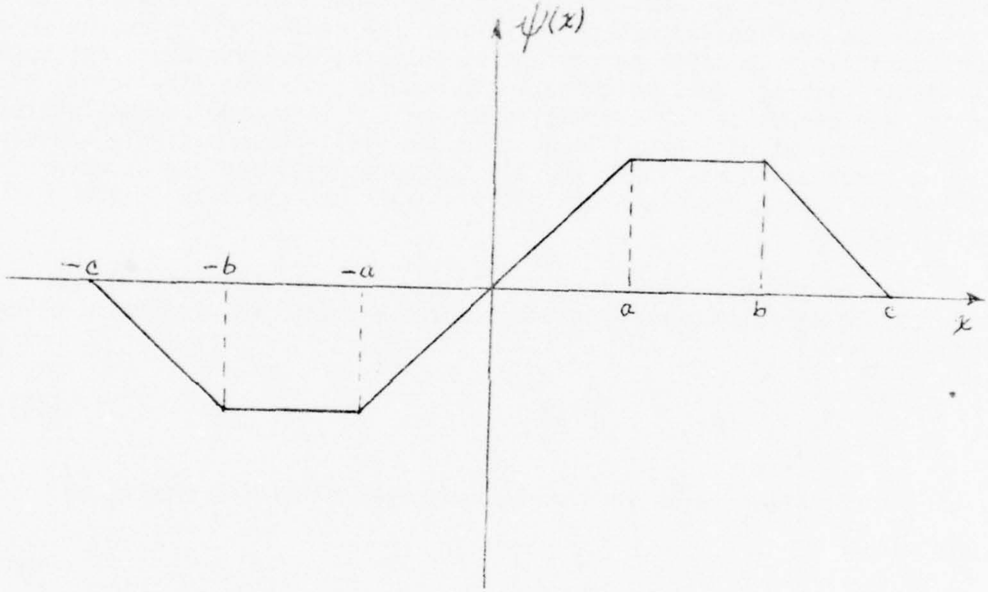
where θ_j are unknown parameters to be estimated. The M-estimates of θ_i minimize

$$\sum_{i=1}^{n} \rho \left(\frac{y_i - \sum_j x_{ij}\theta_j}{s} \right) , \tag{67}$$

where $\rho(\cdot)$ is some suitable function and s is a measure of dispersion of the residuals. Rather than specifying the function ρ, M-estimates are usually specified in terms of their ψ function which is the derivative of ρ. Several ψ functions have been proposed in the literature [15], but we will only consider a ψ function proposed by Hampel [20]. This ψ is given by

$$\psi(x) = \begin{cases} x & |x| \leq a \\ a \ \mathrm{sgn}\ (x) & a \leq |x| \leq b \\ a \dfrac{x - c\ \mathrm{sgn}(x)}{b - c} & b \leq |x| \leq c \\ 0 & |x| > c \end{cases} \tag{68}$$

A graphical representation of this function is given below



In order to minimize (67) we differentiate with respect to θ, where θ is a p-vector, $\theta^T = [\theta_1, \theta_2 --- \theta_p]$. This gives

$$\sum_{i=1}^{n} x_i^T \psi\left(\frac{y_i - x_i \hat{\theta}}{s}\right) = 0 \tag{69}$$

where $\hat{\theta}$ is the M-estimate of θ and $x_i^T = \mathrm{col}(x_{i1}, x_{i2}, \cdots, x_{ip})$. (69) is the analog of the normal equations in least squares estimation. Specifically, if $\psi(x)$ is linear for all x, we have a least square estimation process. (69) can be rewritten as

$$\sum_{i=1}^{n} w_i(\hat{\theta}) x_i^T (y_i - x_i\theta) = 0 \tag{70}$$

where

$$w_i(\hat{\theta}) = \frac{\psi\left(\dfrac{y_i - x_i\theta}{s}\right)}{\left(\dfrac{y_i - x_i\hat{\theta}}{s}\right)} \tag{71}$$

(70) is solved iteratively as follows. Starting at an arbitrary point $\hat{\theta}^{(k)}$ of the iteration sequence, we replace (70) by

$$\sum_{i=1}^{n} w_i(\hat{\theta}^{(k)}) x_i^T (y_i - x_i\hat{\theta}^{(k+1)}) = 0 \tag{72}$$

Solving (72) for $\hat{\theta}^{(k+1)}$ gives

$$\hat{\theta}^{(k+1)} = \left(\sum_{j=1}^{n} w_j(\hat{\theta}^{(k)}) x_j x_j^T\right)^{-1} \sum_{i=1}^{n} w_i(\hat{\theta}^{(k)}) x_i^T y_i \tag{73}$$

(73) is just a weighted least squares solution. Thus, we apply a weighted least squares algorithm iteratively to obtain an M-estimate. Examining (71) we see that for large residuals, $y_i - x_i\theta >> s$, the weight w_i goes to zero. Thus, large residuals are weighted out of this M-estimate solution. The robust dispersion measure s which is used is the MAD (Median of the Absolute Deviations) estimate given by

$$s = \underset{i}{\mathrm{median}}|r_i| \Big/ .6745 \tag{74}$$

where $r_i = y_i - x_i\theta$, Hampel has shown that the MAD estimate is the most robust measure of dispersion. The above method for computing an M-estimate is iterative and therefore requires a starting solution to be specified. The required closeness of the starting solution to the final solution depends on the application and the type of ψ function used. Often an ordinary least squares solution provides a sufficiently good starting solution. In some cases it is necessary to use a starting solution which is more robust, see [16].

The application of robust estimation in the BET preprocessor provided our original motivation for the development and application of robust methods in data reduction. In the preprocessor we have the time history of each measurement function for its entire span of observation on a trajectory. The preprocessor divides this interval of observation into equal segments of T seconds except for a final segment either longer or shorter than T. Over each of these segments a polynomial, usually a quadratic, is fit to the measurements. Alternatively, a cubic spline might be fit to the entire span of measurement data using the end points of the T second segments as knot times. Using an M-estimation procedure we estimate the parameters of the polynomial and delete those points which are given zero weights in the M-estimate process or whose residuals are greater than $k \cdot s$ where k is a suitable constant. The processing variance is computed as the average squared error of the remaining residuals. The following data set is from a real preprocessor application. The data are a sequence of azimuth measurements from a cine.

Observations	Residuals from Robust Fit	Residuals from Least Squares Fit
-1.70987	.000012	-.157774
-1.70942	-.000004	-.000204
-1.70893	.000003	.105480
-1.70845	-.000015	.159227
-1.70793	-.000010	.161087
-1.70741	-.000021	.111021
-1.70682	.000022	.009099
-1.70626	.000019	-.144780
-1.70571	-.000010	-.350595
-1.70510	.000005	-.608277
-1.70449	.000004	-.917885
1.43777	3.141637	1.862231
1.44602	3.149243	1.456410
-1.70257	-.000007	-2.158177
1.44667	3.146558	.473139

The residuals from the robust fit which were obtained using a Hampel ψ with breakpoints (2.5, 5.0, 7.5) show exactly which points are outliers. The least square residuals provide no information about the outliers.

Another application in BET processing in which we have successfully used M-estimates is in calibration. For example, a reasonable error model for a laser tracker is

$$\Delta R_{ij} = \theta_1 + \theta_2 R_{sj}$$

$$\Delta A_{ij} = \theta_3 - \theta_4 \tan E_{sj} \cos A_{sj} - \theta_5 \tan E_{sj} \sin A_{sj} - \theta_6 / \cos E_{sj}$$

$$\Delta E_{ij} = \theta_7 + \theta_4 \sin A_{sj} - \theta_5 \cos A_{sj}$$

where R_{sj}, A_{sj}, E_{sj} are the surveyed range, azimuth, and elevation of the j^{th} target calibration target. We have multiple range, azimuth, and elevation observations, denoted by R_{ij}, A_{ij}, E_{ij}, $i=1, N_j$, $j=1, m$, for each of the M calibration targets, then $\Delta R_{ij} = R_{ij} - R_{sj}$, $\Delta A_{ij} = A_{ij} - A_{sj}$, and $\Delta E_{ij} = E_{ij} - E_{sj}$. The unknown parameters θ_j, $j=1,7$ are to be estimated.

The following example illustrates the application of M-estimates to the calibration of a laser tracker. There were approximately 250 observations for each calibration target. The estimation used a Hampel ψ function with breakpoints (2.5, 5.0, 7.5). The results of this calibration are summarized in the table on the following page by tabulating the number of residuals for each target lying in each of the four regions of the Hampel ψ. We only show the positive side of the Hampel ψ with the number of residuals in each region being the sum of the numbers of residuals in the positive and corresponding negative side of the ψ function. Targets 13-20 are dumped readings of the first eight targets. From the table it can be seen that most of the readings from several target boards are outliers, particularly for the dumped readings. A least squares estimation of the calibration parameters failed miserably on this example. Although this example is extreme for this application, having about 22% contamination by outliers, it well illustrates the power of the M-estimate method in dealing with outliers.

NUMBER OF RESIDUALS

	RANGE				AZIMUTH				ELEVATION			
TARGET POLE #	<2.5 s_r	(2.5 s_r, 5 s_r)	(5 s_r, 7.5 s_r)	>7.5 s_r	<2.5 s_a	(2.5 s_a, 5 s_a)	(5 s_a, 7.5 s_a)	>7.5 s_a	<2.5 s_e	(2.5 s_e, 5 s_e)	(5 s_e, 7.5 s_e)	>7.5 s_e
1	230	0	0	3	230	0	0	3	230	0	0	3
2	252	0	0	0	251	0	0	1	252	0	0	0
3	237	0	0	0	237	0	0	0	237	0	0	0
4	270	0	0	0	270	0	0	0	270	0	0	0
5	241	1	0	0	242	0	0	0	242	0	0	0
6	242	0	0	0	242	0	0	0	242	0	0	0
7	237	0	0	0	237	0	0	0	237	0	0	0
8	215	0	0	9	215	0	0	9	222	2	0	0
9	9	0	0	241	7	2	0	241	193	57	0	0
10	269	15	0	0	243	40	1	0	284	0	0	0
11	250	1	0	0	245	6	0	0	251	0	0	0
12	161	2	0	118	127	35	55	64	217	0	0	64
13	118	103	0	5	224	0	0	2	38	186	0	2
14	135	86	25	4	248	0	0	2	234	13	0	3
15	2	0	0	0	2	0	0	0	2	0	0	0
16	126	59	9	33	221	0	0	6	21	200	0	6
17	138	96	20	39	248	0	0	45	248	44	1	0
18	2	0	0	0	0	0	0	2	0	2	0	0
19	137	71	9	17	0	0	0	234	8	226	0	0
20	81	86	18	111	0	0	0	296	296	0	0	0

DISTRIBUTION OF CALIBRATION RESIDUALS

The robust M-estimates of Huber can also be developed to apply to recursive filtering. The development of these ideas for filtering are described in [18] and [26]. Let $x(t_k)$ be the state vector of a linear dynamic model

$$x(t_{k+1}) = \Phi(k)x(t_k) + w(k) \tag{75}$$

where $\Phi(k)$ is the state transition matrix and $w(k)$ is an n-vector of zero mean Gaussian noise with covariance $Q(k)$. Let scalar measurements

$$z(t_k) = H\, x(t_k) + v(k) \tag{76}$$

be available where $v(k)$ is a random error term which may be contaminated by outliers. A robust, pseudo-maximum likelihood filter for the case is given by the equations

$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k) + \frac{P_{k+1/k}H^T}{s_{k+1}}\,\psi\left(\frac{z(t_{k+1}) - Hx(k+1/k+1)}{s_{k+1}}\right) \tag{77}$$

$$P_{k+1} = P_{k+1/k} - \left(\psi'\left(\frac{z(t_{k+1}) - H\hat{x}(k+1/k)}{s_{k+1}}\right) \Big/ s_{k+1}^2\right)P_{k+1/k}H^T H P_{k+1/k} \tag{78}$$

where ψ' is the derivative of ψ and $P_{k+1/k} = \Phi P_k \Phi^T + Q_k$

$$\hat{x}(k+1/k) = \Phi\hat{x}(k/k) \tag{79}$$

and we use a robust measure of dispersion obtained from the residuals

$$s_{k+1} = \underset{j=0,n-1}{\text{median}}\left|z_{k+1-j} - H\hat{x}(k+1-j/k-j)\right| \Big/ .6745 \tag{80}$$

(77) is nonlinear in $\hat{x}(k+1/k+1)$ and is iterated till convergence. We are testing this robust filtering scheme on simulated and real data to determine its ability to filter measurement data with large amounts of contamination by outliers. We are also working at extending the above ideas to smoothing.

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RIDGE REGRESSION WITH
UNDERSPECIFIED MODELS

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RIDGE REGRESSION WITH UNDERSPECIFIED MODELS

1.0 INTRODUCTION

Well known to analysts associated with regression studies are the problems related to ill-conditioned matrices – problems leading to loss of precision, grossly inaccurate (inflated) estimates of the parameters and gross underestimates of the errors in the estimates. Inflated estimates are those which depart unrealistically (> 3 sigmas) from a priori estimates. The authors have analyzed four hundred experiments with real and simulated data to determine the circumstances associated with inflated estimates. The results of the study may be summarized briefly and qualitatively.

For purposes of discussion it is convenient to consider two general classes of problems. Class A is defined by the presence of a relatively small quantity of measurements accompanied by somewhat large random errors and high correlation among the errors in the adjusted parameters. Class B encompasses a broad family of regression problems (notably orbit determination) wherein the adjustment exercise has available an enormous redundancy of measurements; the random errors in the measurements are relatively unimportant; and the errors in the estimates are highly correlated. Difficulties with computational precision were excluded from this investigation. Perhaps the two most practical types of regression models are correctly specified and underspecified. Underspecification may, for example, result from deficient knowledge or in some cases from limitations in a computer program. With correctly specified models and Class A problems, some degree of inflation was observed occasionally, and the occurrence depended largely upon the vagaries of the random errors in the measurements. We were unable to induce or discover inflated estimates when correctly specified models were applied to Class B problems. Inflation could be induced at will in both Class A and Class B problems with underspecified models. A common situation resulting in rather dramatic inflation is characterized by the triple correlation: two highly correlated parameters in the specified model and a third parameter (missing from the model) whose errors correlate with those of the other two. In the triple correlation a small error in the unmodeled parameter may be magnified many times into errors in the ordinary-least-squares estimates of both the modeled parameters. Our premise regarding ill-conditioned normal matrices tends to preclude consideration of the simple one-to-one or double correlation between the error in an unmodeled parameter and the error in a modeled parameter, because this type of correlation is only accidentally associated with ill-conditioned matrices; however, it may be noted in passing that in this circumstance the error in the estimate is at most comparable in size to the unmodeled error, and if the estimate is "inflated", then the unmodeled error is so large as to be generally detected and removed or else modeled. In any case, the simple one-to-one correlation between a modeled error and an unmodeled error results in an ordinary-least-squares estimate which is relatively unchanged by subsequent ridge regression.

One of the most fruitful approaches to the problem of inflated estimates was developed by Hoerl and Kennard [6] [7] and labeled by them "ridge regression." Hoerl and Kennard point out an important deficiency in ordinary-least-squares, point-estimation procedure: In the case of high correlation among the errors in the parameter estimates, there may be a gross inflation of the adjustment vector in order to achieve a final minuscular reduction in the sum of squares of the residuals. The Hoerl-Kennard (HK) estimation process is inherently Bayesian in nature. It assumes expected values of zero for the adjustable parameters and tends to constrain the adjusted values as close as possible to zero without unduly enlarging the measurement residuals. The HK estimator is biased but offers potential reduction in mean square error. The mathematics supporting the HK estimator is based upon a known fixed regression model and classical unweighted least squares. The HK estimator is not compatible with standard Bayesian minimum variance regression programs such as those associated with orbit determination. In this paper we develop and discuss the properties of an unbiased ridge estimator with application to problems where the regression model is underspecified. The resulting estimator is readily compatible with standard Bayesian minimum variance regression computer programs. For purposes of comparison it will be helpful to discuss the standard estimator.

2.0 STANDARD BAYESIAN MINIMUM VARIANCE ESTIMATOR

We assume the standard linear approximation to the general nonlinear multiple regression problem:

$$Y = X \beta + \epsilon_m \tag{2.1}$$

where Y is (nxl) and denotes the measurement vector; X is (nxj) and of rank j and represents the partial derivative matrix of nonstochastic elements relating the mean values of the measurements to the adjusted parameters; β is (jxl) and designates the true fixed but unknown parameter vector; ϵ_m is (nxl) and constitutes the measurement error vector. We assume $E(\epsilon_m)$ is zero and $E(\epsilon_m \epsilon_m') = VAR(\epsilon_m) = \Sigma_m$, where Σ_m is (nxn) and known.

In addition, we have prior information consisting of a k element parameter vector β_o, which unbiasedly estimates $R\beta$ and a k element parameter error vector ϵ_p. β_o is known from introspection or from previous independent measurements. Therefore

$$\beta_o = R\beta + \epsilon_p \tag{2.2}$$

where R, originally suggested by Thiel [16], is (kxj) of rank k and consists of known nonstochastic elements. If, for example, $R = [I \quad 0]$, where I is a (kxk) unit matrix and 0 is a $[k \times (j-k)]$ zero matrix, then β_o represents estimates of the first k elements of β. Equation (2.2) assumes β_o is random and hence represents a departure from the Bayesian approach, which assumes a prior distribution on β, here considered fixed. In addition, $E(\epsilon_p)$ is zero and $E(\epsilon_p \epsilon_p') = VAR(\epsilon_p) = \Sigma_p$, where Σ_p is (kxk) and known. Furthermore, we assume COV (ϵ_m, ϵ_p) is zero. In order to include the prior information in the estimation of β, we combine equations (2.1) and (2.2) as follows:

$$\begin{bmatrix} Y \\ \beta_o \end{bmatrix} = \begin{bmatrix} X \\ R \end{bmatrix} \beta + \begin{bmatrix} \epsilon_m \\ \epsilon_p \end{bmatrix} \quad ; \tag{2.3}$$

or in an obvious change of notation

$$\tilde{Y} = \tilde{X} \beta + \tilde{\epsilon} \tag{2.4}$$

where

$$VAR(\tilde{\epsilon}) = \begin{bmatrix} \Sigma_m & 0 \\ 0 & \Sigma_p \end{bmatrix} = \tilde{\Sigma} \quad .$$

The prior information has thus assumed the role of measurements. Applying generalized least squares to (2.4), we obtain the following relation for the estimator $\hat{\beta}$ of the parameter vector β:

$$\hat{\beta} = (\tilde{X}' \tilde{\Sigma}^{-1} X)^{-1} (\tilde{X}' \tilde{\Sigma}^{-1} \tilde{Y}) \quad . \tag{2.5}$$

This converts by simple substitution to

$$\hat{\beta} = (X' \Sigma_m^{-1} X + R' \Sigma_p^{-1} R)^{-1} (X' \Sigma_m^{-1} Y + R' \Sigma_p^{-1} \beta_o) \tag{2.6}$$

or in an obvious change in notation

$$\hat{\beta} = P (X' \Sigma_m^{-1} Y + R' \Sigma_p^{-1} \beta_o) \quad . \tag{2.7}$$

Application of the expectation operator to Equation (2.6) gives $E(\hat{\beta}) = \beta$, and hence $\hat{\beta}$ is an unbiased estimator. Application of the law of covariance propagation gives

$$\mathrm{VAR}(\hat{\beta}) = P \ . \tag{2.8}$$

For computational convenience, Equation (2.7) may be linearized to obtain the following iterative form:

$$\Delta\hat{\beta} = P\left[X' \Sigma_m^{-1} (\Delta Y) + R' \Sigma_p^{-1} (\Delta\beta_o)\right] , \tag{2.9}$$

where $\Delta\hat{\beta}$ is the vector of corrections to the current estimates of the adjusted parameters; ΔY is the vector of measurement residuals; and $\Delta\beta_o$ is the vector of differences between current and a priori estimates of the parameters. It can be shown that $\hat{\beta}$ is the best linear unbiased estimate in the sense that it has the smallest variance. Also, if the measurement errors are normally distributed, then $\hat{\beta}$ has minimum variance among all unbiased estimates. Equation (2.9) is widely used in orbit determination programs and elsewhere.

3.0 RIDGE ESTIMATOR

Highly correlated errors in the parameter estimates result in poor conditioning of the $(X' \Sigma_m^{-1} X + R' \Sigma_p^{-1} R)$ matrix in Equation (2.6). The poorer the condition of this matrix, the greater the expected discrepancy between $\hat{\beta}$ and the true vector β. On the other hand, the worse the conditioning, the less is the sensitivity of the residual sum of squares to small departures from $\hat{\beta}$. Following the concept of Hoerl and Kennard, we impose an accessory condition upon the least-squares criterion, thereby restraining the vector $(\hat{\beta} - \beta_o)$ without greatly influencing the residual sum of squares.

Let B be any estimate of the vector β. Then the sum of squares of the weighted measurement residuals is given by $(\tilde{Y} - \tilde{X}B)' \tilde{\Sigma}^{-1} (\tilde{Y} - \tilde{X}B)$. Using Lagrangian constraints, we minimize

$$F = (B-\beta_o)' \Sigma_p^{-1} (B-\beta_o) + (1/h)\left[(\tilde{Y} - \tilde{X}B)' \tilde{\Sigma}^{-1} (\tilde{Y} - \tilde{X}B) - \phi\right] \tag{3.1}$$

where $(1/h)$ is the multiplier and ϕ is the total sum of squares.

We obtain, therefore,

$$\frac{\partial F}{\partial B} \qquad 0 = \Sigma_p^{-1} (B-\beta_o) - (1/h)\ \tilde{X}'\ \tilde{\Sigma}^{-1}\ (\tilde{Y} - \tilde{X}B) \ . \qquad (3.2)$$

This reduces to

$$B = \hat{\beta}* = \left[X'\ \Sigma_m^{-1}\ X + (h+1)\ R'\ \Sigma_p^{-1}\ R\right]^{-1}$$

$$\cdot \left[X'\ \Sigma_m^{-1}\ Y + (h+1)\ R'\ \Sigma_p^{-1}\ \beta_o\right] \ ; \quad h \geq 0 \qquad (3.3)$$

or in an obvious change of notation

$$\hat{\beta}* = Q\left[X'\ \Sigma_m^{-1}\ Y + (h+1)\ R'\ \Sigma_p^{-1}\ \beta_o\right] \ ; \quad h \geq 0 \ . \qquad (3.4)$$

Application of the expectation operator to Equation (3.3) gives $E\ (\hat{\beta}*) = \beta$, and hence $\hat{\beta}*$ is an unbiased estimator. Application of the law of covariance propagation gives

$$VAR(\hat{\beta}*) = Q\left[X'\ \Sigma_m^{-1}\ X + (h+1)^2\ R'\ \Sigma_p^{-1}\ R\right]Q' \ . \qquad (3.5)$$

Clearly, when $h = 0$, $\hat{\beta}* = \hat{\beta}$ and $VAR(\hat{\beta}*) = VAR(\hat{\beta})$.

$\hat{B}*$ is related to $\hat{\beta}$ as follows:

$$\hat{\beta}* = QP^{-1}\ \hat{\beta} + Q\ h\ R'\ \Sigma_p^{-1}\ \beta_o \ . \qquad (3.6)$$

Linearizing Equation (3.3) results in the following iterative form:

$$\Delta\hat{\beta}* = Q\left[X'\ \Sigma_m^{-1}\ (\Delta Y) + (h+1)\ R'\ \Sigma_p^{-1}\ (\Delta\beta_o)\right] \ ; \ h \geq 0 \qquad (3.7)$$

where $\Delta\hat{\beta}*$ is the vector of corrections to the current estimates of the adjusted parameters. Equation (3.7) is the same as Equation (2.9) with the exception that the input constant $\left[(h+1)\ \Sigma_p^{-1}\right]$ has replaced the input constant (Σ_p^{-1}) and hence Equation (3.7) is compatible with a broad class of regression programs.

Subtracting Equation (2.8) from Equation (3.5) gives, not unexpectedly, a positive definite matrix for $h > 0$, and therefore $\hat{\beta}*$ with $h > 0$ is less "efficient" than $\hat{\beta}$ for application with exact regression models. Consider, however, application of the $\hat{\beta}*$ estimator to underspecified regression models.

Suppose we fit the model in Equation (2.1) to data which obey the model

$$Y = X\beta + Z\gamma + \epsilon_m \tag{3.8}$$

where Z is (nxs) and of rank s and represents the partial derivative matrix of nonstochastic elements relating the mean values of the measurements to the unadjusted parameters; γ is (sx1) and designates a true fixed but unknown parameter vector. The vector β will be estimated using the estimator given in Equation (3.3). Applying the expectation operator, we now obtain

$$E(\hat{\beta}*) = \beta + \left[X' \Sigma_m^{-1} X + (h+1) R' \Sigma_p^{-1} R\right]^{-1} X \Sigma_m^{-1} Z \gamma , \tag{3.9}$$

and hence $\hat{\beta}*$ is biased. $VAR(\hat{\beta}*)$ is still given by Equation (3.5), and the total mean square error — sum of variances and squared biases — is given by

$$MSE(\hat{\beta}*) = \text{Trace } VAR(\hat{\beta}*) + \left[E(\hat{\beta}*) - \beta\right]' \left[E(\hat{\beta}*) - \beta\right] \tag{3.10}$$

which for $h = 0$, is identical with $MSE(\hat{\beta})$ for the same underspecified model. In investigating the behavior of $MSE(\hat{\beta}*)$, we need the partial derivative of $MSE(\hat{\beta}*)$ with respect to h. Because the partial derivative of Trace $VAR(\tilde{\beta})$ is the same as the trace of the partial derivative of $VAR(\hat{\beta}*)$, we take the partial derivative of $VAR(\tilde{\beta}*)$ in a straightforward manner and note that it is a continuous function of h and is symmetric positive definite for all $h > 0$. (It is a null matrix for $h = 0$.) Consequently, the trace of $VAR(\hat{\beta}*)$ is monotone increasing with increasing h and in fact approaches infinity as h approaches infinity. The derivative of $\left[E(\hat{\beta}*) - \beta\right]' \left[E(\hat{\beta}*) - \beta\right]$ is also continuous, and the matrix of the quadratic form of the derivative is symmetric negative definite for all $h \geq 0$, approaching the null matrix as h approaches infinity. Therefore $\left[E(\hat{\beta}^*) - \beta\right]' \left[E(\hat{\beta}^*) - \beta\right]$ is monotone decreasing with increasing h but can never be negative. Hence, we have a situation where at $h = 0$, $MSE(\hat{\beta}^*) = MSE(\hat{\beta})$, and at very large values of h, $MSE(\hat{\beta}^*) > MSE(\hat{\beta})$. We inquire whether there is any intermediate positive value of h which will result in $MSE(\hat{\beta}^*) < MSE(\hat{\beta})$. Apparently there is, because the partial derivative of $VAR(\hat{\beta}^*)$ evaluated at $h = 0$ is the null matrix, whereas the partial derivative of $\left[E(\hat{\beta}^*) - \beta\right]' \left[E(\hat{\beta}^*) - \beta\right]$ evaluated at $h = 0$ is negative definite. Thus for some $0 < h < \infty$, $MSE(\hat{\beta}^*)$ is a minimum and less than $MSE(\hat{\beta})$. The authors have not attempted to determine explicitly the value of h corresponding to this minimum because the value will depend upon γ which is presumably completely unknown and unknowable. Rather we adopt the empirical Ridge-Trace approach used by Hoerl and Kennard; i.e., h is chosen to have that value at which the parameter estimates appear to have reached stability.

4.0 APPLICATION IN ORBIT DETERMINATION

The interested reader will find a brief discussion of orbit determination in Appendix A. In order to better illustrate the performance of the ridge estimator with an underspecified model, we have chosen to present a simulation exercise rather than a treatment of real data. This application involves a standard satellite orbit determination (Cowell, special perturbations, batch processing) in which the adjustable parameters include orbital elements (initial conditions) and radar coefficients. The orbital elements define the position and velocity of the satellite at epoch. The measurements are radar track data: range, azimuth and elevation.

The radar track data are characterized by certain errors which may be expressed as linear terms in the so-called radar measurement equations. The radar measurement equations, truncated so as to contain only terms of present interest, are as follows:

$$\underset{\text{measurement}}{A} = \underset{\text{true}}{A_t} + \underset{\text{zero set}}{a_1} + \underset{\text{collimation}}{a_2 \sec E_t}$$

$$+ \underbrace{u \sin A_t \tan E_t - v \cos A_t \tan E_t}_{\text{mislevel}}$$

$$+ \underset{\text{non-orthogonality}}{a_3 \tan E_t} + \underset{\text{random error}}{\epsilon_A} \tag{4.1}$$

$$\underset{\text{measurement}}{E} = \underset{\text{true}}{E_t} + \underset{\text{zero set}}{e_1} + \underset{\text{droop}}{e_2 \cos E_t}$$

$$+ \underbrace{u \cos A_t + v \sin A_t}_{\text{mislevel}} + \underset{\text{residual refraction}}{e_3 \operatorname{ctn} E_t}$$

$$+ \underset{\text{random error}}{\epsilon_E} \tag{4.2}$$

Equations (4.1) and (4.2) plus the equations of motion of the satellite constitute the exact model. The underspecified model does not include terms in non-orthogonality and residual refraction. In Equations (4.1) and (4.2), A represents azimuth; E, elevation. The zero-set errors are constant bias or offset values. Collimation represents the lack of perpendicularity between the radar beam and the elevation axis. Non-orthogonality denotes the lack of perpendicularity between the azimuth and elevation axes. Mislevel represents the tilt of the azimuth plane — u being the northward component and v being the eastward component. This tilt is defined with respect to the local horizontal to the geodetic spheroid. Droop represents the sag of the radar beam axis. Residual refraction refers to the error remaining after approximate corrections have been made for the bending of the radar beam traversing the atmosphere. The random errors represent noise in the data and have zero means.

The primary purpose of this experiment is to estimate the radar coefficients. In order to accomplish this, it is necessary incidentally to determine the orbit. Determining the orbit constitutes estimating the orbital elements at epoch. Since the orbital elements correlate only weakly with the radar coefficients in the geometrical environment we have chosen, we shall not apply the principles of ridge analysis specifically to the elements, although the elements will be estimated each time the radar coefficients are estimated. The true values of the radar coefficients, the a priori estimates of these values and the uncertainties (standard deviations) in the prior estimates are given in Table I.

TABLE I

NUMERICAL VALUES OF RADAR COEFFICIENTS

RADAR COEFFICIENTS	TRUE VALUE (β_i)	A PRIORI ESTIMATE (β_{oi})	S.D. (σ_{oi})
a_1 (deg)	+0.005	0	0.003
a_2 (deg)	-0.003	0	0.003
u (deg)	-0.002	0	0.002
v (deg)	-0.003	0	0.002
e_1 (deg)	+0.005	0	0.003
e_2 (deg)	+0.002	0	0.002
a_3 (deg)	-0.002	–	–
e_3 (deg)	+0.002	–	–

The mathematical adjustment procedure is iterative and is based upon Equation (3.7), the equations of motion of the satellite, and the radar measurement equations. Initially, with $h = 0$, the procedure is the standard one in orbit determination. After a converged solution with $h = 0$ has been obtained, then ten or so additional solutions with increasing values of h are computed in order to define the curves comprising the ridge trace. A pertinent sub-set of the correlation matrix with $h = 0$ is shown in Table II with elements rounded to three digits.

TABLE II

CORRELATION COEFFICIENTS

a_1	a_2	u	v	e_1	e_2
1					
-0.983	1				
-0.008	0.005	1			
-0.053	0.054	-0.127	1		
0.001	0.001	-0.064	0.017	1	
0.000	-0.001	0.078	-0.031	-0.974	1

In the ridge trace, Figure 1, we plot $(\hat{\beta}_i^* - \beta_{oi})/\sigma_{oi}$ vs h and also show the root-mean-square of the weighted measurement residuals (ϕ*) vs h. The symbol σ_{oi} is used for the a priori standard deviation in β_{oi}, where the subscript i designates the i^{th} element in the parameter vector. Figure 1 shows graphically the discrepancies between the individual estimates ($\hat{\beta}_i^*$) and the a priori individual estimates (β_{oi}). Since this is a simulation exercise, we may also present graphically the discrepancies between the estimates ($\hat{\beta}_i^*$) and the true values of the parameters (β_i). In Figure 2, therefore, we plot $(\hat{\beta}_i^* - \beta_i)/\sigma_{oi}$ vs h.

It is observed that the weighted rms in the measurement residuals (ϕ*) has increased less than 1% as h has increased from 0 to 30, the point at which stability in the estimates is effectively reached. The inflation characteristic of the original estimates of a_1, a_2, e_1 and e_2 has been largely removed at h = 30. Since the errors in the estimates of u and v are relatively uncorrelated with each other or with those of any other parameter, their estimates are relatively unaffected by changes in the value of h. Not so obvious in the curves is the fact that the estimate of e_1 corresponding to standard least squares (h=0) not only was grossly in error but also had the wrong sign.

Composites of Figures 1 and 2 are shown in Figure 3, in which plots of root-mean-square of $(\hat{\beta}_i^* - \beta_{oi})/\sigma_{oi}$ and $(\hat{B}_i^* - \beta_i)/\sigma_{oi}$ are given as a function of h. It is particularly encouraging that not only do these curves show marked reduction in discrepancies as a result of ridge regression but also the discrepancies relative to true values are smaller than discrepancies relative to a priori estimates.

5.0 CONCLUDING REMARKS

At first glance the reader might be alarmed at the rather large value of 30 arrived at for h in the numerical example. It appears that the prior information has been given (nearly) full weight. Actually this is not the case. If the prior information had been given full weight, then the curves in Figure 1 would show a general tendency to be tangent to the zero line at h = 30, whereas most of them show a strong disinclination to approach zero even at h = 100. Furthermore, if h had been increased to the point where prior information was given full weight, the residual sum of squares would – except in a prohibitively unlikely coincidence – have shown a marked increase. It should also be recalled that the prior information entered the adjustment process in the role of measurements (only six), and in spite of the large weight attached to them at h 30, they could not dominate a solution which included over three thousand other measurements. In reality, the ridge estimation procedure has a significant effect only upon the parameter estimates whose errors are mutually correlated, and with these the adjustments are minimized and portioned out inversely according to their a priori variances so far as possible, without unduly enlarging the residuals.

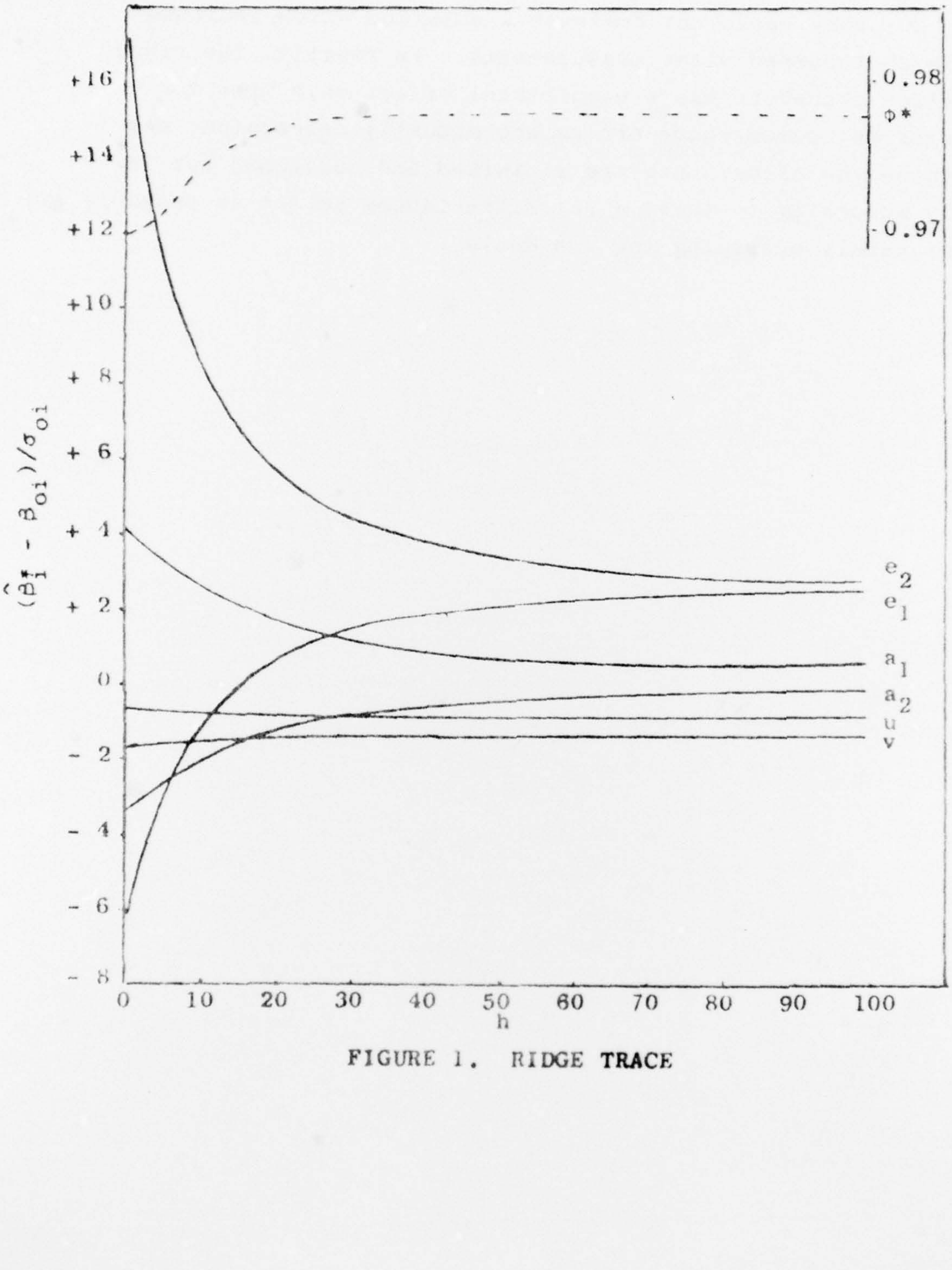

FIGURE 1. RIDGE TRACE

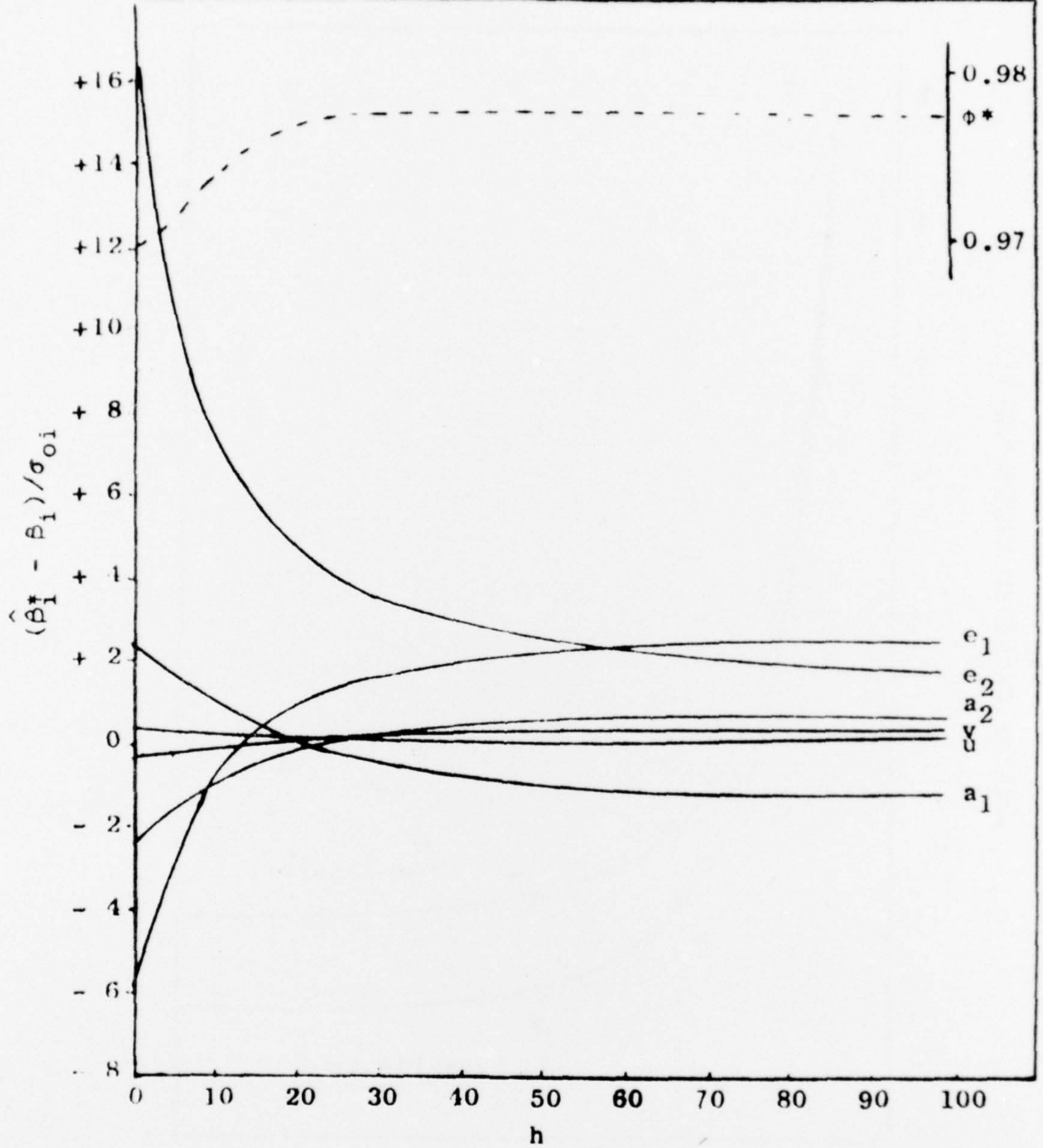

FIGURE 2. SIMULATION ANALOG TO RIDGE TRACE

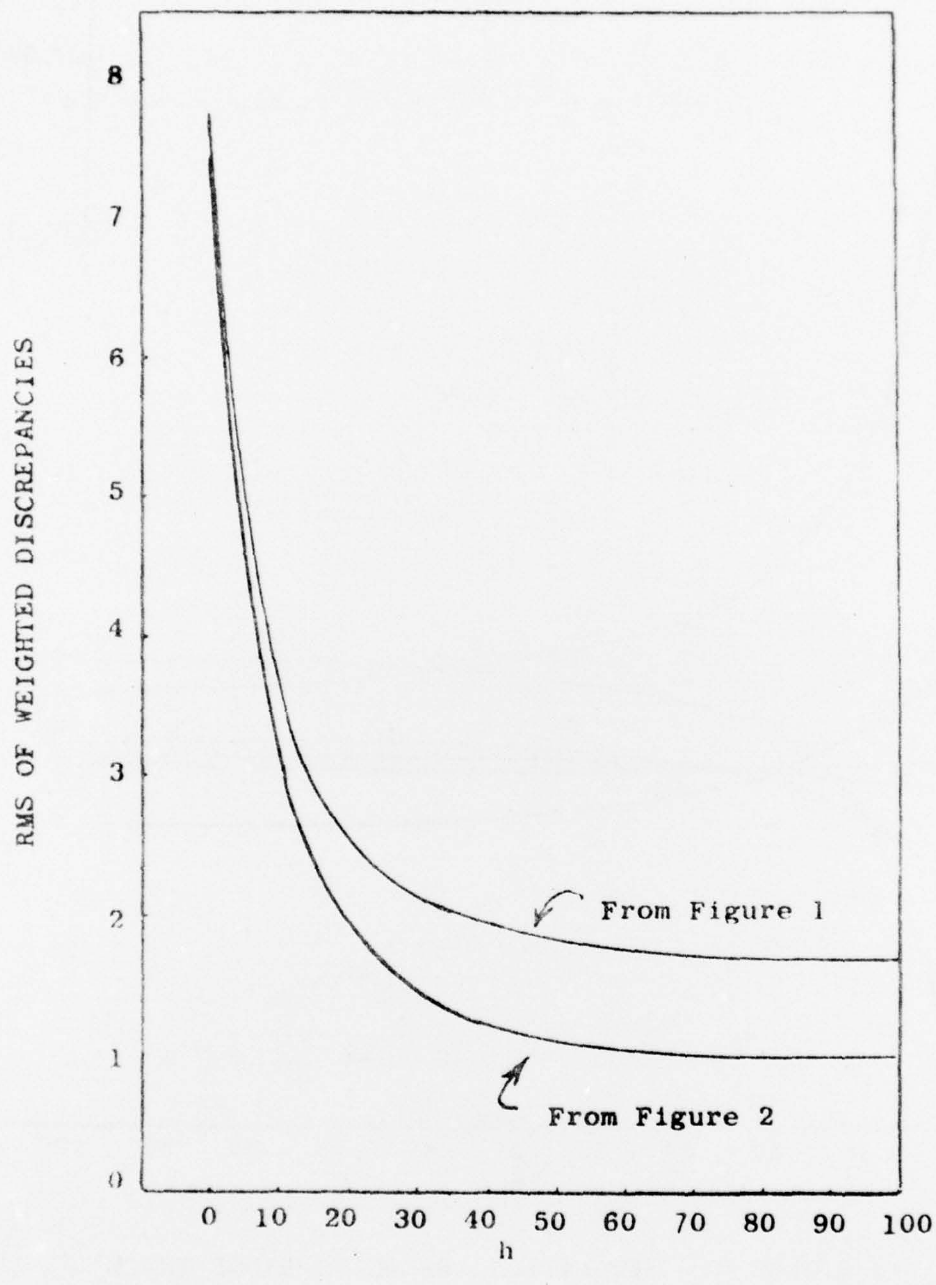

FIGURE 3. COMPOSITE TRACES

APPENDIX A

NOTES ON ORBIT DETERMINATION

Preliminary orbit determination uses only a few observations and is usually based upon simple geometrical and dynamical principles of Kepler's laws. Historically this type was developed to keep track of various planets and comets sufficiently well to point telescopes and to recognize the objects on their reappearance. Preliminary orbit determination in some form is also usually necessary to obtain approximate initial conditions for starting the final orbit determination. Escobal [3] gives a complete and modern treatment of the mathematics of the various methods of preliminary orbit determination. Huseonica [8] evaluates the methods of [3] in terms of limits of applicability, accuracy, storage requirements and computation time.

Final orbit determination makes use of considerable redundant data and is basically nothing more than well-known regression analysis. See, for example, Solloway [13]. In the so-called "batch" process all the data are fitted at once, and a single set of initial conditions is estimated along with a few dozen other parameters relating to the environment and the tracker characteristics. Equation (2.9) is characteristic of the differential correction process. The steps are briefly as follows:

1. An orbit is generated using the approximate initial conditions and the equations of motion.

2. The required partial derivatives are obtained by various means — typically by analytical methods, variational equations, or finite differences.

3. At time points coinciding with actual observations, position points along the generated trajectory are transformed to computed observations.

4. Differencing the actual and computed observations provides residuals.

5. Information from Steps 2 and 4 when supplied to Equation (2.9) permits an improved estimate of the initial conditions and the other parameters, and the process starts over at Step 1.

6. These iterations continue until some convergence criterion is satisfied. Modern computer programs contain a flexible bounding procedure to assist in achieving a converged solution.

The final orbit determination makes use of a wide variety of orbit generators. If numerical integration of the acceleration equations is used, the process is called "special perturbations". If analytical integration of series expansions of the perturbative accelerations is used, the process is "general perturbations".

Cowell's special perturbation technique involves the direct, step-by-step integration of the total acceleration, central as well as perturbative, of the satellite. Encke's method differs from Cowell's in that differential accelerations or the deviations from a two-body reference orbit rather than the total accelerations are integrated. The variation-of-parameters method differs from Encke's in that there is a continuous rectification of the reference orbit. All three of these special perturbation methods are described lucidly by Baker [1]. At the present time, using the most refined techniques in special perturbations, earth satellite orbits are frequently computed to an rms position error of only thirty feet relative to the center of the earth in a single fitting of up to sixteen days of tracking data.

General perturbation schemes are particularly valuable for predicting over long time periods. Their advantages are reduced computer time and the fact that they allow a clearer interpretation of the sources of the perturbative forces. Some of the better known general perturbations schemes are Brouwer's [2], Kozai's [10], Musen's [11] and Frazer's [5]. There are many others. The accuracy of general perturbation methods suffers to some extent because these methods are all limited to a very simple description of the earth's gravity field. They are, however, very widely used because of compensating advantages.

One of the more recent developments in orbit determination is sequential processing, typified by the use of the Kalman filter [9]. This procedure gives an orbit update as each new measurement is received. In accuracy it is essentially equivalent to the batch process. It has been used quite successfully in concert with the Encke process [14]. In computational efficiency it is superior to the batch processor if frequent intermediate answers are required, but it is less efficient if only a single final answer is required.

Finally, there are the sequential-batch processes, which take many forms and applications. Originally proposed by Swerling [15], the sequential-batch process can give normal weight to all prior information, or it can, by reducing the weight on prior information, act like a fading memory filter. Small randomly varying parameters may be excluded from the estimation vector or may be assumed to be piece-wise constant and estimated [4].

Characteristic of all of these orbit determination methods is an extraordinary number and variety of transformations. Reference [12] provides a treatment of many of these transformations along with details of orbit generators, coordinate systems, equations of motion, special and general perturbations, earth gravitational field, variational equations, sequential processing, error analysis, geodetic datums, systems of time, etc.

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